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Modulated inflation

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ABSTRACT

We have studied modulated inflation that generates curvature perturbation from light-field fluctuation. As discussed in previous works, even if the fluctuation of the inflaton itself does not generate the curvature perturbation, fluctuation of a light field may induce fluctuation for the end-line of inflation and this may lead to generation of cosmological perturbation "at the end of the inflation". Our scenario is different from this kind of modulated scenario, as clearly explained in this Letter by using δN formalism. We also explain the crucial difference from the standard multi-field inflation model. We show concrete examples of the modulated inflation scenario in which large non-gaussianity can be generated. We also discuss the running of the non-gaussianity parameter.

1. Introduction

During inflation the vacuum fluctuations of all light scalar fields $\mathcal{M}_i$ are unstable and appear as classical random Gaussian inhomogeneities with an almost scale-free spectrum of amplitude $\delta\mathcal{M}_i \simeq H_I/2\pi$, where H_I is the Hubble parameter during inflation. Inflation significantly stretches the wavelength of the fluctuations over the Hubble horizon after inflation. Thus one can relate $\delta\mathcal{M}_i$ in many different ways to the cosmological curvature perturbation observed in the present Universe. The standard form for the number of e-foldings elapsed during inflation is given by

$$N = \frac{1}{M_p^2}\int_{\phi_e}^{\phi_N} \frac{V}{V_\phi}\, d\phi, \qquad (1.1)$$

where ϕ_N and ϕ_e are the values of inflaton field ϕ, corresponding to N e-foldings and the end of inflation, respectively. In fact, if there is fluctuation of a common spectrum $\delta\phi_N = H_I/2\pi$, we can calculate the spectrum of the density perturbation by using δN-formalism:

$$\delta_H^2 = \frac{4}{25}(\delta N)^2 = \frac{4}{25}\left(\frac{V}{M_p^2 V_\phi}\frac{H_I}{2\pi}\right)^2, \qquad (1.2)$$

which reproduces the standard result; the density perturbation generated during single-field inflation.[1] More generically, one may however expect several scalar fields playing similar roles during inflation. Assuming ϕ_e depends on such a light field, one encounters an alternative mechanism for generating curvature perturbation. This possibility has been discussed by Bernardeau et al. [2] for modulated couplings in hybrid inflation,[2] and by Lyth [4] for a multi-field model of hybrid inflation, and more recently, by us [5] for trapping inflation combined with inhomogeneous preheating. The multi-field model [4] has been applied to brane inflation in a throat as a solution to the serious η-problem in string theory [6]. In fact, finding a light inflaton field whose mass is protected by symmetry is rather hard in string theory models, especially when masses are determined by calculable mechanism of moduli stabilization. It is therefore very important to find a light field other than the inflaton, which can contribute to the curvature perturbation. Moreover, it is obvious that the standard single-field inflationary scenario cannot explain large non-gaussianity [7]. The clear difference from the standard multi-field inflation is explained in Appendix B. There are many "alternatives" in which such light fields play crucial roles in generating the cosmological perturbation, such as cosmological perturbation generated (1) long after inflation (curvatons) [8,9], (2) during preheating (inhomogeneous preheating) [10], or (3) during reheating (inhomogeneous reheating) [11]. Moreover, by combining these ideas it is possible to generate the initial perturbation of the curvaton from inhomogeneous preheating [12].[3]

In summary, the standard inflationary scenario uses genuine inflaton fluctuation $\delta\phi_N$ to generate the curvature perturbation during inflation, while in recent works [2,4] where perturbation is generated "at the end of inflation", curvature perturbation is

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[1] In this Letter we follow the notations given in the textbook [1].

[2] The word "modulated fluctuations" was introduced by Kofman in Ref. [3].

[3] Large non-gaussianity can be generated during inhomogeneous preheating. Even if significant non-gaussianity is not generated at preheating, small ratio $r \equiv \rho_\chi/\rho_{\mathrm{total}} \ll 1$ at the decay can lead to large non-gaussianity, as has been discussed in Ref. [13] for curvatons. Here ρ_χ is the energy density of the preheat field. Note that f_{nl} in these inhomogeneous preheating scenarios can take either sign.

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generated from $\delta\phi_e$ that is indirectly generated by fluctuation of another light field.[4] Looking at the original equation (1.1), one may however find that fluctuations induced by other components (i.e., M_p^{-2} or V/V_ϕ) may generate curvature perturbation if these components are modulated during inflation due to their dependence on a light field (moduli). Based on this simple idea, in this Letter we consider an alternative mechanism for generating the curvature perturbation that relies neither on $\delta\phi_N$ nor $\delta\phi_e$. This is the crucial difference from the previous scenarios of modulated fluctuations [2].

In Section 2, we first consider hybrid inflation with a moduli-dependent inflaton mass $m^2(\mathcal{M}) \equiv m_0^2(1+\beta\log(\mathcal{M}/M_*))$, which induces fluctuation related to (V/V_ϕ). Then in Section 3, we examine the possibility of generating the curvature perturbation in the brane inflation model with a modulated coupling. Note that our examples are based on typical moduli-dependences in conventional models, although we will not specify the model because of the generality of our argument. In Section 4, we consider the generation of the cosmological perturbation from modulated fluctuation of the effective Planck scale. The possibility discussed in Section 4 is in a sense very natural, because the Planck scale always appears in the equation. In some specific examples light fields can be identified with volume of extra dimensions that evolves slowly during inflation. In Appendix A, we consider MSSM inflation that has been advocated by Allahverdi et al. [14]. In this appendix, we consider a rather peculiar source for the moduli-dependence, the fluctuation of ϕ_0 that denotes the point where the secondary minimum appears. Our argument for MSSM inflation is that if couplings depend on moduli, the modulation can be mediated to ϕ_0 that determines the number of e-foldings.

We calculate explicit forms of the non-gaussianity parameter f_{nl} and show how large non-gaussianity can be generated. We show that the running of the non-gaussianity parameter does not vanish in these models. Conclusions and discussions are presented in Section 5.

2. Modulated hybrid inflation

Let us start with the original hybrid inflation model. Hybrid inflation has the effective potential for the two fields (ϕ,σ),

$$V(\phi,\sigma) = \lambda\left(\sigma^2 - v^2\right)^2 + \frac{1}{2}g^2\phi^2\sigma^2 + V(\phi), \tag{2.1}$$

where ϕ is the inflaton and σ is the trigger field. In this section we consider a specific example of the model, whose inflaton potential is given by $V(\phi) = m^2\phi^2/2$. Here the end of inflation expansion occurs at

$$\phi_e = \frac{\sqrt{\lambda}v}{g}. \tag{2.2}$$

The number of e-foldings is given by

$$N = \frac{\lambda v^4}{M_p^2 m^2}\log\frac{\phi_N}{\phi_e} = \frac{1}{\eta_\phi}\log\frac{\phi_N}{\phi_e}, \tag{2.3}$$

where the definition of the slow-roll parameter is $\eta_\phi \equiv m^2/3H_I^2 = m^2M_p^2/(\lambda v^4)$. Modulated fluctuations of couplings $\lambda(\mathcal{M})$ or $g(\mathcal{M})$ are discussed in Ref. [2], which may (or may not) lead to the fluctuation $\delta\phi_e$, and therefore to indirect generation of δN at the end of inflation.

On the other hand, since modulated fluctuation of m^2 does not lead to $\delta\phi_e$, δN is not generated from modulated m at least at the end of inflation. Alternatively, it leads to another kind of the fluctuation

$$\delta N_{\mathcal{M}} = \frac{\lambda v^4\log(\phi_N/\phi_e)}{M_p^2} \times \left(-2\frac{\partial m/\partial\mathcal{M}}{m^3}\right)\delta\mathcal{M} = -2N\left(\frac{m'}{m}\right)\delta\mathcal{M}, \tag{2.4}$$

where m' is the derivative of m with respect to $\mathcal{M}$.

Let us we consider a concrete example of the moduli-dependent mass

$$m^2(\mathcal{M}) \equiv m_0^2\left[1+\beta\log(\mathcal{M}/M_*)\right], \tag{2.5}$$

where β and M_* are model-dependent parameters.[5] This leads to

$$\delta N_{\mathcal{M}} \simeq -\beta N\left(\frac{\delta\mathcal{M}}{\mathcal{M}}\right). \tag{2.6}$$

The inflaton fluctuation may be negligible if inflation is fast-roll [15] and where inflaton mass is as large as $m \sim H$. In this specific case, the perturbation generated by the modulated mass can always dominate the curvature perturbation. However, in other cases, inflaton fluctuation may be significant. The modulated perturbation can thus dominate when the condition

$$\left(\frac{\mathcal{M}}{\phi_N}\right) < \beta\eta_\phi N \tag{2.7}$$

is satisfied.[6] Moreover, the mass of $\mathcal{M}$ (i.e., $m_{\mathcal{M}}$) must be less than H_I during inflation, which leads to an another condition given by

$$m_{\mathcal{M}}^2 \simeq \beta m_0^2\left(\frac{\phi_N}{\mathcal{M}}\right)^2 < H_I^2. \tag{2.8}$$

Combining these equations and assuming that the amplitudes for fluctuations are comparable ($\delta\mathcal{M} = \delta\phi_N$), we obtain

$$\sqrt{\beta\eta_\phi} > \frac{1}{N}. \tag{2.9}$$

Since the right-hand side of Eq. (2.9) is by definition much smaller than unity, the required condition is rather trivial in this model.[7]

It is very important to calculate a non-gaussianity parameter of our modulated inflation scenario, since the non-gaussianity parameter is expected to distinguish the curvature perturbation generated by alternative mechanisms from the one that is generated by conventional inflaton fluctuation. Here we consider the definition of non-gaussianity parameter f_{nl} through curvature perturbation [1]. By using the δN-formalism, we can write

$$\zeta = N'\delta\mathcal{M} + \frac{1}{2}N''(\delta\mathcal{M})^2, \tag{2.10}$$

where higher terms are dropped because they are not important here. The explicit form of the non-gaussianity parameter is then given by

$$-\frac{3}{5}f_{nl} \equiv \frac{1}{2}\frac{N''}{(N')^2}, \tag{2.11}$$

where the prime denotes the derivative with respect to the moduli. Applying the explicit form of $m^2(\mathcal{M})$ to Eq. (2.11), we find for $|N\beta| \ll 1$,

$$f_{nl} = -\frac{5}{6}\frac{N''}{(N')^2} \sim \frac{1}{N\beta}, \tag{2.12}$$

[4] Note that in other scenarios [8–12] dynamics "after" inflation is used to generate curvature perturbation. In this Letter, we however focus our attention on scenarios that can work "during" inflation.

[5] Here for simplicity we neglect loop corrections related to ϕ. Of course this does not destroy the above argument as far as the original hybrid inflation scenario is successful.

[6] See Appendix C for the case with $p \equiv \dot{\mathcal{M}}/\dot{\phi} \gg 1$.

[7] See also Refs. [16,17] for natural values of η-parameter.

which can be large if the inflaton fluctuation is not significant. Here f_{nl} may take either sign. Moreover, since f_{nl} in our modulated inflation scenario depends explicitly on N^{-1}, a running of non-gaussianity may hopefully be used to distinguish modulated inflation from other models of large non-gaussianity. The explicit form of the running of the non-gaussianity is [1]

$$\frac{d\ln f_{nl}}{d\ln k} \simeq -M_p^2 \frac{V'}{V}\frac{d\ln N^{-1}}{d\phi} \simeq N^{-1}. \tag{2.13}$$

3. Modulated brane inflation

Brane inflation is one of the specific accomplishments of the inflationary universe within the brane world framework in the string theory. Inflation potential during brane inflation is typically given by

$$V(\phi) = \frac{1}{2}\gamma H_I^2\phi^2 + V_0\left(1-\lambda\frac{M^n}{\phi^n}\right), \tag{3.1}$$

where γ, λ and M are model-dependent parameters. For simplicity we consider the case in which only λ depends on the light moduli. Here $|\gamma| \ll 1$ is required to make enough e-foldings during inflation. The inflaton field ϕ measures the brane distance, and the end of inflation is induced by brane collision that occurs when two branes come closer than the string scale. Note that the situation in this inflationary model is identical to the conventional hybrid inflation model in so far as the first term (mass term) dominates the evolution of the inflaton field. Therefore, modulated inflation may occur in the same way as the conventional hybrid inflation if the mass term dominates the inflaton evolution. On the other hand, if the second term (Coulomb-like potential) dominates the inflaton potential, modulation will appear through the dimensionless coupling constant λ and the situation of modulated inflation will be different. Again, inflaton fluctuation leads to

$$\delta N_\phi = N(n+2)\times\frac{\delta\phi_N}{\phi_N}, \tag{3.2}$$

while modulation related to moduli-dependent coupling $\lambda(\mathcal{M})$ leads to

$$\delta N_{\mathcal{M}} = -N\times\frac{\lambda'}{\lambda}\delta\mathcal{M}, \tag{3.3}$$

where λ' is the derivative with respect to $\mathcal{M}$. For a specific concrete example let us consider $\lambda(\mathcal{M}) \equiv \lambda_0 e^{-c\mathcal{M}/M_*}$ during inflation, where c is a dimensionless constant and M_* is a scale parameter. Values of these parameters are model dependent. Then we obtain

$$\delta N_{\mathcal{M}} = cN\times\frac{\delta\mathcal{M}}{M_*}. \tag{3.4}$$

To take into account the condition where the modulated perturbation exceeds the conventional cosmological perturbation originating from inflaton fluctuation, we consider the condition $\delta N_\phi \ll \delta N_{\mathcal{M}}$ that leads to

$$\frac{\phi_N}{M_*} \gg \frac{n+2}{c}. \tag{3.5}$$

Another condition is needed to ensure that moduli field $\mathcal{M}$ is light during inflation. The effective mass induced by the inflaton potential is

$$m_{\mathcal{M}}^2 \simeq \lambda V_0\frac{M^n}{\phi_N^n}\left(\frac{c}{M_*}\right)^2, \tag{3.6}$$

which must be less than the Hubble parameters during inflation, $H_I \equiv \frac{V_0}{3M_p^2}$. Thus, we find a condition

$$\phi_N \gg M\times\left(\frac{\sqrt{\lambda}cM_p}{M_*}\right)^{2/n}. \tag{3.7}$$

Now we may conclude that our modulated inflation scenario works with typical brane inflation potential provided that there is the light field and Eqs. (3.5) and (3.7) are satisfied.

Again, a non-gaussianity parameter is important in distinguishing curvature perturbations. Applying a more generic form $\lambda(\mathcal{M}) \equiv \lambda_0 e^{\alpha(\mathcal{M})}$ to Eq. (2.11), and assuming that the dominant component during inflation is $\alpha(\mathcal{M}) \simeq \alpha_n\mathcal{M}^n/M_*^n$, we find for $|N\alpha_n| \ll 1$:

$$f_{nl} = -\frac{5}{6}\frac{N''}{(N')^2} \simeq \frac{1}{N\alpha_n}\frac{M_*^n}{\mathcal{M}^n}, \tag{3.8}$$

which can be large and may take either sign.

4. Inflation with modulated Planck scale

The radial mode $\mathcal{M}_e$ of extra compact space may satisfy the slow-roll condition if the effective potential during inflation satisfies the slow-roll condition. In this case, there can be δN perturbation generated by the modulated Planck scale during inflation,

$$\delta N = -2N\times\left(\frac{M_p'}{M_p}\right)\delta\mathcal{M}_e, \tag{4.1}$$

where M_p' denotes the derivative of M_p with respect to $\mathcal{M}_e$.[8] A specific example of the slow-roll condition for the radion is discussed in Ref. [19]. The condition given in Ref. [19] is apparent in the original frame, but in the Einstein frame the rescaling of the inflaton potential is crucial. For example, in Brans–Dicke theory, the exponential factor induced by the metric rescaling can be canceled by the scalar-field rescaling if the potential is quartic. This cancellation may happen if the inflaton potential is given by $V(\phi) = \lambda\phi^4/4$ for chaotic inflation. Note that the rescaling of the inflation potential is highly model-dependent and also may be shifted by the quantum effect. In this respect, one cannot simply assume that the "constant" vacuum energy during inflation is not affected by the rescaling, especially when the scale is generated dynamically. In any case, it is however possible to assume slow-roll evolution of the radial mode $\mathcal{M}_e$ during inflation that rapidly settles to the minimum after inflation [19].

Note that our mechanism for modulated Planck scale works in that the Planck scale determines the number of e-foldings. The required condition for generating the curvature perturbation is the slow-roll condition for the corresponding moduli field, which is highly model-dependent.

5. Conclusions and discussions

We have studied a new class of modulated fluctuations that generates curvature perturbation from light-field fluctuation. According to Refs. [2] and [4], fluctuation of a light field may induce fluctuation of the end-point of inflation expansion ($\delta\phi_e \neq 0$), which may lead to generation of cosmological perturbation at the end of the inflation. Contrary to these previous scenarios of modulated fluctuations, the origin of the cosmological perturbation in our scenario is not $\delta\phi_e$. As far as we know, this is the first realization of a modulated scenario without $\delta\phi_e$. Moreover, we showed a concrete example of modulated inflaton in which large non-gaussianity can be generated. The possibility of large non-gaussianity has been suggested in Ref. [2], but there was no specific example.[9] As we

[8] A non-trivial kinetic term may cause deviation from the standard Gaussian perturbation, which leads to another kind of non-gaussianity [18]. However, the result will be highly model-dependent. For simplicity, the kinetic term is assumed to be minimal.

[9] Large non-gaussianity in the multi-field model of hybrid inflation has been discussed in Ref. [4] as a specific example.

have discussed in Ref. [16] for curvatons, it is rather easy to generate the required value of spectral index in these alternative scenarios, since in these models the form of the light-field potential is not constrained by the kinematic requirements of the inflation expansion. In fact, in hilltop scenarios [20] of these alternatives [16], the spectral index may be related to the negative η-parameter of a light field during inflation, which can be justified in conventional supergravity models [16,17,20].

An obvious deficit of modulated scenarios is the famous moduli problem, as these scenarios always require moduli fields displaced from their true minimum at least at the time when the perturbation is generated. Late-time entropy production such as thermal inflation [1] may solve this problem, but thermal inflation may not work if the energy scale of the primordial inflation is very low. Of course it is extremely hard to build an inflationary model at low scales [21][10], but low-scale inflation may become very important if the gravitational effect is observed in the Large Hadron Collider (LHC). Moreover, many inflationary models that are based on single $\delta\phi_N$-fluctuation may be excluded if large non-gaussianity is confirmed by the observation [7]. Note that the moduli problem does not appear in multi-field models such as Refs. [4,6] or inhomogeneous preheating scenarios [10], in which a non-gaussianity parameter can be large and at the same time, low-scale inflation is possible.[11]

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Appendix A. Modulated MSSM inflation

In this appendix, we show another possibility of introducing moduli dependence to the cosmological perturbation theory. We start with the Minimal Supersymmetric Standard Model (MSSM), which is a well motivated extension of the Standard Model (SM).[12] Considering a flat direction ϕ with non-renormalizable superpotential

$$W = \frac{\lambda_n}{n}\frac{\Phi^n}{M_p^{n-3}}, \tag{A.1}$$

where Φ denotes the superfield related to ϕ. We find the scalar potential

$$V = \frac{1}{2}m^2\phi^2 + A\cos(n\theta+\theta_A)\frac{\lambda_n\phi^n}{nM_p^{n-3}} + \frac{\lambda_n^2\phi^{2(n-1)}}{M_p^{2(n-3)}}, \tag{A.2}$$

where m and θ_A come from the soft supersymmetry (SUSY) breaking mass and the A-term, respectively. Here ϕ and θ denote the radial and angular coordinates of the scalar component of the superfield Φ. The potential has a secondary minimum at

$$\phi_0 \sim \left(\frac{mM_p^{n-3}}{\lambda_n}\right)^{1/(n-2)}, \tag{A.3}$$

but the potential barrier, however, disappears if the coefficient of the A-term (A) satisfies the condition

$$A^2 = 8(n-1)m^2. \tag{A.4}$$

Around this secondary minimum with the coefficient $A^2 \simeq 8(n-1)m^2$, the field only feels the third derivative of the potential. Then inflation may start near ϕ_0, which leads to the number of e-folds

$$N \simeq \frac{\phi_0^3}{2n(n-1)M_p^2(\phi_0-\phi)}. \tag{A.5}$$

As we are interested in fluctuation related to $\phi_0(\mathcal{M})$, we calculate the derivative of N with respect to $\mathcal{M}$:

$$N_{\mathcal{M}} \simeq 3N\frac{\phi_0'}{\phi_0} - N\frac{\phi_0'}{\phi_0-\phi} \simeq -N\frac{\phi_0'}{\phi_0-\phi}. \tag{A.6}$$

Here $|\phi_0-\phi| \ll |\phi_0|$ is considered. See Ref. [14] for more details. The amplitude of the perturbation is therefore given by[13]

$$\delta N_{\mathcal{M}} \simeq N^2\frac{mM_p}{\phi_0^2}\phi_0', \tag{A.7}$$

where the amplitude is assumed to be

$$\delta\mathcal{M} \simeq H_I/2\pi \simeq \frac{V(\phi_0)^{1/2}}{2\pi M_p} \simeq \frac{(n-2)}{2\pi\sqrt{2n(n-1)}M_p}m\phi_0. \tag{A.8}$$

Comparing our result (A.7) with the standard result, which has been obtained from the fluctuation of the inflaton field [14], we obtain the required condition

$$\phi_0' \propto \phi_0\frac{\lambda_n'}{\lambda_n} \gg 1, \tag{A.9}$$

which is needed for $\delta N_{\mathcal{M}}$ to dominate the cosmological perturbation. The slow-roll condition for the light field $\mathcal{M}$ may put a severe restriction (lower bound) on the inflation energy scale. Note that MSSM inflation requires cancellation of the second derivative of the inflaton field, which makes it possible to construct a low-scale inflation model. On the other hand, the naive estimation of the moduli mass is $O(m)$, if the mass is not protected. In fact, introducing explicit moduli-dependence to MSSM action is not easy at this moment, especially when we need to introduce a moduli field whose mass can be much smaller than the SUSY-breaking mass during inflation.[14]

Appendix B. Source term for the curvature perturbation

The simplest way to see the source term for the curvature perturbation in modulated inflation is to consider the evolution of the curvature perturbation

$$\dot{\mathcal{R}} = -H\frac{\delta P}{\rho+P}, \tag{B.1}$$

where δP is the pressure perturbation. The key idea in the modulated inflation scenario is the explicit $\mathcal{M}$-dependence in the "slow-roll velocity"; $\dot\phi = V_\phi/3H_I$. Here we consider hybrid inflation that leads to $\dot\phi = m(\mathcal{M})^2\phi/3H_I$. The modulated perturbation thus leads to

$$\delta\dot\phi \simeq 2\dot\phi\frac{m'}{m}\delta\mathcal{M}, \tag{B.2}$$

which leads to the pressure perturbation

[10] See also Ref. [9] for the condition for the inflation energy scale in the curvaton scenario.

[11] See also Ref. [23].

[12] MSSM inflation [14] is an attractive idea, but it may be excluded if large non-gaussianity is confirmed by the observation [7]. One way to solve this problem is to generate cosmological perturbation by using modulated inflation.

[13] Numerical factors are neglected for simplicity.

[14] λ_n may be generated from instantons in intersecting brane model. In this case, the moduli may be related to the area bounded by branes, which may be light during inflation [22].

$$\delta P \simeq \dot\phi\delta\dot\phi \simeq 2\dot\phi^2\frac{m'}{m}\delta\mathcal{M}. \tag{B.3}$$

From the integral of $\dot{\mathcal{R}}$, we find the correction from the modulated inflation;

$$\Delta\mathcal{R} \simeq -2N\frac{m'}{m}\delta\mathcal{M}, \tag{B.4}$$

where $\dot{\mathcal{M}} \simeq 0$ is assumed during inflation.

On the other hand, there is no such term in the standard equation for the multi-field inflation [24], in which the source term is proportional to the bent parameter $\dot\theta$. Obviously, "bent" in the inflation trajectory is not important in modulated inflation. For example, considering flat potential that leads to $\dot{\mathcal{M}} = 0$, we find the curvature perturbation (B.4), while there is no "bent" in the inflation trajectory.

The key is the explicit $\mathcal{M}$-dependence of the inflaton velocity $\dot\phi(\mathcal{M})$, which has been disregarded in the standard calculation of multi-field inflation.

The source term that is proportional to $\dot\theta$ does not generate a significant correction to the curvature perturbation if there is no significant bend in the trajectory.

Appendix C. Modulated inflaton for $p \equiv \dot{\mathcal{M}}/\dot\phi \gg 1$

In Appendix B, we considered only the limiting case for $p \ll 1$. Note that $\dot{\mathcal{M}} \simeq 0$ represents the ideal situation for the modulated inflation and is very useful to explain the origin of the curvature perturbation in modulated inflation. However, to attain $\dot{\mathcal{M}} = 0$ we have to consider a fine-tuning (i.e. careful cancellation with an additional potential for $\mathcal{M}$). Moreover, considering a more general situation the energy landscape during inflation can be changed by the motion of $\mathcal{M}$, and therefore can participate in the inflaton trajectory. In this respect, the significance of modulated inflation should be considered for arbitrary p. In this appendix, we discuss what happens for $p \gg 1$.

Let us consider the slow-roll velocity of the fields at horizon crossing:

$$\dot{\mathcal{M}} \simeq \frac{V_\mathcal{M}}{3H_I} \simeq \frac{\beta m_0^2\phi^2}{6H_I\mathcal{M}}, \tag{C.1}$$

$$\dot\phi \simeq \frac{V_\phi}{3H_I} \simeq \frac{m^2\phi}{3H_I}, \tag{C.2}$$

which leads to the ratio

$$p \equiv \frac{\dot{\mathcal{M}}}{\dot\phi} \simeq \beta\frac{\phi}{2\mathcal{M}}. \tag{C.3}$$

The curvature perturbation generated at the horizon crossing is given by [24]

$$\mathcal{R} = H_I\left(\frac{\dot\phi Q_\phi + \dot{\mathcal{M}}Q_\mathcal{M}}{\dot\phi^2 + \dot{\mathcal{M}}^2}\right) \simeq \frac{1+p}{1+p^2}\mathcal{R}_0, \tag{C.4}$$

where Q_ϕ and $Q_\mathcal{M}$ are the Sasaki–Mukhanov variables and $\mathcal{R}_0$ denotes the curvature perturbation for the conventional hybrid-type inflation (i.e. for $p = 0$). For $p \ll 1$, we may disregard the effect of the field $\mathcal{M}$ in the curvature perturbation generated at the horizon crossing. However, for $p \geqslant 1$, we should consider the curvature perturbation caused by the field $\mathcal{M}$ *at the horizon crossing*.

Let us consider modulated inflation for $p \gg 1$. The perturbation related to the modulated inflation (i.e. the perturbation caused by $\delta\dot\phi$ and $\delta\dot{\mathcal{M}}$) gives

$$\dot{\mathcal{R}} = -H\frac{\delta P}{\rho + P} \simeq \frac{-H}{(1+p^2)}\left[\beta\frac{\delta\mathcal{M}}{\mathcal{M}} + 2p^2\frac{\delta\phi}{\phi}\right]. \tag{C.5}$$

Note that the second term related to $\delta\dot{\mathcal{M}}$ becomes important for $p \gg 1$.

Using the above results, we can examine the conditions for modulated inflation. For $p \gg 1$, the main source of the curvature perturbation at the horizon crossing is caused by the motion of $\mathcal{M}$, which is multiplied by a factor $(1+p)/(1+p^2) \sim p^{-1}$. A factor $1/(1+p^2) \sim p^{-2}$ appears for modulated inflation caused by $\delta\dot\phi$, while the factor is $p^2/(1+p^2) \sim 1$ for $\delta\dot{\mathcal{M}}$. In addition to the above conditions, we must consider the condition (2.8). Finally, we conclude that modulated inflation can be significant for $p \gg 1$, but in this case the roles played by ϕ and $\mathcal{M}$ are exchanged.

References

[1] A.R. Liddle, D.H. Lyth, Cosmological Inflation and Large Scale Structure, Cambridge Univ. Press, Cambridge, UK, 2000.
[2] F. Bernardeau, L. Kofman, J.P. Uzan, Phys. Rev. D 70 (2004) 083004, astro-ph/0403315;
F. Bernardeau, J.P. Uzan, Phys. Rev. D 67 (2003) 121301, astro-ph/0209330.
[3] L. Kofman, astro-ph/0303614.
[4] D.H. Lyth, JCAP 0511 (2005) 006, astro-ph/0510443.
[5] T. Matsuda, JHEP 0707 (2007) 035, arXiv: 0707.0543.
[6] T. Matsuda, JCAP 0609 (2006) 003, hep-ph/0606137;
D.H. Lyth, Phys. Rev. Lett. 97 (2006) 121301, astro-ph/0607326.
[7] A.P.S. Yadav, B.D. Wandelt, arXiv: 0712.1148.
[8] A.D. Linde, V.F. Mukhanov, Phys. Rev. D 56 (1997) 535, astro-ph/9610219;
D.H. Lyth, D. Wands, Phys. Lett. B 524 (2002) 5, hep-ph/0110002;
T. Moroi, T. Takahashi, Phys. Lett. B 522 (2001) 215, hep-ph/0110096.
[9] T. Matsuda, Class. Quantum Grav. 21 (2004) L11, hep-ph/0312058;
K. Dimopoulos, D.H. Lyth, Y. Rodriguez, JHEP 0502 (2005) 055, hep-ph/0411119;
Y. Rodriguez, Mod. Phys. Lett. A 20 (2005) 2057, hep-ph/0411120.
[10] E.W. Kolb, A. Riotto, A. Vallinotto, Phys. Rev. D 71 (2005) 043513, astro-ph/0410546;
T. Matsuda, JCAP 0703 (2007) 003, hep-th/0610232;
T. Matsuda, JHEP 0703 (2007) 096, astro-ph/0610402;
T. Matsuda, JCAP 0706 (2007) 029, hep-ph/0701024.
[11] G. Dvali, A. Gruzinov, M. Zaldarriaga, Phys. Rev. D 69 (2004) 023505, astro-ph/0303591;
F. Vernizzi, Phys. Rev. D 69 (2004) 083526, astro-ph/0311167;
K. Enqvist, A. Mazumdar, M. Postma, Phys. Rev. D 67 (2003) 121303, astro-ph/0304187;
A. Mazumdar, Phys. Rev. Lett. 92 (2004) 241301, hep-ph/0306026.
[12] T. Matsuda, Phys. Rev. D 72 (2005) 123508, hep-ph/0509063;
T. Matsuda, JHEP 0709 (2007) 027, arXiv: 0708.4098;
T. Matsuda, JCAP 0708 (2007) 003, arXiv: 0707.1948.
[13] D.H. Lyth, C. Ungarelli, D. Wands, Phys. Rev. D 67 (2003) 023503, astro-ph/0208055;
Q.G. Huang, arXiv: 0801.0467.
[14] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Mazumdar, Phys. Rev. Lett. 97 (2006) 191304, hep-ph/0605035;
A. Mazumdar, arXiv: 0707.3350;
R. Allahverdi, A. Kusenko, A. Mazumdar, JCAP 0707 (2007) 018, hep-ph/0608138;
R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen, A. Mazumdar, JCAP 0706 (2007) 019, hep-ph/0610134.
[15] A. Linde, JHEP 0111 (2001) 052, hep-th/0110195;
K. Dimopoulos, M. Axenides, JCAP 0506 (2005) 008, hep-ph/0310194.
[16] T. Matsuda, Phys. Lett. B 659 (2008) 783, arXiv: 0712.2103.
[17] M. Dine, L. Randall, S.D. Thomas, Phys. Rev. Lett. 75 (1995) 398, hep-ph/9503303.
[18] D. Babich, P. Creminelli, M. Zaldarriaga, JCAP 0408 (2004) 009, astro-ph/0405356.
[19] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, J. March-Russell, Nucl. Phys. B 567 (2000) 189, hep-ph/9903224;
J.M. Cline, Phys. Rev. D 61 (2000) 023513, hep-ph/9904495.
[20] L. Boubekeur, D.H. Lyth, JCAP 0507 (2005) 010, hep-ph/0502047;
D.H. Lyth, T. Moroi, JHEP 0405 (2004) 004, hep-ph/0402174.

[21] D.H. Lyth, Phys. Lett. B 466 (1999) 85, hep-ph/9908219;
A.M. Green, A. Mazumdar, Phys. Rev. D 65 (2002) 105022, hep-ph/0201209;
R.N. Mohapatra, A. Perez-Lorenzana, C.A. de S. Pires, Phys. Rev. D 62 (2000) 105030, hep-ph/0003089;
T. Matsuda, Phys. Rev. D 66 (2002) 107301, hep-ph/0209214;
T. Matsuda, Phys. Rev. D 67 (2003) 083519, hep-ph/0302035;
A. Mazumdar, arXiv: 0707.3350.
[22] M. Cvetic, G. Shiu, A.M. Uranga, Phys. Rev. Lett. 87 (2001) 201801, hep-th/0107143;
T. Matsuda, Gen. Relativ. Gravit. 37 (2005) 1297, hep-ph/0309314.
[23] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, A. Vaihkonen, Phys. Rev. Lett. 94 (2005) 161301, astro-ph/0411394;
K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, A. Vaihkonen, JCAP 0503 (2005) 010, hep-ph/0501076;
A. Jokinen, A. Mazumdar, JCAP 0604 (2006) 003, astro-ph/0512368.
[24] C. Gordon, D. Wands, B.A. Bassett, R. Maartens, Phys. Rev. D 63 (2001) 023506, astro-ph/0009131.