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Analogies in fracture mechanics of concrete, rock and ice

A. P. Fantilli\textsuperscript{a},*, B. Chiaia\textsuperscript{a}, B. Frigo\textsuperscript{a}

\textsuperscript{a}Dept. od Structural, Building and Geotechnical Engineering, Politecnico di Torino, Corso Duca degli Abruzzi, 24, 10129 Torino, Italy

Abstract

Both in architecture and arts, the golden ratio has been taken into consideration most exclusively for its geometrical properties. Specifically, among all the proportions, the golden ratio can inspire beauty and aesthetic pleasure. Indeed, it has driven, in an implicit or explicit manner, the construction of buildings for centuries. Nevertheless, as discussed in the present paper, also fracture mechanisms in brittle and quasi-brittle materials call the golden ratio into play. This is the case of fracture energy and fracture toughness, in which the irrational number 1.61803 recurs when the geometrical dimensions vary. This aspect is confirmed by the results of different experimental campaigns performed on concrete and rock beams and ice sheets. In other words, it can be argued that the centrality of the golden ratio for quasi-brittle structures has profound physical meanings, as it can bring together the aesthetic of nature and architecture, and the equilibrium of stress flow in solid bodies.

Keywords: fracture toughness; concrete; rock; ice; golden section.

1. Introduction

As is well known (Rilem TC QFS 2004), size effect represents an important physical character of cracked members made of plain or lightly reinforced concrete. It affects mechanical properties (i.e., tensile strength, shear strength, fracture energy, etc.) because of the non-homogeneous character of the material and the occurrence of fractures.

A wide literature on size effect has considered the decrease of material strength, and the increase of fracture energy, with structural size. Weibull (1939) defined size effect as the result of statistical distribution of defects...
inside the material, whereas Bazant (1984) adopted a deterministic approach based on the energy release rate. More recently, Carpinteri et al. (1995) combined the statistical and energetic approaches within the framework of fractal geometry, which permitted to take into account the intimate nature of energy dissipation during fracture propagation. As a result, the following Multifractal Scaling Law (MFSL) can be assumed for the fracture energy (Carpinteri and Chiaia 1996):

\[
\frac{G_F}{G^\infty_F} = \left(1 + \frac{l_{ch}}{D}\right)^{-1/2}
\]

where \( G_f \) = nominal fracture energy; \( D \) = external characteristic structural size; \( G^\infty_F \) = nominal asymptotic fracture energy \((D \to \infty)\); \( l_{ch} \) = internal microstructural length.

According to Eq.(1), fracture energy, and consequently fracture toughness, increase with the dimension \( D \) of the specimen, up to the asymptotic value \( G^\infty_F \). Several experimental analyses, performed on concrete, rock and ice, seem to confirm the reliability of such law. Although concrete is a man-made material while ice and rock are natural materials, they show grain size depending behaviour and present a structurally similar brittle response. However, the properties of concrete, rock and ice show, at different scales, show the typical peculiarities of a fractal, characterized by scale invariant fractal porosity (Carbone et al. 2010, Carpinteri and Chiaia 1996).

Nevertheless, to apply the MFSL in brittle and quasi-brittle materials, the parameters \( G^\infty_F \) and \( l_{ch} \), which cannot be measured through direct specific tests, have to be computed using a best-fitting algorithm. Obviously, the regression procedure is more statistically reliable if the experimental measures of \( G_F \) are performed on a wide size range. In other words, a large number of tests is required to use correctly the MFSL (Carpinteri et al. 1995), as well as the Bazant’s (1984) size effect law.

A new size-effect model is therefore introduced in the present paper with the aim of predicting the fracture energy of concrete, and the fracture toughness of rock and ice, starting from only a few tests performed on a single size-scale.

2. The size effect law

If the magnitude of a physical property of a body increases with the geometrical dimensions of the body, a size effect law having the form of a power function (Rilem TC QFS 2004) can be introduced:

\[
\frac{s_r}{s_{\text{ref}}} = \left(\frac{D}{D_0}\right)^\beta
\]

where \( D_0 \) = reference dimension; \( D \) = dimension at a generic scale; \( s_r \) = magnitude of the physical property at reference dimensions; \( s_{\text{ref}} \) = magnitude of the physical property at a generic scale; and \( \beta \) = exponent that has to be defined by fitting experimental and/or numerical data. In some circumstances, when \( D = n D_0 \), and \( n \) is integer, \( s_r \) increase of a factor \( \phi \) with respect to \( s_{\text{ref}} \) (i.e., \( s_r / s_{\text{ref}} = \phi = 1.61803 \)).

In mathematics, \( \phi \) is generally known as the golden ratio, or the divine proportion. A first definition of the golden ratio was proposed by Euclid (300 b.C.). In particular, in the VI book of the Elements, the third definition states: “A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less”.

In other words, in the line segment depicted in Fig. 1, it is possible to localize a point where the ratio of the whole line (A) to the large segment (B) is the same as the ratio of the large segment (B) to the small one (C). Only when this ratio is equal to the golden ratio (i.e., the irrational number \( \phi \)), can such a division be possible.

In physics, this number, intimately interconnected with the Fibonacci sequence (1, 2, 3, 5, 8, 13…), controls growth in several natural patterns. In fact, the limit of the ratios of two successive terms of the series tends to the golden ratio.
When golden ratio recurs in a size effect law, the exponent $\beta$ in Eq.(2) can be computed as follows:

$$\beta = \frac{\log \phi}{\log n}$$

For instance, in the case of reinforced concrete (RC) members subjected to pure bending or tensile loads, crack spacing increases with the size. In particular, as observed by Fantilli and Chiaia (2013), when the geometrical dimensions of the RC beams or ties are doubled (i.e., $n = D/D_0 = 2$), crack spacing increases of $I$ and therefore $\beta = 0.7$. Results of different experimental campaigns validate the tension stiffening model by Fantilli and Chiaia (2013).

In the opinion of the authors, the evaluation of $\beta$ based on the golden ratio, can also be used to define the size effect on fracture energy and fracture toughness in quasi-brittle materials.

3. Experimental observations on the fracture of concrete, rock and ice

Similarly to the value of crack spacing of reinforced concrete ties and beams, the fracture energy of concrete, and the fracture toughness of rock and ice, increase with the structural size. Some experimental campaigns, performed on both the materials and described in the following sections, evidenced such behaviour.

3.1. Tests on concrete beams

The procedure proposed by Rilem TC-50 FCM (1985) is generally used to evaluate the fracture energy of cement-based composites. Perdikaris and Romeo (1992) adopted such procedure in an experimental campaign performed on concrete beams. Specifically, the notched specimens depicted in Fig. 2a, and tested in three point bending, were investigated at different scales. Table 1 shows the geometrical properties of these samples.

<table>
<thead>
<tr>
<th>$SF$</th>
<th>Concrete</th>
<th>$H$ (mm)</th>
<th>$L$ (mm)</th>
<th>$a/H$</th>
<th>$B$ (mm)</th>
<th>$G_f$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C1</td>
<td>63.5</td>
<td>254</td>
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<td>127</td>
<td>76</td>
</tr>
<tr>
<td>2</td>
<td>C1</td>
<td>127</td>
<td>508</td>
<td>0.3</td>
<td>127</td>
<td>94</td>
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<td>254</td>
<td>1016</td>
<td>0.3</td>
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<td>117</td>
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<tr>
<td>1</td>
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<td>63.5</td>
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<td>0.3</td>
<td>127</td>
<td>69</td>
</tr>
<tr>
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<td>127</td>
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<td>0.3</td>
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<td>254</td>
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<td>118</td>
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</tbody>
</table>

The beams were cast with two different concrete mixtures: C1 was a normal strength concrete ($f_c = \text{concrete strength} = 35 \text{ MPa}, E_c = \text{elastic modulus} = 33 \text{ GPa}$), whereas C2 was a high strength concrete ($f_c = 75 \text{ MPa}, E_c = 40 \text{ GPa}$). In both the concretes, the maximum aggregate size was 6.4 mm.

The external load $P$ (Fig. 2a) was applied by means of a servo-hydraulic testing machine, so that the peak was
reached in approximately 10 minutes. Local LVDTs were also adopted to measure deflection $\eta$ in the midspan of the specimen. Fig. 2b shows a complete load midspan deflection curve measured in a single beam, and used to evaluate $G_f$. To be more precise, the value of fracture energy can be experimentally measured through the following formula (Hillerborg 1985):

$$G_f = \frac{W}{B(H - a)}$$

where, $W$ = total amount of absorbed energy (i.e., the area below the measured load-deflection curve in Fig.2b). An increase of $G_f$ with the size factor $SF$, which varies from 1 to 4, can be observed in Table 1.

Nevertheless, such variation seems to differ from that measured in the crack spacing of RC ties and beams. As reported in Fantilli and Chiaia (2013), crack distance becomes $\phi$ times higher when the geometrical dimensions of the specimen are doubled. Here, fracture energy shows the same increase only when $SF = 4$ (see Table 1). This difference can be ascribed to the geometry of the beams adopted in the three point bending tests (Fig. 2b). In fact, only the height $H$, the notch $a$, and the span $L$ of the beam were scaled. Conversely, in the case of Perdikaris and Romeo (1992), the width $B$ of the cross-section was kept constant, and equal to 100 mm, for all the specimens. Therefore, if also the width $B$ had been scaled, the same $\phi$-factor scaling for $G_f$ would have been found.

Finally, the size effect on $G_f$ seems to be the same for both the types of concrete. Indeed, as observed by Perdikaris and Romeo (1992), at all scales the fracture energy of higher strength concrete mix (C2) is either similar or slightly lower than that of the lower strength mix (C1).

![Fig. 2. Three point bending tests on beams: a) Geometrical dimensions of the specimens; b) Typical applied load vs. midspan deflection diagram in the case of concrete (Rilem TC-50 FCM 1985).](image)

### 3.2. Tests on rock beams

To study the fracture behaviour in rocks, Bazant et al. (1991) and Bazant et al. (1993) tested four sizes of notched beams in three-point bending. Test setup is similar to that of concrete beams (Fig.2a), but the scale factor varies from 1 to 7.85 (see Table 2). The beams were made with the Indiana (Bedford) limestone, having a mass density equal to 2.2 g/cm$^3$, an elastic modulus $E = 15.3$ GPa (coefficient of variation equal to 25%), a Poisson’s ratio $\nu = 0.15$ and an average tensile strength equal to 3.45 MPa.

The three-point (single edge-notched) specimens were cut from the same block of limestone with four orders
of magnitude in depths ($H = 13, 25, 51, 102$ mm). Conversely, the thickness ($B = 13$ mm) and notches width (1.3 mm) are the same at each scale. The ratio height of the notch is $0.4H$, whereas the span of the span $L$ is equal to $4H$. The beams were mined in order to have natural bedding plane of the rock normal to the load and to the notches. From the tests, conducted under constant crack-mouth opening displacement (CMOD), only the maximum applied load $P_{\text{max}}$ was measured by Bazant et al. (1991) and Bazant et al. (1993).

<table>
<thead>
<tr>
<th>SF</th>
<th>$H$ (mm)</th>
<th>$L$ (mm)</th>
<th>$a/H$</th>
<th>$B$ (mm)</th>
<th>$K_1$ (N/mm$^{3/2}$)</th>
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</table>
However, to measure the size effect of the stress intensity factor, the values of \( K_I \) can be easily computed through the following formulae (Tada et al. 1985):

\[
K_I = \sigma \sqrt{\pi a} F(\alpha) \tag{5}
\]

where

\[
\alpha = \frac{a}{H} \tag{6}
\]

\[
\sigma = \frac{6M}{BH^2} \tag{7}
\]

\[
M = \frac{P_{\max} L}{4} \tag{8}
\]

and

\[
F(\alpha) = \frac{1.99 - \alpha (1-\alpha) \left[ 2.15 - 3.93\alpha + 2.7\alpha^2 \right]}{\sqrt{\pi (1+2\alpha)(1-\alpha)^{3/2}}} \tag{9}
\]

All the geometrical properties of the beams, and the corresponding values of \( K_I \) as well, are reported in Table 2. As for the fracture energy of concrete specimens, the fracture toughness of rock increases with the size factor \( SF \). Also in the present case, the values of \( K_I \) appears to be \( \phi \) times higher when the geometrical dimensions of the beams are scaled with a size factor \( SF = 4 \), due to constant thickness of the specimens.

3.3. Tests on ice plates

Ice is a porous medium consisting of air and a random assemblage of grains of solid water. It could be generated from metamorphosis of snow or direct freezing of water. In Nature, several types of ice, having different physical and mechanical properties, can be observed (e.g., sea and saline ice, equi-axed granular, macro-crystalline S1 and columnar S2 freshwater ice, glaciers, etc.). In all the cases, density can vary from 800 to 917 kg/m³.

![Fig. 2. In-situ base-edge-notched reverse-tapered plate tests on S1 ice (Dempsey et al. 1999a).](image)
To investigate the fracture properties of ice, a set of samples are herein taken into consideration (Fig. 3). The specimens, generally called base-edge-notched reverse-tapered plate (Mai et al. 1975, Defranco and Dempsey 1994, Dempsey et al. 1995), are those of the *in-situ* experimental campaign performed by Dempsey et al. (1999a, 1999b) on S1 freshwater lake ice of Spray Lakes Reservoir (Alberta - Canada). The aim was to investigate the size effect on fracture toughness by testing several plates, whose geometry (see Table 3) varies from laboratory to structural-scale (1.41 m < \( L < 28.64 \) m).

Table 3. *In-situ* base-edge-notched reverse-tapered plate tests by Dempsey et al. (1999a).

<table>
<thead>
<tr>
<th>( SF )</th>
<th>Ice</th>
<th>( L ) (mm)</th>
<th>( a ) (mm)</th>
<th>( a/L )</th>
<th>( B ) (mm)</th>
<th>( K_Q ) (kPa m(^{0.5}))</th>
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</thead>
<tbody>
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<td>0.43</td>
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<td>0.5</td>
<td>97</td>
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<td>144</td>
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</tbody>
</table>

S1 freshwater ice is a secondary congregation ice that usually grows under, or on, the primary ice (the first thin layer to form) or snow. It is typically composed by massive irregularly shaped crystals extended on vertical, or near vertical, c-axes (Michel and Ramseier 1971). The physical and mechanical properties of Spray Lakes Reservoir ice have been measured by Dempsey et al. (1999a) at Clarkson University.

The plates were cut from the parent ice sheet, having uniform and stable thickness (\( B = 0.5 \) m), by using chainsaws. The test temperatures were above \(-5^\circ C\), and the ambient weather did not affect the ice sheet thickness during the tests. In this campaign, Dempsey et al (1999a) kept the thickness \( B \) of the specimens constant. In fact, due to the crystalline microstructure of ice, it is necessary to have a 2D similarity to compare each specimen tested. Moreover, according to van Vliet and van Mier (1998), in quasi-brittle fracture tests, measurements do not depend on the length of the crack front.

The load \( P_i \) (see Fig. 3) was applied by means of flatjack loading device, controlled by a servo-hydraulic control system coupled with a displacement measuring gauge located on the crack (Shapiro et al. 1979). The specimen size and the loading rate are scaled to consider the homogeneity of the ice during the tests. In fact, as the specimen size increases, the material behaviour is expected to become more homogeneous, and eventually the length of the fully developed process zone will be lower in comparison with the specimen size (Dempsey et al. 1999a).

As no standard test exists, the theory of Linear Elastic Fracture Mechanics is generally applied to freshwater ice. Accordingly, the critical stress intensity factor \( K_{lc} \) is generally substituted by \( K_Q \) (Dempsey 1991):

\[
K_Q = K_{Q,\text{apparent}} = K_I(a, \rho, P_Q)
\]

where \( K_Q \), which symbolically considers all the assumptions involved and the lack of a standard test, also depends on the crack tip radius \( \rho \) and on the maximum load \( P_Q \).

When the load \( P_i \) is applied on the crack surfaces by a flatjack, the fracture toughness \( K_Q \) can be easily evaluated through the following equation (Dempsey et al. 1999a):

\[
K_Q = f(a, \rho/L, P/w, h, \sqrt{\pi a})
\]

where \( w \) and \( h \) = width and thickness of the flatjack, respectively; and \( f(a/L) \) = function defined by Dempsey et al. (1995). For each plate, the related values of \( K_Q \) are reported in Table 3. As for the fracture properties of concrete and rock specimens, the fracture toughness of ice becomes \( \phi \) times higher when the geometrical dimensions of the specimen are scaled with a size factor \( SF = 4 \) (i.e., due to constant thickness of the specimens).
4. Size effect and golden ratio

Similarly to crack spacing (Fantilli and Chiaia 2013), if the fracture energy of concrete and the fracture toughness of rock and ice specimens increase with the size, a size effect law having the general form of a power function (Rilem TC QFS 2004) can be introduced:

\[
\frac{G_F}{G_F^0} = \frac{K_I}{K_I^0} = \frac{K_Q}{K_Q^0} = \left( \frac{D}{D_0} \right)^\beta = SF^\beta
\]

(12)

where, \(G_F^0\) (or \(K_I^0\) and \(K_Q^0\)) = average fracture energy (or fracture toughness) measured in the reference member \((SF = 1)\); \(G_F\) (or \(K_I\) and \(K_Q\)) = average fracture energy (or fracture toughness) at a generic scale \((SF \neq 1)\); and \(\beta\) = exponent that has to be defined by fitting experimental data. According to the experimental results reported in the previous section, when \(SF = D/D_0 = 4\), fracture energy of concrete, and fracture toughness of rock and ice, increase of a factor \(\phi\) (i.e., \(G_F/G_F^0 = K_I/K_I^0 = K_Q/K_Q^0 = \phi\)), and therefore \(\beta = 0.35\).

The new model for fracture energy and toughness can be used to predict the experimental data previously described. In the case of concrete beams (Fig.4), the proposed size effect law of fracture energy [i.e., Eq.(12) with \(\beta = 0.35\)] seems to be in good agreement with the results obtained by Perdikaris & Romeo (1992). This is true for both normal (C1 in Fig.4) and high strength concrete (C2 in Fig.4).

![Fig. 4. Comparison between the measured fracture energy of concrete and the proposed size effect law.](image)

The proposed size effect law also appears a suitable model for predicting the size effect of fracture toughness in natural materials. Specifically, as shown in Fig.5, Eq.(12) with \(\beta = 0.35\) fits the experimental values of \(K_I\) measured by Bazant et al. (1991) and by Bazant et al. (1993) in the three point bending beams made with the Indiana (Bedford) limestone.

Similarly, the fracture toughness of the reverse-tapered plates, tested by Dempsey at al. (1999a) and made with S1 freshwater ice, varies according to the proposed size effect law. As Fig.6 shows, Eq.(12) with \(\beta = 0.35\) gives a reasonable approximation of \(K_Q\) also in the case of large specimens (i.e., \(SF = 20\)).In Fig.4, Fig.5 and Fig.6, the results of MFSL have been reported. In general, the predictions given by Eq.(1) are in good agreement with the experimental data, and results of the proposed size effect law as well, only when \(SF \leq 4\).

On the contrary, at larger scales, the fracture energy of concrete, or the fracture toughness of natural materials, seems to be better evaluated by Eq.(12) with \(\beta = 0.35\).
Fig. 5. Comparison between the measured fracture toughness of rock and the proposed size effect law.

Fig. 6. Comparison between the measured fracture toughness of ice and the proposed size effect law.
5. Conclusions

A new size effect law for the fracture energy and the fracture toughness of quasi-brittle materials has been presented, based on the golden ratio $\phi$. From the comparison between the results of the proposed model and some experimental data, the following conclusions can be drawn:

- similarly to the crack pattern of reinforced concrete ties and beams, the golden ratio also recurs in the fracture mechanism of concrete, rock and ice;
- a single size effect law can be used for the fracture energy of concrete and the fracture toughness of rock and ice;
- with respect to the traditional size effect models, the proposed law can predict $G_F$, $K_I$, and $K_Q$ with good accuracy, by using a reduced number of tests.

Nevertheless, only two types of specimens (i.e., beams and reverse-tapered plates) have been investigated in the present paper. Future works should therefore consider the behaviour of other structures, such as ties, in order to generalize the effectiveness of the proposed size effect law.

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References


