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ScienceDirect
Energy Procedia 95 (2016) 459 – 466
Energy Procedia

International Scientific Conference "Environmental and Climate Technologies", CONECT 2015, 14-16 October 2015, Riga, Latvia

# Comparative evaluation of different computational models for performance of air source heat pumps based on real world data

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**Abstract**

To reduce energy usage and $CO_2$ emission due to heating, heat pumps have turned out a good option. For example, to obtain a net zero house, often a combination of solar panels and a heat pump is used. A computational model of the performance of a heat pump provides a useful tool for prediction and decision making. In this paper, six variations of such computational models are discussed and evaluated. Evaluation was based on real world empirical data for 8 different domestic situations. The evaluation took place by determining the most optimal values for the parameters of each of the models for the given data, and then considering the remaining error.

Peer-review under responsibility of Riga Technical University, Institute of Energy Systems and Environment.
*Keywords:* domestic energy usage; renewable energy; air source heat pump; seasonal performance factor

## 1. Introduction

In designing or renovating houses, often an aim is to come as close as possible to an energy neutral or net zero house; e.g., [1, 2]. In a net zero house, on an annual basis the total amount of energy used by the building is roughly equal to the amount of renewable energy it produces. Usually a photovoltaic (PV) solar energy production system is

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Peer-review under responsibility of Riga Technical University, Institute of Energy Systems and Environment.
doi:10.1016/j.egypro.2016.09.065

used, in combination with a heat pump; e.g., [3–7]. Computational models for the performance of both heat pumps and PV systems are useful as a basis for the decision on the dimensions of both systems.

The energy demand for heating a house depends on the ambient temperature. For traditional gas-, oil- or coal-based heating systems it is common to model the heating demand in a linear manner as a function of the outdoor and indoor temperatures, proportional to the number of degree days. These are based on the integral of the differences of indoor and outdoor temperatures over time, as long as the indoor temperature is higher than the outdoor temperature. Such linear relations between energy usage and indoor and outdoor temperatures make it easy to aggregate and average the usage over time. For example, the average energy usage over some time period can be determined on the basis of average indoor and outdoor temperatures over this period.

For an air-to-water heat pump as considered here, the energy usage also depends on the energy demand and through that on the indoor and outdoor temperatures, similar to the traditional cases of heating. However, an important difference is that also its efficiency of heating (expressed as its Seasonal Performance Factor, SPF) depends on a number of dynamic situational factors such as the outdoor temperature, humidity and the frequency at which the compressor is working. Among these, the outdoor temperature has the most important effect on SPF. Therefore in the overall relation between energy usage and temperatures, these temperatures have a double, nonlinear effect on energy usage: on the one hand through the energy demand expressed via degree days, and on the other hand through the performance factor. As a consequence, taking averages over longer time periods cannot be used as they do not provide adequate estimations. For every occurrence of an ambient temperature the energy usage has to be calculated separately, for example, by simulations per hour or per day.

In this paper, six different variants of computational models for air source heat pump performance are considered. Each of these models has a number of parameters that have to be estimated for a given situation. A data set was obtained from the Website www.liveheatpump.nl. Using simulation over hours and a parameter estimation approach to tune the values for the parameters, the most optimal values of its parameters were determined for each of the models. As error function, the square root of the average of the squares of the deviations for all time points was used. The remaining error after this parameter tuning was used as a comparison measure for the models.

In the paper, first in Section 2 some background theory on heat pumps is presented. Section 3 describes the used dataset. In Section 4, the computational models are presented. Next, in Section 5 it is shown how parameters of models have been estimated based on numerical techniques and empirical data. This provides well-tuned models of the performance of the heat pump in the given house. In Section 6, the results are compared. Section 7 includes a discussion and future directions. The last section provides conclusion.

## 2. Background

### *2.1. Heating by Renewable Energy: Heat Pumps*

In colder areas a substantial amount of domestic energy usage during the winter season is spent on (space) heating of the house; e.g., [8]. Traditional heating systems are usually based on non-renewable resources such as gas, oil, and coal, which are not sustainable as only limited amounts of them are available. Besides, they have serious negative effects on the environment and climate. Therefore alternative domestic heating systems get much attention nowadays, such as the use of a heat pump (e.g., [3–7]). A heat pump takes thermal energy from the environment (from air, water or soil) and uses this to heat the water of a central heating system in the house. To do this, it uses an amount of electrical energy to run the heat pump, which is only a fraction of the provided heating energy. Since surface water (like a lake or a river) is not available everywhere, and installing ground source heat pumps needs a big investment, many domestic heat pumps harvest their energy from the air (air source, or air to water heat pump). This is the type of heat pump considered in this paper. An important issue here is that on the coldest days, when heating needs most energy, the air temperature is low, and due to that an air to water heat pump becomes less efficient in use.

The performance of a heat pump is described by the seasonal performance factor (SPF), which indicates how much electrical energy (in kWh) is needed (to run the heat pump) as input to get a certain amount of heating energy as output for the heat pump over a certain time period or a season (e.g., [6, 7]):

$$SPF = \frac{energy\_provided}{energy\_used} \quad (1)$$

This performance factor mostly varies between 2 and 5 (e.g., for outdoor temperatures between -5 °C and 15 °C); e.g., [5]. Often it is between 3 and 4 (e.g., for ambient temperatures between 2 °C and 10 °C). It strongly depends on the outdoor temperature, and in particular on the difference between the outdoor temperature and the water temperature of the heating system. Manufacturers often only give indications of these performance factors for just a few water and outdoor temperatures. For example, the value of SPF for outdoor/water temperatures 7/35 or 7/45. Such values for SPF are usually based on theoretical analyses or lab experiments. However, such theoretical analyses are often not guaranteed to provide values that occur in realistic situations.

### *2.2. Effects of humidity on performance*

The performance of heat pumps is affected not only by the outdoor air temperature, but by humidity as well. However, it has different effects in different conditions.

By cooling the air, the heat pump will extract heat. Cooling requires the coil temperature to be lower than the ambient air temperature. When the air temperature is lower than 6 °C, the coil can start freezing, and as a result ice will accumulate. The dryer the air, less ice accumulates; at 100 % humidity (fog) and 0 °C air temperature the, ice accumulation will be high. At -10 °C the air is much dryer, so then almost no ice will accumulate. When the air moisture content is high and temperature is lower than 6 °C, frost formation on the surface of the airside heat exchanger may occur. However, when the temperature is lower than -3, usually, the humidity of ambient air is low, and this phenomenon does not take place. Due to frost formation, the heat pump performance can be reduced significantly [9]. Therefore, the frost needs to be removed periodically to improve the efficiency of operation. Defrost cycles are typical for air sourced heat pumps; they are fully automated and will remove the ice by inverting the thermodynamic process, the heat pump will go in cooling mode. By doing this the outdoor coil will become warm and the ice will melt. The required heat is extracted from the indoor system; in many cases the indoor unit will use an electric heater to supply enough heat. However, the units focused in this paper use the heat in their built-in small water tank to defrost the outdoor unit, in this way the negative effect of defrosting on energy consumption is reduced dramatically. However, during defrosting cycles, the heat pump should use electricity just to remove the ice, and not produce heating energy, so the performance of heat pump during that interval drops significantly.

In contrast, for the higher temperatures, the higher the humidity content in the air, the more heat energy available in the ambient air. In these cases, it is easier to extract more heat from the ambient air, so the performance of heat pump is higher in the more humid situation (with the same temperature). However, this is something that just happens in higher temperatures.

## 3. Dataset

Eight different sets of real world data have been used to tune the parameters of the considered models.

Table 1. Overview of used data set.

| Location | Country | From | To | # of Samples | Minimum $T_{out}$ | Maximum $T_{out}$ | |
|---|---|---|---|---|---|---|---|
| Brecht* | Belgium | May-2011 | Nov-2014 | 4038 | 2,01 | 34,88 | In this location, a heat pump is used in a swimming pool, not open during winter. |
| Dikkebus | Belgium | Sep-2010 | Nov-2014 | 14433 | -14,43 | 19,72 | |
| Koolskamp | Belgium | Sep-2012 | Nov-2014 | 9562 | -11,12 | 19,23 | |
| Laar | Belgium | Nov-2011 | Nov-2014 | 17482 | -13,54 | 24,44 | |
| Lembbeek | Belgium | Oct-2011 | Nov-2014 | 10359 | -11,44 | 22,44 | |
| Mormont | Belgium | Sep-2010 | Nov-2014 | 11082 | -17,62 | 28,41 | |
| Schiermonnikoog | Netherlands | Sep-2010 | Nov-2014 | 27721 | -12,92 | 26,06 | This house is located on an island with weather of high humidity |
| Wijnegem | Belgium | Oct-2011 | Nov-2014 | 15316 | -11,39 | 32,1 | |

The empirical data for hourly performance characteristics and outdoor temperature of houses can be found at www.liveheatpump.nl. Table 1 contains some general information about the characteristics of dataset of each house; and Fig.1 visualizes the available date for each location in separated graphs.

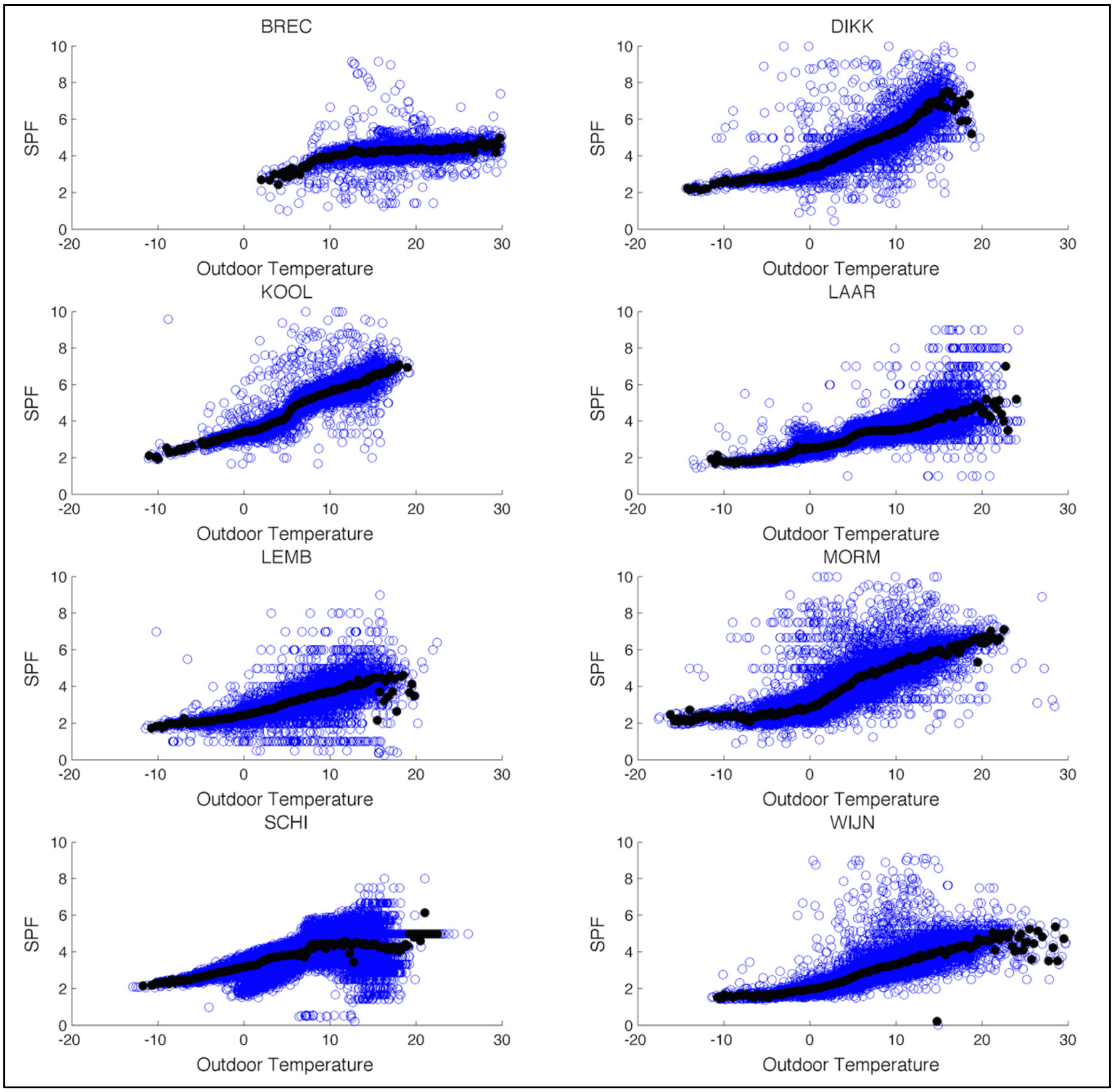


Fig. 1. Hourly value of SPF for different locations. Blue points show the SPF-temperature in one hour. Black points show the average of SPFs for a small interval of temperature.

In Fig. 1, each blue point shows the outdoor temperature vs. resulted performance of the heat pump for an hour.

The effect of the defrosting cycle (see Section 2.2) is visible in Fig. 1, especially in the graph of location SCHI (location with high humidity). In this graph, there are drops of SPF in many cases with temperatures in the range of (-3, 6). In addition, it can be seen, in most cases, the graph is thicker on the higher temperatures. The reason is that in the higher temperature, higher humidity leads to higher SPF (please see section 2.2). However, the humidity in hot days has a big variance, which leads to a large variance also for SPF.

## 4. Different Models for Performance

To model the electricity usage of a heat pump in all different circumstances in a year, with all its variations in outdoor temperature, it is needed to have a more systematic estimation of SPF in a realistic context. An extra complication is that in most cases the water temperature used for space heating is also variable depending on circumstances, by using a dynamic set point for the water temperature which is higher when the outdoor temperature is lower or when the indoor temperature set by the thermostat is higher. Often such values for the space heating water temperature are difficult to predict.

An approach that can be followed is to take empirical data from realistic domestic contexts (real family houses) as a point of departure and make an approximation of SPF as a mathematical function of the outdoor temperature. The parameters of such approximations can be estimated from a real world empirical data set (such as the one explained in Section 3).

In this paper, six different types of functions are evaluated and compared: four polynomial functions, a function inspired from ideal performance and also an exponential function.

### *4.1. Polynomial*

In some articles, for the sake of simplicity it is assumed that performance of a heat pump linearly depends on the outdoor temperature. For instance, in [10], it is assumed as a rule of thumb that by a one-degree increase in outdoor temperature, the performance of a heat pump will decrease for 0.1. The general format of such a linear function is:

$$SPF = A + BT_{od} \tag{2}$$

However, to get a more precise approximation, in some cases (like [11]) a quadratic function is used. In this case, the second-order derivative of the performance relative to outdoor temperature plays an important role. This has the following format:

$$SPF = A + BT_{od} + CT_{od}^2 \tag{3}$$

In this paper, the cubic and forth degree polynomial functions are investigated as well to find out the advantages and disadvantages of modeling the performance with a higher degree polynomial functions:

$$SPF = A + BT_{od} + CT_{od}^2 + DT_{od}^3 \tag{4}$$

$$SPF = A + BT_{od} + CT_{od}^2 + DT_{od}^3 + ET_{od}^4 \tag{5}$$

### *4.2. Inspired from Ideal Performance*

A heat pump is subject to the limitations from the second law of thermodynamics as any other heat engine and therefore an ideal maximum efficiency can be calculated from the Carnot cycle. This provides the function:

$$SPF = \frac{T_w}{T_w - T_{od}} \tag{6}$$

Here $T_w$ indicates the temperature of the water used in the heating system in the house. Please note that in this formula, the temperatures should be in values of Kelvin ([K] = [°C] + 273.15).

Of course, such a theoretically maximal performance factor is not realistic in practice; multiplying it by a factor makes it more realistic [12]. It is assumed here that a more realistic function can be obtained as a linear function of the ideal curve. Thus the rational function type considered here is:

$$SPF = A + B\frac{C + 273.15}{C - T_{od}} \tag{7}$$

### 4.3. Exponential Model

In [13] it is suggested that an exponential function can be used to model the performance of heat pump. So, the next investigated model is an exponential one:

$$SPF = A + Be^{cT_{od}} \tag{8}$$

## 5. Tuning the Models to Empirical Data

This section explains how the parameters of models have been estimated for a given situation.

### 5.1. Sampling

As it is clear in graphs of Fig. 1, for each location there are just few cases with a very low temperature, but there are many hours with higher temperatures, and they do not have a uniform distribution. As a result, if all points from the data are used, the parameter tuning is more accurate for the temperatures with more points. However, because of high amount of usage in cold hours, they are non-negligible points.

To solve this problem, the range of temperatures has been partitioned into intervals of 0.25 °C; the average SPF is calculated for each interval; the average points for the intervals are depicted in black circles in Fig. 1. Finally, the parameter tuning process is based on these averages. In this way, the models are fitted to a uniform distribution of points and all possible temperatures have the same influence.

### 5.2. Determining the parameter values

To tune the parameters of the models in realistic ranges, some limitations were set on parameters of models:

| | | |
|---|---|---|
| Linear | $A > 0$ | $B > 0$ |
| Quadratic | $A > 0$ | $B > 0$ |
| Rational | $C > 20$ | $C < 50$ |
| Exponential | $A+B>0$ | $A+B<10$ |

See Section 2 for the meaning of these parameters. For example, the limitations for C are present because it refers to water temperature, which should always be substantially higher than the indoor temperature. For each model, the Trust Region Algorithm from [14] was used to minimize the sum of square error (SSR) between the model and real data points of each house. To get rid of the local optimum points, this process was conducted several times for each model and the parameters with the best result (lowest SSR) are reported.

## 6. Results

Table 2 shows the average error for different models (after parameter tuning) and each location. As it is expected, in all houses, the $4^{th}$ degree polynomial model leads to the minimum error. However, the order of other models is not fixed. For instance, in location Dikkebus, exponential model is the worst; but in Brecht, exponential is one of the best ones.

Table 2. Best set of parameter values leading to minimal sum of squares of errors for each model.

| Location | Model | Error | Location | Model | Error |
|---|---|---|---|---|---|
| Brecht | Linear | 0,22678 | Lembbeek | Linear | 0,2199 |
| | Quadratic | 0,16273 | | Quadratic | 0,22713 |
| | 3rd Degree | 0,12365 | | 3rd Degree | 0,14465 |
| | 4th Degree | 0,11874 | | 4th Degree | 0,12365 |
| | Rational | 0,27945 | | Rational | 0,26796 |
| | Exponential | 0,13151 | | Exponential | 0,22607 |
| Dikkebus | Linear | 0,35148 | Mormont | Linear | 0,34962 |
| | Quadratic | 0,19599 | | Quadratic | 0,27458 |
| | 3rd Degree | 0,1974 | | 3rd Degree | 0,16152 |
| | 4th Degree | 0,15046 | | 4th Degree | 0,15477 |
| | Rational | 0,22652 | | Rational | 0,32691 |
| | Exponential | 0,3539 | | Exponential | 0,29602 |
| Koolskamp | Linear | 0,17029 | Schiermonnikoog | Linear | 0,19505 |
| | Quadratic | 0,1486 | | Quadratic | 0,17404 |
| | 3rd Degree | 0,11148 | | 3rd Degree | 0,17227 |
| | 4th Degree | 0,10638 | | 4th Degree | 0,14049 |
| | Rational | 0,20251 | | Rational | 0,30788 |
| | Exponential | 0,1723 | | Exponential | 0,17657 |
| Laar | Linear | 0,16361 | Wijnegem | Linear | 0,27678 |
| | Quadratic | 0,15675 | | Quadratic | 0,2753 |
| | 3rd Degree | 0,14729 | | 3rd Degree | 0,15283 |
| | 4th Degree | 0,14178 | | 4th Degree | 0,15471 |
| | Rational | 0,22482 | | Rational | 0,44508 |
| | Exponential | 0,16473 | | Exponential | 0,27718 |

As an example, Table 3 shows the tuned parameter of each model for location Dikkebus; and Fig. 2 shows the resulting curves of these models.

To be able to have an overview of the accuracy of each model, the following bar chart (Fig. 3) shows the average error (over all 8 locations) of each model.

Table 3. Best set of parameter values leading to minimal sum of squares of errors for each model in location Dikkebus.

| | Model | A | B | C | D | E | Error |
|---|---|---|---|---|---|---|---|
| Dikkebus | Linear | 3,8075 | 0,16556 | | | | 0,35148 |
| | Quadratic | 3,5478 | 0,1474 | 0,0033042 | | | 0,19599 |
| | 3rd Degree | 3,4667 | 0,1777 | 0,0049331 | -0,00021579 | | 0,1974 |
| | 4th Degree | 3,3029 | 0,14408 | 0,011359 | 8,71E-05 | -3,21E-05 | 0,15046 |
| | Rational | -3,2418 | 1,0537 | 50 | | | 0,22652 |
| | Exponential | 321,8523 | -318,0411 | -0,00052089 | | | 0,3539 |

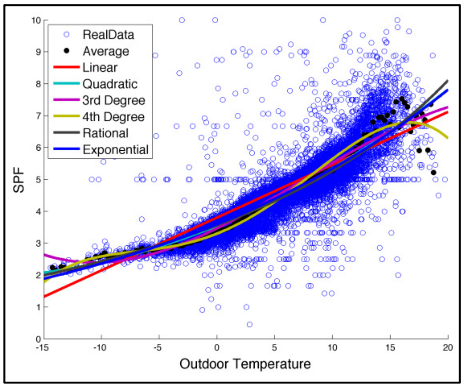


Fig. 2. Resulted curves of each model after setting the parameters according to the real data (location Dikkebus).

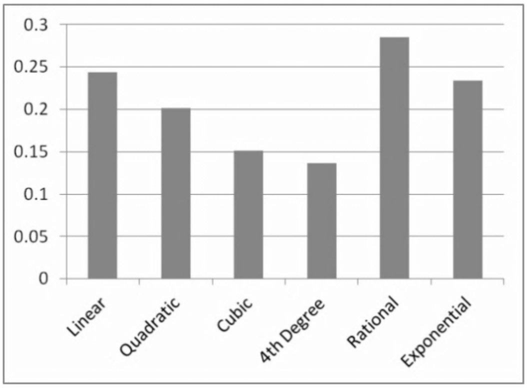


Fig. 3. Average of error for different models.

## 7. Discussion and Future Work

Because of the unique ability of heat pumps in transferring the available heating energy in the ambient environment into the buildings, they are used more and more in houses as their main heating system. Because of this property, the performance of heat pumps is usually much higher than other common heating systems. An important characteristic of air-to-water heat pumps is that their performance depends much on the outdoor temperature.

On the other hand, to use the future smart power grids, prediction about the amount of energy usage in different sectors has an important role for the efficiency of the whole grid. Therefore, as a matter of fact, by having an accurate model to predict the performance of the heat pump, it is more possible to estimate its energy usage in future hours or days. Moreover, it will be possible to control the heating system more smartly.

In this paper, we have explained about different computational models for the performance of such heat pumps depending on outdoor temperature. Different models have been analyzed and evaluated against an extensive dataset. There are some differences in the resulting errors. In general, the fourth degree polynomial model shows the best results, followed by the cubic model. By looking at Fig. 3, it is clear that increasing the complexity of model from cubic to a $4^{th}$ degree polynomial, just has a small effect on the error. On the other hand, even this small improvement in the error may be because of over fitting of the model to the data. So, as a next step, training data can be separated from data in test set. And, by using the techniques like cross validation, we can find the optimum polynomial function which leads to minimum error, but not with the possibility of an over fitting problem.

Moreover, in future work, the effect of the humidity on the performance of heat pump will be analyzed; and also the average and variance of amount of usage in different temperatures will be investigated.

## 8. Conclusion

In this paper, different computational models for the performance of a heat pump depending on outdoor temperature have been assessed based on a large dataset. The outcome is that, overall, the quadratic and higher (third and fourth degree) order polynomial models provide the most accurate results (see Fig. 3), they are much more accurate than the often-used linear model. However, for specific cases other types of models can also be almost as accurate (see Table 2), for example, the exponential model or the broken rational model.

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