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Impact of Gravity on Fluid Mechanics Models

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Abstract

Mathematical formulae which describe fluid mechanics models include the influence of gravity. In the literature and practice, when considering numerical hydraulic models, gravity is taken as constant value ("gravitational constant"). Actually, gravity is not constant and it is changing depending on mass distribution into the body of the Earth, mass density, altitude and topography (relief shape and mas density above the geoid). This paper is focused on the gravity influence on the different hydraulics models and fluid mechanic formulae in order to point out that gravity acceleration should not be treated routinely as "constant".

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1. Introduction

Earth gravity field covers all around the planet and all phenomena on earth are influenced by it. This fact is stated through many hydraulics models which contain gravity as one of parameters which determine the fluid behavior. However almost all hydraulic formulae and models which describe water flows and hydraulic structures consider gravity as constant and its' value is often adopted without accurate determination for certain area. This approach is a consequence of fact that changes of gravity field are small and that influence on wholesale hydraulic models caused

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by error of gravity acceleration were not too significant. None of these reasons could be justified on the today's level of technology development because the gravity acceleration is not highly resource-demanding as it was before (some methods for approximate gravity acceleration determination exists which could provide satisfactory accuracy for some application but measurement of gravity acceleration is not too expensive and does not increase the price of projects significantly if high accurate is necessary).

Gravity acceleration is not constant value and it changes in time which means that calculation made on base of gravity in one moment of time do not have to be the same in other moment of time on the same location (i.e. on the same) point of earth surface. Variation of gravity which is resultant of gravitation and centrifugal forces depending on distribution of masses inside Earth as well as the Earth's position respective to Sun and Moon. Although the influence of Sun and Moon is relatively small it points out the fact that earths' gravity field is changeable with time and that could not be treated as constant. Continuous changes in earth crust and its' interior, distribution of masses continuously changes which affects the Earths' gravity field. Aforementioned reasons have a consequence that Earths' gravity field is not constant in space and time. This fact could be simply expressed by formulae:

$$g = g(x, y, z, t) \tag{1}$$

$$\frac{\partial g}{\partial x} \neq 0; \frac{\partial g}{\partial y} \neq 0; \frac{\partial g}{\partial z} \neq 0; \frac{\partial g}{\partial t} \neq 0 \tag{2}$$

where:

- g – gravity
- x – x – coordinate in Cartesian 3D World coordinate system;
- y – y – coordinate in Cartesian 3D World coordinate system;
- z – z – coordinate in Cartesian 3D World coordinate system and
- t – time.

Gravity is mostly the field of interest of two scientific disciplines, physical geodesy and geophysics [1]. The subject of physical geodesy is the study of gravity field and the figure of the earth [2]. Geophysics could be defined as application of physics to study the interior of Earth [3]. Earth's gravity field exploration has different function when is considered from aspects of two scientific disciplines. From the aspect of physical geodesy the Earths' gravity field is researched with aim to determine geoid (mathematical surface which describes planet Earth and whose every point is perpendicular to the direction of gravity force), while from the aspect of geophysics the main aim of gravity field determination is to find out the Earths' interior. The fact that, at least, two scientific disciplines have the gravity field as their subject, that significant resources are involved in those researches, that a number of scientific papers are devoted to those topics, that a number of technologies and methodologies (terrestrial as well as satellite) for gravity field determination and also practical results obtained by these researches imply the need to investigate the influence of gravity on hydraulic models of river flows.

The research of gravity influence on certain models will be performed by analysis of those models as a function of their arguments, whereby the function shall be linearized and after that the increment of function will be considered depending on the increment of its' arguments. In this paper only terms of first order will be considered.

2. Methodology

Hydraulics models of river flows and models which describe hydraulic structures in many cases are described by formulae based on model, numerical and empirical research, [11-13]. All these researches are based on measurements which contain unavoidable errors and whose influence is propagating through models and affects output quantities. Errors propagation through certain hydraulic model is depending on the form of formulae which describe observed hydraulic phenomenon.

Errors of measurement are unavoidable and they are the consequence of different sources. Sources of errors could be: accuracy of used instruments, applied measurement methodology, external conditions, personal qualities of the surveyor (experience, knowledge) and other influences which were not identified during the measurement.

Formulae which describe certain hydraulic models often contain some coefficients which were determined through empirical research. These coefficients represent values obtained from limited sample and under certain conditions. From this immediately follows that they are valid only for conditions under which they were determined and that in different conditions some deviations in their values could appear. Empirical coefficients also are rounded on the certain number of decimals which means that rounding error also exists and influence output quantity. The consequence of rounding error is that empirical coefficients in hydraulic models also could be considered as erroneous values i.e. that they actually are not a constant. Mathematically it could be expressed in following way:

$$\epsilon' \neq 0 \tag{3}$$

or

$$\epsilon \in [\epsilon - |\Delta\epsilon|, \epsilon + |\Delta\epsilon|] \tag{4}$$

where ϵ is empirical coefficient, ϵ' is first derivative and $\Delta\epsilon$ is possible deviation of empirical coefficient.

Error propagation also depends on the shape of the formula which describes it and on initial conditions i.e. of measured or adopted values of parameters. The simplest explanation is that the error propagation shall not be the same for quadratic and logarithmic function and that some errors will not affect in same way the function value when the argument is close to zero and if it is around some other value.

Concisely is it possible to say that output quantities (searched values) are burdened with different errors as a consequence of measurements errors (measured values deviations from its real value), the initial values of parameters and the form of applied model. Aforementioned facts justify consideration of hydraulic models' parameters as variables and analysis of these influences in model.

Analysis of hydraulic models in this paper is based on linearization of functions around initial values of arguments and on analysis particular influences through variation increments of arguments. After that the variation of output quantities are considered. For following analysis the usual and well known mathematical models will be used.

Considering function dependent of n arguments:

$$\Psi = \Psi(x_1, x_2, \ldots, x_n) \tag{5}$$

then is possible, for small increments of its parameters in certain point, to approximate its value in initial point increased for its increment defined by first order derivative:

$$\Psi \approx \Psi(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) + \sum_{i=1}^{n} \frac{\partial\Psi}{\partial x_i}\Delta x_i \tag{6}$$

From formula (6) follows that first order derivative reads

$$\Delta\Psi = \sum_{i=1}^{n} \frac{\partial\Psi}{\partial x_i}\Delta x_i \tag{7}$$

Insofar as increments of functions' arguments are considered as spans of interval where the difference between initial and real values of arguments (or maximal measurement error) belongs then it is possible to determine their particular influence on functions' increment.

According to low of error propagation the root mean square error for formula (7) shall read:

$$m_{\Delta\Psi} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial \Psi}{\partial x_i} m_{\Delta x_i} \right)^2} \tag{8}$$

where $m_{\Delta x_i}$ are the root mean square errors of increments Δx_i. Here we assume that Δx_i are adopted i.e. exact values and consider only increment of function dependence on increments of its arguments.

The inverse problem i.e. determination of arguments values increments when increment of function is given could be solved under condition that every term in formula (7) has equal influence. Then formula (7) can be written in form:

$$\Delta\Psi = n\delta \tag{9}$$

where

$$\delta = \frac{\partial \Psi}{\partial x_1} \Delta x_1 = \frac{\partial \Psi}{\partial x_2} \Delta x_2 = \cdots = \frac{\partial \Psi}{\partial x_n} \Delta x_n \tag{10}$$

Bearing in mind (7), (9) and (10) immediately follows:

$$\Delta x_i = \frac{\Delta\Psi}{n} \frac{1}{\frac{\partial \Psi}{\partial x_i}} \tag{11}$$

Particular consideration in this paper will be devoted to influence of gravity on some hydraulic models. Gravity force is contained in numerous hydraulic models and mostly is treated as a constant. As stated before this is not a case both in space and time, adopting approximate value could have significant influence on searched values. Gravity acceleration determination on the today's development level of measurement technologies (terrestrial and satellite) is possible with high accuracy. For some models, however, it is possible to use simple models for gravity acceleration determination which could also provide satisfactory accuracy.

Normal value of gravity acceleration is possible to determine on the base of International Gravity Formula 1980 [1] depending on the choice of ellipsoid, latitude and ellipsoidal height of point. Geodetic Reference System 1980 has been adopted at the XVII Assembly of the IUGG in Canberra, December 1979, because the International Union of Geodesy and Geophysics recognized the Geodetic Reference System 1967 at the XIV General Assembly of IUGG, Lucerne, 1967, "no longer represents the size, shape, and gravity field of the Earth to an accuracy adequate for many geodetic, geophysical, astronomical and hydrographic applications and considering that more appropriate values are now available" [4]. The formula for normal gravity is given by the means of conventional series:

$$\gamma = 9.780327(1 + 0.005\,3024\, sin^2\Phi - 0.000\,0058\, sin^2 2\Phi)\mathrm{ms}^{-2} \tag{12}$$

where normal gravity is denoted by γ and latitude is denoted by Φ.

Formula (9) has an accuracy of $1\mu\mathrm{ms}^{-2}$=0.1 mGal. The normal gravity γ belongs to the interval of ($9.780327\mathrm{ms}^{-2}$; $9.832186\mathrm{ms}^{-2}$) when$\Phi \in [0^o, 90^o]$. Average of normal gravity over ellipsoid is $\bar{\gamma} = 9.797\,644\,656\ \mathrm{ms}^{-2}$ and $\gamma_{45} = 9.806\,199\,203\ \mathrm{ms}^{-2}$ at latitude $\Phi = 45^o$. According to literature [5] the candidate locations for extreme values of gravity acceleration are $9.76392\ \mathrm{ms}^{-2}$ at Huascaran, Peru (Φ=-9.12^o, Λ=-77.60^o) - minimum value and $9.83366\ \mathrm{ms}^{-2}$ at Arctic Sea (Φ=86.71^o, Λ=61.29^o) - maximum value, which means that variation range of gravity acceleration on Earth is about $0.07\ \mathrm{ms}^{-2}$.

These values shall differ from the real values depending on the mass distribution inside of the planet Earth and the masses over the geoid [6]. The complexity of the topic caused many researches and measurements of gravity for determination of geoid. Fig. 1 illustrates the shape of geoid [7] and its relationship with ellipsoid, where red color means that geoid is over ellipsoid and blue the opposite situation i.e. that geoid is under ellipsoid. Height deviations of geoid from ellipsoid are approximately ±100 m.

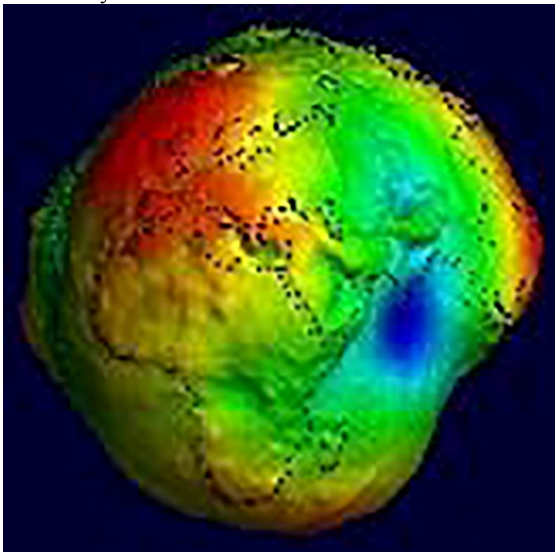

Fig. 1. Shape of geoid

Deviations of real gravity acceleration from normal values could be significant and imply that, in some cases, its value shall be determined very accurately. Bearing in mind pervious considerations it is possible to conclude that gravity is not constant and it could take values approximately $\gamma \in (\gamma_{45} - 0.026\ \mathrm{ms}^{-2}, \gamma_{45} + 0.026\ \mathrm{ms}^{-2})$ or $\gamma \in (\bar{\gamma} - 0.017\ \mathrm{ms}^{-2}, \bar{\gamma} + 0.035\ \mathrm{ms}^{-2})$ for its normal values.

Also the changes of gravity caused by ellipsoidal height (distance between point and ellipsoid) shall be included in normal gravity determination. These changes are given by formula [1]:

$$\delta g_{h1} = 0.3086h\ \mathrm{mGal} \tag{13}$$

where h is ellipsoidal height of point in meters and mGal=10^{-5} ms^{-2}.

This paper will consider the influence of arguments increment on numerical models of bed shear stress and ogee spillway with emphasis of gravity influence.

3. Results

In this paper, the following considerations of models for bed shear stress [8] and ogee spillway [9] are performed according to described methodology.

Model for bed shear stress is:

$$\tau = \rho g h I \tag{14}$$

where:

- τ – bed shear stress;
- ρ – density of water;
- h - water depth and
- I – slope of the water surface.

Applying formula (6) on formula (8) we get:

$$\tau = \rho_0 g_0 h_0 I_0 + g_0 h_0 I_0 \Delta\rho + \rho_0 h_0 I_0 \Delta g + \rho_0 g_0 I_0 \Delta h + \rho_0 g_0 h_0 \Delta I \quad (15)$$

Increment of function τ due to increment of arguments (or their errors) has following form:

$$\Delta\tau = g_0 h_0 I_0 \Delta\rho + \rho_0 h_0 I_0 \Delta g + \rho_0 g_0 I_0 \Delta h + \rho_0 g_0 h_0 \Delta I \quad (16)$$

When initial values and limit increment for Δτ are given it is possible to determine intervals for every argument's increment according to formula (7).

For given values of bed shear stress increment (last column in table 1) the maximum values of uncertainty are shown in table 1.

Table 1. The impact of uncertainties of arguments determination for given bed shear stress function uncertainty $\Delta\tau$

ρ_0 [kg/m^3]	g_0 [m/s^2]	h_0 [m]	I_0	$\Delta\rho$	Δg	Δh	ΔI	$\Delta\tau$
999	9.805	5	0.005	2.550	0.02503	0.013	0.0000127	2.5
999	9.805	5	0.005	2.040	0.02002	0.010	0.0000102	2.0
999	9.805	5	0.005	1.530	0.01502	0.008	0.0000076	1.5
999	9.805	5	0.005	1.020	0.01001	0.005	0.0000051	1.0
999	9.805	5	0.005	0.510	0.00501	0.003	0.0000025	0.5

On the base of results given in table 1 it could be noted that allowed interval of gravity is smaller than its real variation. That implies that there exist cases when the gravity acceleration could not be treated as "constant".

Model for ogee-spillway is:

$$Q = \frac{2}{3} C_0 L \sqrt{2g} H_e^{\frac{3}{2}} \quad (17)$$

where:

- Q – total discharge;
- C_0 – discharge coefficient;
- L – lateral crest length or width;
- g – gravity acceleration ("gravitational constant" [9]) and
- H_e – total head upstream from the crest.

Applying formula (6) on formula (17) we get:

$$Q = \frac{2}{3} C_0 L_0 \sqrt{2g_0} H_{e0}^{\frac{3}{2}} + \frac{2}{3} L_0 \sqrt{2g_0} H_{e0}^{\frac{3}{2}} \Delta C_0 + \frac{2}{3} C_0 \sqrt{2g_0} H_{e0}^{\frac{3}{2}} \Delta L + \frac{2}{3} C_0 L H_{e0}^{\frac{3}{2}} \frac{1}{\sqrt{2g_0}} \Delta g + \frac{2}{3} C_0 L_0 \sqrt{2g_0 H_{e0}} \Delta_{H_e} \quad (18)$$

Increment of discharge function Q which is consequence of arguments' errors reads:

$$\Delta Q = \frac{2}{3} L_0 \sqrt{2g_0} H_{e0}^{\frac{3}{2}} \Delta C_0 + \frac{2}{3} C_0 \sqrt{2g_0} H_{e0}^{\frac{3}{2}} \Delta L + \frac{4}{3} C_0 L H_{e0}^{\frac{3}{2}} \frac{1}{\sqrt{2g_0}} \Delta g + C_0 L_0 \sqrt{2g_0 H_{e0}} \Delta_{H_e} \quad (19)$$

For given increment of total discharge (ΔQ) the values for the parameters ΔC, ΔL , Δg and ΔH_e are given in table 2.

Considered case highlights that, for given initial values in considered ogee-spillway case, variation interval is significantly smaller than possible variation of gravity acceleration on Earth. This case also justifies the state that gravitational acceleration is not a "constant" on planet Earth.

Table 2. Increment of discharge function for adopted initial values and increments of its arguments

C_0	L_0 [m]	g_0 [m/s^2]	H_{e0} [m]	ΔC	ΔL	Δg	ΔH_e	ΔQ
0.8	14	9.805	10	0.00096	0.01674	0.01172	0.008	5
0.8	14	9.805	11	0.00083	0.01451	0.01016	0.008	5
0.8	14	9.805	12	0.00073	0.01273	0.00892	0.007	5
0.8	14	9.805	13	0.00065	0.01129	0.00791	0.007	5
0.8	14	9.805	14	0.00058	0.01010	0.00708	0.007	5
0.8	14	9.805	15	0.00052	0.00911	0.00638	0.007	5

Bearing in mind that gravity acceleration could be measured with very high accuracy [10] and according to formula (9) immediately follows that, if gravity acceleration is measured (even with two order smaller accuracy than given in [10]) its influence on increment of considered models would be disannulled. In that case formula (9) would have form:

$$\Delta\Psi = (n-1)\delta \tag{20}$$

and consequently, formula (11) will read:

$$\Delta x_i = \frac{\Delta\Psi}{n-1}\frac{1}{\frac{\partial\Psi}{\partial x_i}} \quad (i = 1 \sim n-1) \tag{21}$$

Formulae (20) and (21) shows that, measuring gravity acceleration allow greater uncertainties of left models' arguments or decrease uncertainty of models' total increment.

4. Conclusions

The impact of gravity acceleration participates in numerous models of hydraulic models and structures, but it is usually considered as a constant. Gravity acceleration, however, is not constant because it depends on numerous factors which also change with time.

In this paper a few examples for bed shear stress and ogee-spillway models were considered and it is shown there are cases for hydraulic models and structures where needed variation of gravity for obtaining given models' variation (total increment of function) is smaller than real variation of gravity acceleration. These cases suggest that gravity acceleration shall not be routinely treated as "constant".

Availability of data about gravity acceleration which are satisfactory accurate for some applications justify attitude, that every hydraulic model or structure shall be provided with adequate data of gravity acceleration for geographic location where certain hydraulic model or structure is located. Changes of gravity acceleration in time justifies its measurement (i.e. most accurately determination) because hydraulic models and structures are assumed to last for decades. If future experts are provided with adequate data they will have better insight in changes or events related to certain hydraulic model or structure over a long term.

Increasing accuracy of hydraulic model for impact of gravity acceleration make it possible to decrease the influence of other influences in hydraulic models.

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