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Enabling an Industrial Robot for Metal Cutting Operations

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Abstract

This paper focuses on a cost-effective manufacturing of large frame parts for aerospace industries with an industrial robot. The main challenge is the low stiffness of a serial kinematic, resulting in positioning errors due to gravity and cutting forces. Therefore, an approach is presented to optimize positioning of a robot by compensation of tool deflection. A static deflection model of the robot is built up to calculate the deflection caused by forces acting on the spindle. To detect these forces a suitable measurement device is presented. This sensing spindle holder is calibrated to detect cutting forces.

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Keywords: machining robot; process monitoring; compensation; force measurement

1. Introduction

Within the research project Innoflex, the weight reduction of aircraft parts by use of new materials like AlCuLi-alloy is analyzed. A further advantage of this material is that it can be extruded to a shape close to the final contour. This reduces the cutting volume and enables the reduction of cutting forces to an optimum for cutting with robots. This might enable an industrial robot for the machining task. Large frame parts today are machined on large gantry machine tools that are expensive. Because of its low price and large working area, more and more industrial robots are used for machining operations. Compared to conventional machine tools, robots are cheaper but not accurate enough to compete against them. Main reason for their poor positioning accuracy is their low static and dynamic stiffness due to high joint compliance and long arms. A non-accurate calibration of the load or changes in the load lead to positioning errors, which are pose dependent. Additional process forces acting on the robot structure lead to further displacements of the tool center point (TCP). Weigold presents these and other effects on the positioning accuracy [1].

Tests show, that the positioning accuracy as well as the trajectory accuracy, which includes dynamic effects, need to be improved for the use in cutting operations.

For this reason, much research has been done to optimize the positioning accuracy. Since gravity forces on the robot structure are pose dependent, they cause deviations of the end effector position without process forces acting on the robot. Eastwood and Webb [2] analyzed the effects of gravity and built a simulation model, which reduces at least 70 % of the mass-induced positioning error. Roth et al. divides robot calibration approaches into three levels [3]. Level 1 is a calibration of joint sensors and drives. The calibration of the kinematic transformation is defined as level 2. This kinematic calibration is used to identify geometry errors resulting from tolerances in the robot arms or their assembling. Hollerbach [4] presents an overview of different approaches of kinematic calibration. As an example Duelen and Schröer [5] describe a kinematic calibration and showed that this calibration can improve the absolute accuracy of the robot up to its repeatability. The third level of calibration is described as "non-kinematic" calibration, which summarizes all errors resulting from thermal effects, compliance of joints and links, backlash and friction in the gearing and compliance in the bearing of the joints as well as dynamic effects.

Today, the calibration in level one and two is done by robot manufactures as well as service providers who do a kinematic calibration to achieve higher positioning accuracy in a defined

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working space. Since, the International Organization for Standardization in September 2012 rejected the DIN EN ISO 9283, there are no standards for robot testing and calibration. The ISO 9283 defined important performance characteristics for robots and recommended tests to achieve them [6]. The standards are based on Schröer, who started an approach to define standard tests generalized for different industrial robots in 1998 [7].

While these approaches calibrate the position accuracy for the robot with constant payload, forces acting on the robot are not taken into account, even though measurements show deflections of up to 2 mm during cutting operation. Thus, the compliance of industrial robots is analyzed in different research projects. Abele et al. presented an approach with a Cartesian compliance map for an industrial robot [8]. Based on measurements of the joint stiffness, the compliance in Cartesian coordinate system is calculated. The approach presented by Nubiola and Bonev [9] and Klimchik et al. [10] deals with a robot manipulator calibration of an ABB IRB 1600 and a Kuka KR-270. The second approach focuses on a procedure for use in industrial environment while the first one is more complex to achieve a higher accuracy.

These results show that the stiffness of the industrial robot is the main reason for trajectory errors during milling operation. Because a force-prediction and offline compensation or a feed-forward control is not precise enough, these forces must be detected online. A force sensor can be placed on the side of the tool or on the side of the workpiece. Typically, a force dynamometer is used, which is placed between the working table and the workpiece. These systems are available on the market for several years and are mainly used in laboratory for cutting tests. Most of these dynamometers base on piezo sensors, which measure the forces or torque at the workpiece. Due to their small size the working area is limited. Furthermore, the manual effort for changing the workpiece is very high, which is one reason why they are not established in industrial application. Some research has been done the last years to implement force sensors to the machine side without limiting the working area and the machine operator during his work [11]. These sensors are mainly used for process monitoring tasks. The sensing device SPIKE by pro micron GmbH was developed to measure torque and axial force acting on the tool. This device uses strain gauges placed at a special tool holder to identify measured values from their signal. Because the sensing is placed on the rotating part, the data is transferred wireless to a monitoring system. Kistler Instrumente AG offers a rotating dynamometer Typ 9171A to their customers for force and torque measurement, which is placed in the tool holder of the spindle and offers a standard HSK tool holder shaft to make tool changes easier. Kistler Instrumente AG and the Institute for Machine Tools and Manufacturing at the ETH Zürich developed a sensing unit integrated in the spindle. Though the sensing device works in laboratory tests [12], it is not on the market, yet. All these measurement devices suffer from a low stiffness high additional weight and integration into the machine control for compensation task is complicated. The effort for integration of these systems is very high which might be a main reason why these systems are not used for compensation tasks.

An approach on how the compensation of compliance is done is presented in section 2. The stiffness of an industrial robot is analyzed with help of a modal analysis. Main results are presented in section 3. Based on the knowledge gained from these measurements, a simulation model was built up to calculate the displacement due to forces acting on the tool center point of the milling robot. Furthermore, the model is used in online compensation of trajectory errors. Therefore a force measurement device is needed. A new force-sensing unit is developed to attain more stiffness without limiting the working space. The force sensing spindle holder is presented in section 4. The calibration and validation of the sensing spindle holder, presented in section 5, is done for a six axes industrial robot.

The main challenge of industrial milling robots is their low positioning accuracy and low stiffness in the gears and bearings of the joints and the structure. Thus, the kinematic transformation cannot determine the tool center point (TCP) of the robot precisely without taking forces and torques acting on each axis into account. Cutting forces lead to a quasi-static force load on the robot. This static force has the main effect on the deflection of the TCP and it is possible to compensate this with the robot. A compensation of higher frequency parts of the cutting force is not realizable with the robot itself since the dynamic is limited. Thus a static compliance model is needed to simulate the deviation of the robot according to the force acting on the TCP. This model is used for offline trajectory programming to calculate the stiffest pose and the best position of the workpiece with help of forces from a cutting simulation. However, process forces might differ from simulation results, thus an online compensation is required that measures the actual force acting on the TCP to rise the trajectory accuracy.

The compensation approach presented in Figure 1 consists of two main components: the robot control and the sensing spindle holder. The robot control is a Siemens Sinumerik control connected to the original control KR C4 of the robot. Main task of these controls is the positioning of the milling spindle according to the programmed trajectory in workspace. The Sinumerik control gives a common machine user interface to the operator and enables g-code programming, known from conventional machine tools. The sensing spindle holder measures forces acting on the TCP and uses this information for a communication between the control and the compliance model.

The force measurement is implemented with use of strain gauges placed at the spindle holder. These strain gauges are

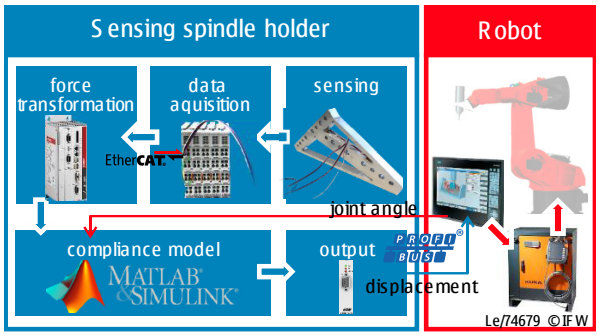

Figure 1: Compensation approach

connected to special precise strain gauge resistor terminals. With help of a measurement matrix the force is calculated from measured strain values with use of an industrial PC running a TwinCAT control. The calculated force is used for compensation of positioning errors resulting from compliance in the bearings and gears of the robot. Since the compliance of the robot depends on the pose of the robot the joint angles are required. To attain this information the industrial PC is connected to the robot control via a Profibus network. The calculated deviation is shared with the robot control to compensate trajectory errors. Since the task sample time of the robot control is 2 ms, the calculation time must be fast enough to complete each calculation in this time.

2. Modeling of robot positioning errors

The industrial robot is a complex, multi body structure. Thus, the main influences on the compliance are analyzed by a modal analysis and a measurement of the static stiffness. While the modal analysis shows which parts of the robot are compliant, the static measurements are used to identify the model parameters like stiffness, mass and center of gravity and thereby optimize the numerical model. The dynamic behavior mainly depends on the mass and the distance to the center of gravity. If the robot joints are positioned in a way that the robot is outstretched horizontally, the center of gravity of each part is located furthermost from its joints. This pose defines the worst case for positioning accuracy since the lever arm of gravity forces are very high. The aim of the modal analysis is to show the most compliant parts of the robot, which need to be identified and simulated for increasing positioning accuracy. Therefor a modal analysis is done at a horizontally outstretched robot. Two dominant eigenmodes of the robot result from compliance in the first joint. The first eigenfrequency is 5.5 Hz where the entire robot from the rotating column up to the wrist is rotating around the A1 axis. This vibration results from the compliance in the gear of the A1 axis. For each joint one mode shape and an according frequency can be identified which result from gear stiffness. The second eigenmode shows that compliance of the bearing leads to a tilting of each axis. This compliance is related to the mode shape at 36 Hz where the rotating column is tilting around an axis normal to the A1 axis. These mode shapes show that the joints do not have only one degree of freedom, as assumed ideally for all joints in the kinematic transformation. Furthermore, the modal analysis shows that the compliance of the bodies are negligible compared to the stiffness of the joints.

For an analytical description of the multi-body simulation approach, which is fast in calculation time for online compensation, a model was built up to calculate the direct kinematic of the machine. Usually the Denavit-Hartenberg notation [13] is used. This notation describes the geometric transformation with help of four parameters for each joint. These are two angles and two lengths describing the transformation from one coordinate system to the adjoined. For the compliance simulation the coordinate systems are defined according to this notation. With the Denavit-Hartenberg notation the transformation is fixed to only 4 degrees of freedom for each axis. For modeling of the robot compliance this is not enough since each joint has six-degrees of freedom. The position and orientation of each joint is described by use of transformation matrices. In general, a transformation matrix is notated as a rotation matrix R (3x3) and a translational vector t:

$$\boldsymbol{T}_i = \begin{bmatrix} & \boldsymbol{R} & & t \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

According to the notation of Denavit-Hartenberg for each joint four basic transformations are performed. These are a rotation around z-axis (θ), a displacement in z-direction (d), a displacement in the new x-axis (a) and a rotation around the new x-axis (α). Summarizing these four transformations for each joint i, leads to one transformation matrix

$$\boldsymbol{T}_i = \begin{bmatrix} c(\theta_i) & -s(\theta_i)c(\alpha_i) & s(\theta_i)s(\alpha_i) & a_i c(\theta_i) \\ s(\theta_i) & c(\theta_i)c(\alpha_i) & -c(\theta_i)s(\alpha_i) & a_i s(\theta_i) \\ 0 & s(\alpha_i) & c(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

Sine and cosine are abbreviated by s respectively c. If the coordinate system defined in the convention of Denavit-Hartenberg is not placed in the center of the joint and their bearing, a transformation to this position is required to calculate the forces and torques acting at the bearing and gear of each joint. A wrong transformation matrix leads to a wrong length of the lever arm for the forces and torques acting on the TCP or joints following on this axis. For the six-axis robot, six transformation matrices are to be defined, which describe the transformation from the TCP to each joint. For this paper, the transformation T_1 defines the transformation to the A6 axis as this is the first axis applied with force and torque acting on the TCP.

The joint stiffness defines six degrees of freedom and thus they are expressed as a vector of translational stiffness and rotational stiffness:

$$k_i = diag\left[k_x k_y k_z k_{rx} k_{ry} k_{rz}\right] \quad (3)$$

The stiffness of each joint can be defined with the jacobian matrix in a general stiffness matrix. For each joint a stiffness matrix can be defined as:

$$\boldsymbol{K_i} = \boldsymbol{J}_i^T \, k_i \, \boldsymbol{J}_i \quad (4)$$

Where J_i is the Jacobi matrix of joint i which is calculated with help of the transformation matrix T_i. The resulting stiffness matrix is calculated as the sum of each joint stiffness matrix.

$$\boldsymbol{K}_{sum} = \left(\sum_{i=1}^{6} \boldsymbol{K}_i^{-1} \right)^{-1} \quad (5)$$

The deviation is calculated by multiplication of the inverse of the stiffnes matrix and a generalized force vector.

$$x = \boldsymbol{K}_{sum}{}^{-1} \mathcal{F} \quad (6)$$

The generalized force vector $\mathcal{F}$ is defined as vector of the forces and torques acting on the TCP.

$$\mathcal{F} = \left[F_x F_y F_z M_{rx} M_{ry} M_{rz}\right]^T \quad (7)$$

The analytical compliance model uses the kinematic transformation described above and the actual joint position achieved by the robot controller to calculate the pose dependent Jacobi matrices. With this information and the force information received from the robot-control, the deviation is calculated.

3. The sensing end effector

The compensation of positioning errors, due to compliance in the joints of the robot, requires information about the force acting on the TCP. To realize a stiff and compact force measurement unit the spindle holder is designed with the help of an analytical static force simulation model. The influence of orientation and location of the spindle at the robot end effector to the deviation at the TCP is analyzed. Measurements on industrial robots show that the stiffness of the hand axes are, compared to the stiffness of the first three axes very low. Assuming this and mainly radial cutting forces acting at the TCP leads to an optimal orientation of the spindle of α=55° and a position where the point of applied force is close to the center of the last three axes. This is the best way of uniform force distribution to the joints and leads to less displacement than placing the spindle vertical (α=90°) to the end effector. The spindle orientation is presented in figure 2.

Forces acting on the TCP are measured indirectly by the local strain at the spindle holder, since this value is proportional to the force. Since the strain of a stiff structure is very low for cutting forces of up to 1000 N notches locally maximize the strain.

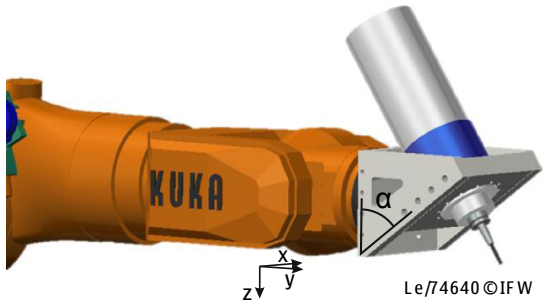

Figure 2: Position and orientation of spindle at end effector

With help of finite element method sensor positions are defined where the strain is high and forces in three Cartesian directions can be calculated independently from each other [14]. Simulations show that the side parts of the spindle holder are particularly suitable for sensor application. To achieve higher strain at suitable measurement positions pockets are designed with different positions and sizes. The original design and the design with pockets are shown in Figure 3. The stress and deformation are simulated and the best design of the pockets is evaluated. Aim of the design of experiment is to achieve a high strain at measurement position and at the same time a high stiffness of the spindle holder. Thus, small notches with a radius of 3 mm are placed in the side-parts of the spindle holder. Figure 3 compares the equivalent strain for a simple notch (Figure 3c) to a smaller notch where the sensing is placed on a bridge (Figure 3d). The results are achieved with 1 kN force load acting in x-direction of the hand coordinate system in figure 2 at the TCP. The simulation shows a two times higher sensitivity for the small bridged notch at a force load in x-direction.

This design is suitable for sensor application, since the local strain is high enough for force detection. Still the stiffness of the spindle holder in x- and y-direction are not affected significantly and the stiffness in z-direction is still higher than in the other directions. Table 1 shows the results for the compliance of the spindle holder in its original design without modification for sensing and the stiffness achieved with the modifications for sensing.

Six Strain gauges are used for force measurement, since they are able to measure the strain, which is proportional to the static

Table 1: Compliance of spindle holder at TCP

	Spindle holder [µm/N]	**Spindle holder with strain gauges** [µm/N]
x-direction	0.125	0.137
y-direction	0.137	0.168
z-direction	0.034	0.048

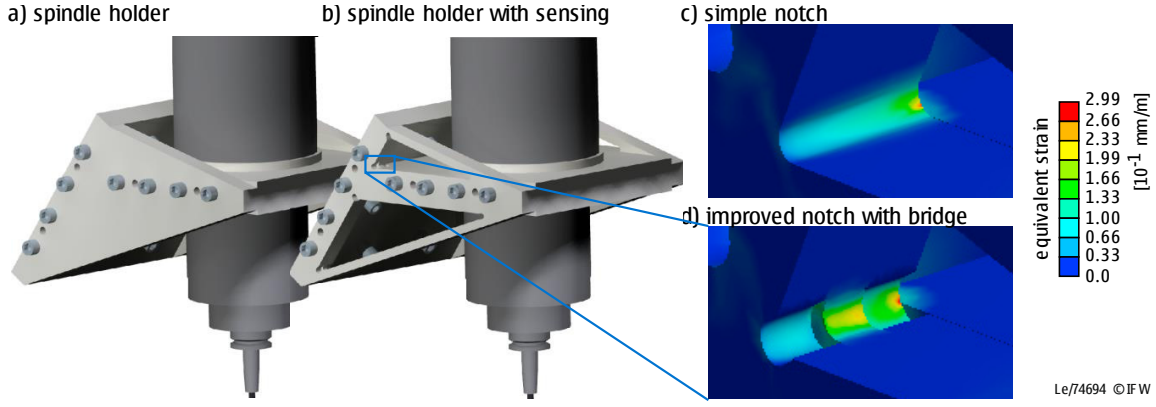

Figure 3: Equivalent strain at spindle holder in a notch (a) and small bridged notch (b)

forces, with high reliability. For each measurement position four strain gauges are connected to form a Wheatstone bridge circuit [15]. To detect a change in resistance of the bridge circuit a constant power supply of in this case U_A=5 V is used. Thermal drifts due to thermal expansion are compensated using a Wheatstone bridge presented in [16, 17]. This is required to measure low changes in resistance. The strain gauge full bridge circuit is presented in figure 4. The strain gauges R_1 and R_3 are placed in the notch to measure the strain, which is linear to the force applied to the TCP. For thermal compensation two more strain gauges R_2 and R_4 are used, which are placed close to the measurement position, but oriented orthogonally to the direction of maximum strain.

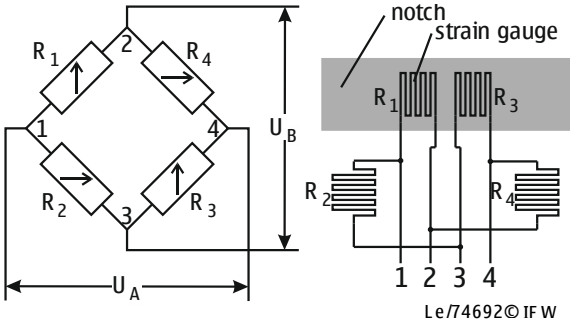

Figure 4: Wheatstone bridge circuit for strain measurement

For this sensing spindle holder HBM strain gauges 1-DY13-3/350 are placed into the notches and 1-LY73-3/350 are used for thermal compensation. The data acquisition is done with six Beckhoff load cell terminal for resistor bridges connected to a TwinCAT control. Figure 5 shows the final design where the spindle holder is mounted to the robot.

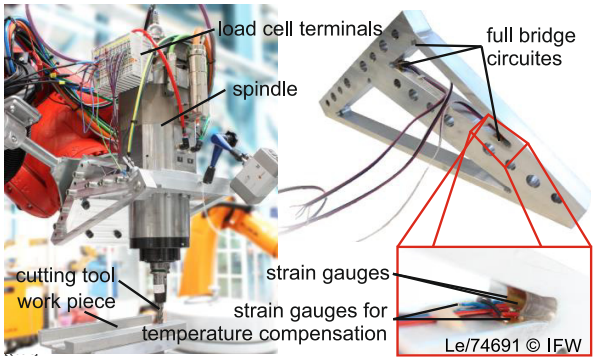

Figure 5: Setup of spindle holder and use on a KR 500 industrial robot

4. Calibration of sensing robot end effector

The calibration of the six strain gauge full circuit bridges is done with a Cartesian three axis dynamometer from Kistler. Since the measuring device is sensitive to the position of force acting on the tool the sensing unit is in a first stage calibrated for one tool. To avoid damage to the tool a tool dummy from carbide is used with the same length as the original tool. For calibration of the sensing head, the force as well as signals from strain gauges are measured. Thus, a solid workpiece is mounted to the dynamometer and the robot is moving the TCP into collision with the workpiece to apply a force of up to 300 N, which is low enough to avoid damage to the spindle bearing and high enough to attain good calibration results. This measurement is done for all Cartesian directions. Furthermore, it is repeated for positive and negative direction in the plane and only in negative direction normal to the working plane. This data is used for a linear regression with help of the least square method [18] to calculate the measurement matrix $\boldsymbol{M}$ according to

$$f = \boldsymbol{M}\, s, \tag{8}$$

where f is the measured three dimensional force vector and s is a vector of the six strains measured for each time sample.

In this case the identified measurement matrix is:

$$\boldsymbol{M} = \begin{bmatrix} -2.17 & 1.58 & -0.26 & 3.45 & 13.23 & 0.46 \\ -3.01 & -0.41 & 0.27 & 2.32 & -2.34 & -0.23 \\ -5.08 & -3.31 & 0.83 & 6.62 & 17.61 & 2.45 \end{bmatrix} \tag{9}$$

To validate the force sensing unit a cutting test was performed with a force dynamometer clamped between machine table and workpiece. An aluminum cutting operation was performed with a spindle speed of 12,000 rpm and a feed rate of 3,000 mm/min and 1 mm radial and 10 mm axial depth of cut. The cutting tool was a three fluted endmill with 12 mm diameter. Figure 6 shows the cutting forces detected with help of the force dynamometer and sensing presented above.

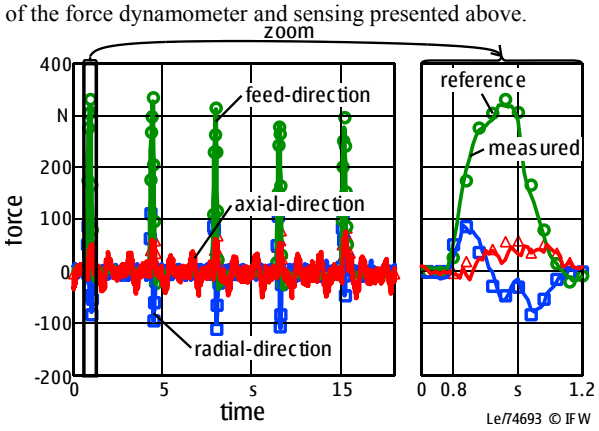

Figure 6: Validation of force sensing spindle holder

The validation shows that the forces in feed and radial-direction are detected with an accuracy of +/- 10 N while forces in axial-direction have an accuracy of +/- 50 N. Nevertheless during cutting operation the force prediction is good. Static force and deviation measurements for one pose of the robot in center of working space where the cutting operation is performed, show a compliance of 1.8 µm/N in feed-direction (tangential to A1 axis), 0.5 µm/N in radial-direction and 1.1 µm/N in axial direction for vertical milling operation. The compliance in the working area of the robot leads to a deviation of about 0.6 mm in feed-direction and 0.2 mm in radial direction. These deviations can be compensated with an accuracy of 0.02 mm by the approach presented.

5. Summary and outlook

An approach for online compensation of machine deviations is presented. Based on measurement of a six-axis robot a static compliance model is presented that simulates six degrees of compliance for each joint. With help of a new sensing spindle holder the cutting forces during machining with a conventional industrial six-axis robot is measured. A calibration procedure is described and the achieved measurement matrix is used for the validation. The calibration is performed for one tool and vertical cutting operations. An aluminum cutting operation is used for validation of the sensing device. The validation shows that the sensing device is able to measure process forces in a respective accuracy good enough for the purpose of compliance compensation.

To achieve even higher accuracy in force measurement further research is done on the sensing itself. The force transformation needs to be implemented and the use of this sensing unit for compensation must be analyzed to show their potential for achieving higher trajectory accuracy during cutting operations with industrial robots.

The sensing spindle holder can be optimized to achieve higher strain at the measurement position. This is achieved by a spindle holder design realizing higher strains at measurement positions without losing stiffness. This higher sensitivity is needed to reduce noise in the signals and thus achieve a higher accuracy in force measurement. A second approach on reducing the noise is the use of a Kalman observer or a Gauß Markov prediction.

In a further step, the sensing spindle holder might be used for process monitoring and control tasks. Thus, the cutting forces need to be transformed to a Cartesian coordinate system, which is aligned to the feed direction. This orientation changes during operation since the trajectory can be a three dimensional path. A communication with the robot controller, the kinematic transformation and a detection of feed direction is required for this transformation.

The approach shown here is realized for one tool length for a vertical orientation of the spindle holder to prove the concept. A new calibration matrix adapted to the tool length and orientation is part of further research to realize a force measurement with different tools. A new model describing the dependence of tool length and force transformation matrix is needed.

In a final step the compliance model of the robot identified for the Kuka KR-500 robot is combined with the force measurement to realize an online compensation of displacements resulting from low stiffness of the robot joints and their bearings.

Acknowledgements

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References

[1] Weigold, M.: Kompensation der Werkzeugabdrängung bei der spanenden Bearbeitung mit Industrierobotern. Dr.-Ing. Dissertation, TU Darmstadt, 2008.
[2] Eastwood, S.: A gravitational deflection compensation strategy for HPKMs. Robotics and Computer-Integrated Manufacturing. 26 (6), 2010, pp. 694-702. DOI: 10.1016/j.rcim.2010.03.009.
[3] Roth, Z.; Mooring, B.; Ravani, B.: An overview of robot calibration. IEEE J. Robot. Automat. 3 (5), 1987, pp. 377–385, DOI: 10.1109/JRA.1987.1087124.
[4] Hollerbach, J. M.: A Survey of Kinematic Calibration. In: The Robotics review 1. Cambridge, Mass.: MIT Press, 1989, pp. 207–242.
[5] Duelen, G.; Schröer, K.: Robot calibration—Method and results. In: Robotics and Computer-Integrated Manufacturing, 8 (4), 1991, pp. 223–231. DOI: 10.1016/0736-5845(91)90034-P.
[6] ISO 9283: Manipulating industrial robots — Performance criteria and related test methods. 1998.
[7] Schröer, K.: Handbook on Robot Performance Testing and Calibration. Fraunhofer IRB, Stuttgart, 1998
[8] Abele, E.; Weigold, M., Rothenbücher, S.: Modeling and Identification of an Industrial Robot for Machining Applications. Annals of the CIRP, Vol. 56 (1), 2007, DOI: 10.1016/j.cirp.2007.05.090.
[9] Nubiola, Albert; Bonev, Ilian A.: Absolute calibration of an ABB IRB 1600 robot using a laser tracker. In: Robotics and Computer-Integrated Manufacturing 29 (1), 2013, pp. 236–245. DOI: 10.1016/j.rcim.2012.06.004.
[10] Klimchik, Alexandr; Furet, Benoit; Caro, Stéphane; Pashkevich, Anatol: Identification of the manipulator stiffness model parameters in industrial environment. Mechanism and Machine Theory 90, pp. 1–22, 2015. DOI: 10.1016/j.mechmachtheory.2015.03.002.
[11] Rizal, M.; Ghani, J. A.; Nuawi, M. Z.; che Haron, Che Hassan: A review of Sensor System and Application in Milling Process for Tool Condition Monitoring. Research Journal of Applied Sciences, Engineering and Technology, 7 (10), 2014, pp.2083-2097
[12] Scheer, C.; Hoffmann, P.; Kirchheim, A.; Schaffner G.: Spindle-integrated force sensors for monitoring drilling and milling processes. Sensor ‘99, 9th International Conference for Sensors, Transducers & Systems, Nürnberg: May 18th – 20th, 1999.
[13] Denavit, J.; Hartenberg, R.: A kinematic notation for lower-pair mechanisms based on matrices. In: Journal Of Applied Mechanics, 1955, pp. 215–221.
[14] Litwinski, K.M.: Sensorisches Spannsystem zur Überwachung von Zerspanprozessen in der Einzelteilfertigung, Berichte aus dem IFW, Band 05/2011, PZH Produktionstechnisches Zentrum GmbH, 2011. ISBN 978-3-941416-97-0
[15] Hoffmann, Karl: Eine Einführung in die Technik des Messens mit Dehnungsmessstreifen, Hottinger Baldwin Messtechnik GmbH, Darmstadt, 1987
[16] Wheatstone, Charles: An account of several new instruments and processes for determining the constants of a voltaic circuit, Philosophical Transactions of the Royal Society of London, vol. 133, 1843, pp 303 – 327
[17] Parthier, R.: Messtechnik. Friedrich Vieweg & Sohn Verlag, Wiesbaden, 2008, pp 14-21.
[18] Strutz, T.: Data Fitting and Uncertainty: A practical introduction to weighted least squares and beyond. Vieweg+Teubner, 2010, ISBN 978-3-8348-1022-9