Strength and deformation characteristics of cement-mixed gravelly soil in multiple-step triaxial compression

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Abstract

When backfill is mixed with cement and compacted in the field, it is very difficult to obtain the multiple similar samples needed to determine a Mohr–Coulomb (M–C) failure envelope by a set of triaxial compression (TC) tests at different levels of effective confining pressure ($s_0$), due to the inevitably large heterogeneity. The effects of the intermediate loading histories, with and without large stress-amplitude unload/reload cycles, on the stress–strain properties of compacted cement-mixed well-graded gravelly soil, particularly the peak strength, were evaluated in multiple-step loading (ML) drained TC tests using a single specimen. A set of single-step loading (SL) tests at different $s_0$ were also performed. Similar specimens, prepared in the laboratory, were used for the tests in this study. Although the M–C failure envelope can be quite accurately determined by relevant ML tests, the peak strength may be underestimated if (a) TC loading is ceased far before the peak stress state or (b) TC reloading is started after the peak stress state has been passed at previous loading steps. Cases (a) and (b) may take place in both ML tests, with increasing and decreasing $s_0$, while Case (b) may take place in ML tests with decreasing $s_0$. The reloading stress–strain relation at the intermediate stage in ML tests may become very soft due to the additional damage caused by a negative irreversible shear strain increment occurring in the immediately preceding unloading process. This effect gradually decreases during reloading, while it totally disappears once large-scale yielding has started, with essentially no effect on the peak strength. It is of the first priority to perform ML tests by increasing $s_0$. It is recommended that ML tests be performed by decreasing $s_0$ if possible, because the peak stress may be closely reached at some intermediate steps. An upper envelope of results from a pair of ML tests, with increasing and decreasing $s_0$, could be more representative of the true M–C failure envelope.

Keywords: Cement-mixed gravel; Deformation; Mohr–Coulomb failure envelope; Multiple-step loading; Strength; Triaxial compression test (IGC: D6/E14)

1. Introduction

The use of a cement-mixed granular material, with a much smaller amount of cement than ordinary concrete employed in civil engineering construction projects, is now gaining wide acceptance. A number of critical civil engineering projects have become feasible due to the application of various compacted cement-mixed soil technologies (e.g., Hansen and Reinhardt, 1990; Schrader, 1996; Tatsuoka et al., 1997). It is not usually difficult to produce compacted cement-mixed soil with mechanical properties close to those of natural sedimentary soft rock (Tatsuoka and Kohata, 1995; Tatsuoka et al., 1997). The construction of bridge abutments for high-speed railways, which requires a high level of ultimate stability against high...
seismic loads while allowing for only very small instantaneous and residual deformation at working loads, using well-compacted cement-mixed well-graded gravelly soil (CMG), has recently become one of the standardized bridge construction technologies in Japan (Aoki et al., 2003; Watanabe et al., 2003a; Tatsuoka et al., 2005).

Despite the above circumstances, the design of ordinary cement-mixed soil structures in routine practice is still based on unconfined compression strengths, not shear strengths expressed in the Mohr–Coulomb (M–C) failure criterion determined by a set of consolidated drained (CD) triaxial compression (TC) tests, while design based on deformation is not popular. For wider applications of cement-mixed soil, it is certainly necessary to better understand the strength characteristics as well as the pre-peak and the post-peak stress–strain properties under field confined conditions. These characteristics are a complicated function of the grading and the particle characteristics of the original granular materials, the cement content, the mixing water content, the compacted dry density, the curing period, the stress conditions (i.e., the confining pressure and the shear stress) and the water content/temperature conditions during curing and so on. The results from CD TC tests, using a set of similar specimens prepared in the laboratory, show that the effects of the confining pressure on the strength and the deformation characteristics of compacted cement-mixed granular materials are significant. Watanabe et al. (2003a, 2005) and Tatsuoka et al. (2005) showed that the angle of internal friction of CMG could become nearly the same as, or even larger than, the value of the original unbound gravelly soil compacted similarly. To evaluate the M–C failure envelope for a given type of soil, we need at least three, preferably four or five, very similar specimens.

One of the problems that make it very difficult to evaluate the M–C failure envelope of a given CMG, mixed and compacted in the field, is the high heterogeneity resulting from the inevitably high inhomogeneous on-site mixing of the soil with cement and water and the non-uniform vertical distribution of compacted dry density in the respective lifts. Although a much more homogeneous CMG can be obtained by mixing the cement at a plant on-site, this method is usually not adopted in ordinary scale construction projects from the viewpoint of cost-effectiveness. For this reason, core samples retrieved from a given site usually exhibit a large variance in strength and deformation characteristics. The following is a typical case. Fig. 1 shows the relationships between the deviator stress and the axial strain obtained from consolidated drained (CD) TC tests on four core samples (12.8 cm in diameter and 22.4–25.4 cm in height) retrieved by rotary core tube sampling immediately after construction from a 12.55 m-high compacted backfill of the CMG of a bridge abutment in Takada for a newly constructed Bullet Train line (Fig. 2: Watanabe et al., 2005). These TC tests were performed 28 day after mixing and compacting in the field. The original backfill material is a gravelly soil, relatively angularly crushed gravelly soil of gabbro from a quarry (\(G_s = 3.03, D_{50} = 5.4\) mm, \(D_{max} = 37.5\) mm, fines content=6% and \(U_c = 61\)). This gravelly soil was mixed with ordinary Portland cement at a cement/gravel ratio in weight equal to 4% at the optimum water content (4.9%) to an averaged degree of compaction equal to 97% for the maximum dry density=2.60 g/cm\(^3\) based on laboratory compaction tests using the modified Proctor. The backfill was compacted in a lift equal to 15 cm. Details of the field compaction control are reported in Tatsuoka et al. (2005) and Watanabe et al. (2003b, 2005). It can be seen from Fig. 1 that, although all

<table>
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<th>Nomenclature</th>
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<td>(\sigma'_s)</td>
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<td>(\sigma'_3)</td>
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<td>(c)</td>
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<td>(\phi')</td>
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<td>(D_{50})</td>
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\[D_{max}\] maximum particle size

\[G_s\] specific gravity of solid particles

\[U_c\] uniformity coefficient

\[(\rho_d)_{max}\] maximum dry density

\(w_{opt}\) optimum water content

\(\rho_d\) dry density

\(w_i\) water content

\(c/g\) cement content per gravel weight

\(E_0\) energy level for standard Proctor compaction test

\(n\) coefficient of proportionality

\(m\) scaling factor

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**Fig. 1.** Deviator stress-axial strain relations from CD TC tests (\(\sigma'_d = 29\)kPa) on four core samples retrieved from same compacted backfill of CMG of bridge abutment shown in Fig. 2 (Watanabe et al., 2005).
the compressive strengths are larger than the specified required minimum compressive strength, equal to 2 MPa, the strength and the deformation characteristics scatter very largely. The respective samples exhibited a highly non-uniform deformation (Fig. 3), indicating non-uniform very largely. The respective samples exhibited a high non-uniformity of stony boulder clay (Anderson, 1974), for the effective shear strength of both unsaturated (De Beer, 1950) and saturated soils (Kenney and Watson, 1961), for the shear strength of rock materials (Kovari and Tisa, 1975) and for the shear strength of rock joints (Goodman, 1976). The ML TC test is now one of the standardized tests for rock materials. However, their effects on the results have not been reported. Kenny and Watson (1961) performed ML TC tests by fully removing the deviator stress and by allowing the excess pore pressure to dissipate before the start of isotropic consolidation. Sorenzon (1988) used the same method on alluvial clay. Due to the strong trend of the strain-hardening behaviour of the soil, the specimens were strained significantly to produce a sufficiently high shear stress at the respective TC loading steps. In both studies, the effect of the intermediate loading history on the results was not evaluated. In the study by Schoenmann and Marvin (1988) on residual-colluvial soils, the consolidation of the specimens was performed with and without removing the deviator stress. They reported that both test procedures produced almost identical results.

Various intermediate loading histories and associated testing sequences were also employed in ML tests on rock materials. However, their effects on the results have not been reported. For example, Kim and Ko (1979) reported that the relevance of a given ML TC test method depends on the type of stress–strain property of a given rock type. They used three different types of rock, namely, Pierre shale, Raton shale and Lyons sandstone. In their tests, after an initial confining pressure was applied, TC loading was continued until the specimen showed signs of failure. After TC loading was halted at that point, the confining pressure was increased toward the next higher level without removing the deviator stress. They described that the
amount of underestimation of the shear strength parameters depends on the failure mode that the rock is expected to have. As the rock becomes stronger, it generally becomes more brittle, which may result in the specimen losing its integrity with an increase in damage during multiple-step loading tests, and consequently, an increase in the margin between the results of single-step and multiple-step tests. Akai et al. (1981) found that in a number of ML TC tests on siltstone and tuff, it often became difficult to continue the ML TC testing, because soon after the specimen approached the failure state, it exhibited abrupt post-peak strain-softening before the increase in confining pressure. They controlled the lateral strain during TC loading to avoid abrupt failure. Cain et al. (1986) and Crawford and Wylie (1987) modified the ML test procedure to detect imminent failure in respective TC loading stages. They suggested that the point at which the volumetric strain returned to zero in the volumetric strain versus axial strain relation was the point of the axial load reversal. In both studies, the confining pressure was increased under isotropic conditions before the start of the next TC loading. Pagoulatos (2004) also employed this loading method, while the TC loading was reversed when the volumetric characteristics of the specimen changed from contraction to dilation. Taheri and Tani (2008) not only increased, but also decreased, the confining pressure in their ML TC tests on sedimentary soft rock.

In summary, the previous researchers have suggested that the following three factors may have a significant effect on the results of the ML TC tests on soft and medium-hard rock:

(a) TC loading may have been halted too early before the peak stress state during strain-hardening.

(b) In the ML TC tests on samples that exhibit significant strain-softening, the peak strength may be underestimated at the second (or later) TC step and subsequent steps. This possibility increases with an increase in the brittleness (i.e., the ratio of residual strength to peak strength). This underestimation is due to the fact that the peak stress state has been passed during previous TC steps, applied before the start of the TC loading at the current step.

(c) A bound specimen is continuously damaged by the shearing that is applied after the bonding has developed. A specimen may be damaged by excessive shearing at previous TC steps and/or by negative shearing during unloading to a higher extent than the one that would be experienced by the same loading history in a corresponding SL TC test. This additional damage may result in the under-estimation of the peak strength evaluated by the SL TC tests at the second (or later) TC loading step and subsequent steps in the ML TC tests.

To alleviate the last problem with sedimentary soft rock (i.e., mudstone and siltstone), a damage model was proposed to correct the ML test results for a predicted amount of damage (Tani, 2007; Taheri and Tani, 2009a,b). Yet, it is not well understood when and how the damage develops during the whole loading histories applied in a given ML TC test on natural hard and soft rock. Taheri and Tani (2009b) also showed that with mudstone that exhibits significant strain-softening behaviour at lower $\sigma_s'$, an upper bound for $c'$ and a lower bound for $\phi'$ are obtained from the ML TC tests with increasing $\sigma_s'$, while an upper bound for $\phi'$ and a lower bound for $c'$ result from the ML TC tests with decreasing $\sigma_s'$. This is likely to be because in the ML TC tests with decreasing $\sigma_s'$, the underestimation of the peak strength by factor b), and possibly also by factor c), becomes more significant at later steps at lower $\sigma_s'$, when the material becomes more brittle.

The strength and deformation characteristics of the CMG prepared in the laboratory were studied rather comprehensively by Watanabe et al. (2003a), Kongsukprasert and Tatsuoka (2003, 2005, 2007), Lohani et al. (2004), Kongsukprasert et al. (2005, 2007) and others. These studies showed that, like hard soil/soft rock, well compacted CMG exhibits the trend of strain-hardening behaviour that becomes stronger at higher $\sigma_s'$ and the trend of strain-softening behaviour that becomes stronger at lower $\sigma_s'$. This implies that the M–C failure criterion of CMG from a given ML TC test may be subjected to the effects of two factors, (a) and (b) described above, like hard soil/soft rock. The effect of factor (c) should also be taken into account if necessary. No study on the relevance of the ML TC tests on CMG can be found in the literature. Therefore, the particular concerns presented here include the following:

1. When should the TC compression be ceased at a given TC loading step?
2. How much should the deviator stress be decreased toward the isotropic stress state before changing $\sigma_h'$ to the value at the next TC loading step?
3. How should the stress path during the consolidation stage be?
4. How much should $\sigma_h'$ be changed to move to the next TC loading stage?
5. Should $\sigma_h'$ be increased or decreased in order to move to the next TC loading step?

In view of the above, the following three series of CD TC tests of compacted CMG were performed in the present study:

1. a set of single-step loading (SL) CD TC tests toward the ultimate failure at different $\sigma_s'$s to evaluate the stress–strain relations under continuous monotonic loading with no previous shearing,
2. a set of multiple-step loading (ML) CD TC tests in which the effective confining pressure ($\sigma_s'^{'}$) is either increased or decreased to move to the next TC loading step and
3. a set of TC tests at fixed $\sigma_h'$ applying several unload/reload cycles to increase the stress amplitude from different fixed shear stresses during otherwise monotonic loading at a constant strain rate to evaluate the
effects of damage by such cyclic loading histories on the stress–strain behaviour and the peak strength during subsequent TC loading.

The levels of strength from Series 2 and 3 were compared with those from Series 1. The effect of the damage to the specimen that may have been additionally produced during previous intermediate loading histories, in particular the immediately preceding unloading in Series 2, was inferred by a decrease in the stiffness during the reloading in Series 3. Based on these results, a relevant ML TC test procedure is suggested to obtain a M–C failure envelope as evaluated by the SL tests at different $\sigma_N'$s.

### 3. Test methods

The methods to prepare the rectangular prismatic and solid cylindrical specimens, the triaxial test systems and the loading methods to perform the SL and ML tests employed in the present study are briefly described below. Their details are reported in Kongsukprasert et al. (2005) and Kongsukprasert and Tatsuoka (2005).

**Specimen preparation:** particles larger than 10 mm were removed from a well-graded gravelly soil of crushed sandstone ($G_i = 2.71$), so-called Chiba gravel, to obtain the test material (called “model Chiba gravel” with $G_i = 2.74$). Fig. 4 shows the grading curve. Compaction tests of the material were newly performed using a mould with an inner diameter of 10 cm and an inner volume of 1000 cm$^3$ at an energy level of $E_0 = 550$ kJ/m$^3$ (the standard Proctor). The obtained maximum dry densities, $\rho_d$ at optimum water contents, $w_{o}$, of the original material without cement and the cement-mixed material with a cement-to-gravel ratio by dry weight, $c/c_g$, equal to 2.5% are nearly the same (Fig. 5). These results are consistent with those reported by Kongsukprasert et al. (2007) and Tatsuoka et al. (2008).

The gravelly soil was thoroughly mixed with cement powder and then water. The target initial water content, $w_i$ (in the ratio to the dry weight of the soil plus cement), was 8.75%, which is slightly lower than $w_{opt}$ for $E_i$ (Fig. 5). The target compacted dry density, $\rho_d$ was 2.0 g/cm$^3$. Table 1 lists the CD TC test conditions. The actual values for $w_i$ and $\rho_d$ were scattered to some extent, which may be partly the cause of the scatter of the data discussed later. The mixture was compacted manually to produce either rectangular prismatic specimens (72 mm × 72 mm in cross-section and 150 mm high), shown in Fig. 6(a), or solid cylindrical specimens (100 mm in diameter and 200 mm in height) shown in Fig. 7(a). The compaction was made in five layers in a rectangular prismatic or cylindrical mould, both having the same inner dimensions as the respective specimens. Each compacted specimen was sealed and cured inside the compaction mould under atmospheric pressure at a constant water content for 25°C for five days. Then, the specimen was removed from the mould and sealed for further curing under atmospheric pressure at a constant water content for 4 days (i.e., the total curing time was 9 days).

Accurate lateral strain levels, free from bedding errors at the interface between the latex rubber membrane and the lateral surface of the test material when the effective confining pressure changes, were obtained by measuring the compression and the extension of the linear lateral segment arranged on the planar lateral surface of the rectangular prismatic specimen (Fig. 6(b)). Only one triaxial apparatus could accommodate a rectangular prismatic specimen. As this factor was a strong restraint to the experiment programme, an ordinary triaxial cell that could accommodate a solid cylindrical specimen was also used to perform CD TC tests at a fixed confining pressure (Fig. 7). The specimen was subjected to drained TC loading at the same water content as when it was prepared.

**Triaxial test systems:** a high pressure triaxial cell with a pressure capacity of 3 MPa was used for the rectangular prismatic specimens, while a low pressure triaxial cell with a pressure capacity of 700 kPa was used for the solid cylindrical specimens. With these two advanced triaxial test systems, the axial loading was displacement-controlled using a precision gear system and the confining pressure was applied using a pneumatic loading system (Tatsuoka et al., 1994, 1999; Hayano et al., 1997; Suntucci de Magistris et al., 1999). Prescribed loading histories were applied in an automated way. To minimise the effects of inhomogeneous deformation and the associated non-uniform stress distribution within the specimen due to bedding errors and high end friction, the top

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**Table 1:**

<table>
<thead>
<tr>
<th>Water Content, $w_i$ (%)</th>
<th>$\rho_d$ (g/cm$^3$)</th>
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<td>8.75%</td>
<td>2.0</td>
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**Fig. 4.** Grain size distribution curve for model Chiba gravel.
Table 1
Test conditions and part of test resultsa.

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<tr>
<th>Test no.</th>
<th>Test name</th>
<th>Test series</th>
<th>Loading method a) increasing $\sigma'_s$</th>
<th>Specimen shape</th>
<th>$q$ (MPa) at the start of unloading</th>
<th>Water content ($%$)</th>
<th>$\rho_d$ (g/cm$^3$)</th>
<th>$D_c$ ($%$)</th>
<th>$\sigma'_s$ (MPa)</th>
<th>$q_{max}$ (MPa)</th>
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<td>1</td>
<td>CB3-1</td>
<td>1</td>
<td>Single step loading (SL)</td>
<td>Rectangular</td>
<td>NA</td>
<td>9.45</td>
<td>1.98</td>
<td>94.3</td>
<td>0.02</td>
<td>1.94</td>
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<td>2</td>
<td>CB3-6</td>
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<td></td>
<td></td>
<td>10.26</td>
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<td>CB3-8</td>
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<td>93.3</td>
<td>1.50</td>
<td>5.85</td>
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<td>CMG-D04</td>
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<td>ML w/oU b) increasing $\sigma'_s$</td>
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<td>2.00 and 3.60</td>
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<td>1.99</td>
<td>94.8</td>
<td>0.50</td>
<td>2.76</td>
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a) For all tests: cement content ($c/g$)=2.5%, basic compaction energy ($E_c$): 1.0 and total curing time: 9 day.

b) With full-unloading of $q$ before increasing or decreasing $\sigma'_s$.

c) Without unloading of $q$ before increasing or decreasing $\sigma'_s$. 
and the bottom of the specimen were made smooth by pasting a thin layer of gypsum slurry and, after it had solidified, lubricated by smearing a thin layer of silicon grease. Thrust force, induced by non-symmetric deformation due to shear banding in the specimen, may damage the apparatus (particularly a load cell fixed between the specimen cap and the loading piston). To alleviate the above, the high pressure triaxial cell has a specimen pedestal placed on the bearing system that allows free lateral displacements in any direction. In tests using the low pressure triaxial cell, the axial loading was stopped before noticeable unsymmetric deformation had developed in the specimen.

Axial strains were sensitively and accurately measured locally with a pair of 160-mm-long LDTs (Goto et al., 1991) set on opposite lateral faces of the rectangular prismatic specimen (Fig. 6(b)) or at opposite ends of the diameter of the solid cylindrical specimen (Fig. 7(b)). Lateral strain levels of the rectangular prismatic specimen were also measured locally with two sets of three lateral LDTs (six in total) arranged on the opposite lateral faces of the specimen. Lateral strains of the solid cylindrical specimen were measured locally by three clip gauges arranged at three heights (5/6, 3/6 and 1/6 of the specimen height from the specimen bottom) (Fig. 7(b)). The local lateral strain of the solid cylindrical specimen, measured as above, is free from bedding errors at the specimen lateral face if the effective lateral confining pressure is kept constant.

Axial and lateral strain levels obtained by averaging the readings of the respective sets of local gauges are presented in this paper.

**Loading methods**: the specimen was isotropically consolidated at $\sigma'_h = 20$ kPa by means of partial vacuum, and LDTs and other instruments were set; this took about 2 h. Then, the following three series of CD TC tests were performed. The absolute value of the axial strain rate during the TC loading, unloading and reloading (including ten-minute unload/reload cycles) was equal to 0.03%/min in all the tests. Since the specimens were unsaturated as they were when they were compacted, it is very likely that the specimens were essentially under drained conditions at this relatively low strain rate:

(1) **Series 1** (*SL TC tests at different levels of constant effective confining pressure, $\sigma'_h$; Fig. 8(a))**: the specimen was isotropically consolidated at $\sigma'_h = 0.02$ or 0.50 or 1.00 or 1.50 MPa and cured for two hours before the start of the continuously drained TC loading toward ultimate failure to determine the original stress–strain properties at constant $\sigma'_h$ that are free from any effects of previous TC loading history at the same $\sigma'_h$ or other $\sigma'_h$s. To evaluate the quasi-elastic properties, ten-minute unload/reload cycles were applied at several intermediate stages during otherwise continuous TC loading.

(2) **Series 2**: the following different ML tests were performed to specifically evaluate the possible effects of factors (a), (b) and (c), explained before, on the stress–strain behaviour at respective steps in the ML tests:

**Fig. 7.** Typical solid cylindrical specimen: (a) after curing for 9 day in compaction mould and (b) instrumented under partial vacuum with rubber membrane in triaxial cell.

**Fig. 6.** Typical rectangular prismatic specimen: (a) after curing for 5 day in compaction mould and (b) instrumented under partial vacuum with rubber membrane in triaxial cell.

**Fig. 8.** Schematic diagrams showing effective stress paths in (a) single-step loading (SL) tests, (b-1) and (b-2) multiple-step loading (ML) tests with full unloading—increasing and decreasing $\sigma_h$ and (c-1) and (c-2) ML tests with partial unloading—increasing and decreasing $\sigma_h$. 


132
(2a) A ML test with full unloading of the deviator stress $q = \sigma_v - \sigma_h$ followed by an increase in $\sigma_h^*$ (Fig. 8(b-1)): at the first step at $\sigma_h^* = 0.02$ MPa, when it was judged that sufficient irreversible deformation had taken place to closely approach the peak stress state, the drained TC loading was terminated. Then, $q$ was reduced to zero and the specimen was left for 40 min to reach the equilibrium state. Then, $\sigma_v^*$ was increased to 0.25 MPa under isotropic stress conditions to perform the next step of TC loading. This procedure was repeated to perform TC loading at $\sigma_h^* = 0.02$, 0.25, 0.50, 0.75 and 1.0 MPa.

(2b) A ML test with full unloading of $q$ followed by a decrease in $\sigma_h^*$ (Fig. 8(b-2)): at the first step at $\sigma_h^* = 1.0$ MPa, when it was judged that some large irreversible deformation had taken place to closely approach the peak stress state, the TC loading was terminated. Then, $q$ was reduced to zero and the specimen was left for 40 min to reach the equilibrium state. Then, $\sigma_v^*$ was decreased under isotropic stress conditions to 0.75 MPa to perform the next step of TC loading. This procedure was repeated to perform the TC loading at $\sigma_h^* = 1.0$, 0.75, 0.50, 0.25 and 0.02 MPa.

(2c) A ML test without full unloading of $q$ before increasing $\sigma_h^*$ at a fixed axial strain (Fig. 8(c-1)): at the first step at $\sigma_h^* = 0.02$ MPa, when it was judged that some large irreversible deformation had too closely approached the peak stress state, the TC loading was stopped and $\sigma_h^*$ was increased to 0.25 MPa at a fixed axial strain. The specimen was left at fixed values of axial strain and $\sigma_h^*$ for 40 min to reach the equilibrium state allowing the axial stress to relax. This procedure was repeated to perform the TC loading at $\sigma_h^* = 0.02$, 0.25, 0.50, 0.75 and 1.0 MPa.

(2d) A ML test without full unloading of $q$ before decreasing $\sigma_h^*$ at a fixed axial strain (Fig. 8(c-2)): at the first step at $\sigma_h^* = 1.0$ MPa, when it was judged that some large irreversible deformation had taken place to closely approach the peak stress state, the TC loading was stopped and then $\sigma_h^*$ was decreased to 0.75 MPa at a fixed axial strain. The specimen was left at the fixed values of axial strain and $\sigma_h^*$ for 40 min to reach the equilibrium state allowing the axial stress to relax. This procedure was repeated to perform TC loading at $\sigma_h^* = 1.0$, 0.75, 0.50, 0.25 and 0.02 MPa.

In these ML tests, the TC loading at the respective steps was stopped and reversed so that the total axial strain that would have taken place by the end of the final TC loading would be smaller than the measurement range of axial LDTs (about 2.5%). Under this restriction, at each step, some large irreversible deformation was allowed to take place to closely approach the peak stress state (in the strain-hardening regime), or only some irreversible deformations were allowed to take place after the peak stress state has been passed (in the strain-softening regime). With an increase in $\sigma_h^*$, the specimen exhibited more ductile behaviour. Therefore, in the ML tests with decreasing $\sigma_h^*$, the specimens were strained more at higher $\sigma_h^*$ and less at lower $\sigma_h^*$.

(3) Series 3 (cyclic loading tests varying the stress amplitude at a fixed $\sigma_h^*$ (Fig. 9): to evaluate the effects of the unload/reload cycles of $q$ applied between consecutive TC loading steps in the ML tests on the strength and deformation characteristics during subsequent TC loading, multiple unload/reload cycles with unloading from the same shear stress level followed by full reloading increasing the deviator stress amplitude with cycles were applied during otherwise continuous TC loading at a fixed $\sigma_h^*$ (0.02 or 0.50 or 1.0 MPa). In one of the tests, the deviator stress was decreased to reach the triaxial extension stress state.

4. Experimental results

In this section, the effects of the intermediate stress path (i.e., $\sigma_h^*$ is increased or decreased and then $q$ is partially or fully reduced between consecutive TC loading stages) on the peak strength in a given ML test were firstly examined by comparing the results from test Series 1 and 2. Then, the effects of the unloading of $q$ on the stress–strain behaviour during subsequent reloading toward ultimate failure were examined by analysing the results from Series 3. Finally, a relevant ML test method is suggested based on these results.

4.1. Series 1 (SL tests) and Series 2 (ML tests)

Comparison of stress–strain behaviour: Fig. 10 shows the relationships between deviator stress $q$ and the axial strain and between the volumetric and the axial strain from four SL TC tests at different $\sigma_h^*$s (Series 1). It may be seen that, at the lowest $\sigma_h^* (= 0.02$ MPa), significant strain-softening takes place after the peak stress state is reached at a relatively small axial strain. With an increase in $\sigma_h^*$, the axial strain at the peak stress state increases with the strain range in pre-peak strain-hardening becoming larger. The strain-hardening is associated with the contraction of the
specimen, while the post-peak strain-softening is associated with dilatation. These stress–strain properties are used below as the reference for the ML TC tests (Series 2).

It is known with rocks that the dilatation of a specimen during TC loading is associated with the growth of micro-cracks in the direction dominantly parallel to the axial stress, which means an increase in damage to the micro-structure. As the CMG specimen has an initially large amount of small pores, the mechanism described above may also be relevant. One of the major issues of the present study is whether or not additional damage is induced during intermediate loading histories and this additional damage noticeably decreases the pre-peak stiffness and peak strength at subsequent TC steps in ML tests. Fig. 11(a)–(d) shows the results from four drained ML TC tests (Series 2). Fig. 11(a) shows the results from the ML test with an increasing $s_0^h$ and with the full unloading of $q$ (Test 2a). After having passed a large-scale yield point at the second and subsequent steps, the stress–strain relation tends to rejoin the original relation at respective $s_0^h$s as obtained from the SL tests. Furthermore, the stress–strain behaviour during the first primary loading at $s_0^h = 0.02$MPa, as well as those after the start of large scale yielding at the later steps at higher $s_0^h$s, all exhibit strain-hardening with volume contraction until the end of the TC loading. That is, at all the steps, the TC loading was terminated before reaching the respective peak stress states. Correspondingly, the maximum deviator stress $q_{\text{max}}$ at each step increased with an increase in $s_0^h$.

Fig. 11(b) shows the results from the ML test with decreasing $s_0^h$ and full unloading of $q$ (Test 2b). The following trends may be seen:

1. The stress–strain relations after the start of large-scale yielding at the second and subsequent steps tend to rejoin the original relation at respective $s_0^h$s. At $s_0^h = 0.02$MPa, the dilatation rate starts increasing when large-scale yielding starts. However, contraction restarts after some strain has taken place (Point a). It seems that this peculiar behaviour is due to the start of the significant non-uniform deformation of the specimen associated with shear banding, and the changes in volume obtained from locally measured axial and lateral strains after Point a have become reliable.

2. Despite a decrease in $s_0^h$, the maximum deviator stress ($q_{\text{max}}$) at the second step ($s_0^h = 0.75$MPa) is larger than that at the first step ($s_0^h = 1.0$MPa). This is due to the fact that at the first step, this $q_{\text{max}}$ value was measured far before the peak stress state during strain-hardening and, at the second step, the strain-hardening regime continues for some large axial strain increment after the start of large-scale yielding.

3. At the third and subsequent steps, the stress–strain behaviour exhibits noticeable trends of strain-softening after the start of large-scale yielding. Correspondingly, the $q_{\text{max}}$ value at each step decreases with a decrease in $s_0^h$. In particular, at the last step ($s_0^h = 0.02$MPa), the trend of strain-softening is most significant in association with the dilatation of the specimen. The $q_{\text{max}}$ value at the last step is considerably lower than the one observed at the step immediately before. The decrease in $q_{\text{max}}$ with a decrease in $s_0^h$ from 0.25 MPa to 0.02 MPa is considerably larger than the increase when $s_0^h$ increases from 0.02 MPa to 0.25 MPa in the ML test increasing $s_0^h$ (Fig. 11(a)). This trend could be explained by the
following two factors: (1) in the SL tests, the axial strain at the peak stress state decreases and the trend of post-peak strain softening becomes stronger with a decrease in $\sigma'_h$ (Fig. 10); (2) in the course of the ML test, the cumulative axial strain becomes larger at later steps. Therefore, the peak stress state has been already passed at the start of the TC loading at this last step and the stress–strain behaviour after the start of large scale yielding is already post-peak strain-softening behaviour.

It will be examined later whether or not additional damage was induced during the unloading processes prior to the last step (in particular, during the one immediately before) and it could be another factor for a particularly low $q_{\text{max}}$ value at the last step.

The changes in volume during unloading, seen in Fig. 11(a) and (b) are an expansion reflecting the dominant trend of elastic expansion associated with a decrease in the effective axial stress. On the other hand, the changes in volume during reloading are contraction due to both elastic contraction and in-elastic contraction associated with slight strain-hardening yielding. Sharp changes in the tangential slope of the volumetric strain–axial strain relation cannot be seen at the start of large-scale yielding, unlike the deviator stress–axial strain relation. Therefore, it is difficult to find the start of the large-scale yielding from these volumetric strain–axial strain relations.

Fig. 11(c) shows the results from the ML test without the full unloading of $q$ while increasing $\sigma'_h$ fixing the axial strain (Test 2c). Fig. 12(a) shows the stress path (i.e., the $\sigma'_h$–$\sigma_v$ relation) measured in this test. In the process of increasing $\sigma'_h$, the axial loading device was locked. Despite the above, the axial strain of the specimen slightly increased, due likely to the following two mechanisms: (a) compression of the specimen by an increase in the confining pressure was not perfectly restrained as the loading system was not perfectly rigid; (b) by its viscous properties, some axial creep deformation of the specimen took place associated with a release of elastic strain energy stored inside the specimen during TC loading, but the effects of this additional damage are fully recovered after small large-yielding has taken place. These points are examined in detail below.

Fig. 12(a)–(e) compares the $q$–$\varepsilon_v$ curves during primary loading or reloading at $\sigma'_h=0.02$, 0.25, 0.50, 0.75 and 1.0MPa from the ML tests increasing and decreasing $\sigma'_h$ with full unloading of $q$ at intermediate stages (Series 2a and 2c). In this plot, the axial strain at the start of the TC loading at each step was redefined as zero. The results from the SL tests at $\sigma'_h=0.02$, 0.50 and 1.0MPa are also plotted. Under the same primary loading condition, the strength at $\sigma'_h=0.02$MPa from the SL test is slightly lower than that at the first step in the ML test increasing $\sigma'_h$ (Fig. 14(a)), while the strength at $\sigma'_h=1.0$MPa from the SL test is nearly the same as that at the first step in the ML test decreasing $\sigma'_h$ (Fig. 14(e)). The former result is due very likely to an inevitable scatter among the specimens. Yet, the trend of the stress–strain behaviour from the SL tests is a good reference for the ML tests. It may be seen from these figures that, at the same $\sigma'_h$, the stiffness until the
during the primary loading (Fig. 11(c) and (e)). This trend is most typically seen from the TC tests at \( \sigma_h = 1.00 \text{ MPa} \) (Fig. 14(e)). This trend is due to the fact that the stress–strain behaviour becomes more reversible during reloading than during primary loading, typically observed in cyclic loading tests at the fixed \( \sigma_h \) (as shown later in Fig. 21).

It can also be seen from Fig. 14(e) that, at the last step in the ML test increasing \( \sigma_h \), the deviator stress still noticeably increases with strain, exhibiting the trend of strain-hardening, after the start of large-scale yielding. Yet, it looks like the maximum deviator stress (\( \approx 4.44 \text{ MPa} \)) at this loading step is only slightly lower than the peak strength at this TC step. Based on the fact that this value is slightly larger than the peak strength (\( \approx 4.1 \text{ MPa} \)) observed by the SL test (Table 1), this trend indicates that, at this last step of the ML test increasing \( \sigma_h \), negative effects on the stress–strain behaviour after the start of large scale yielding of the additional damage that has taken place during the preceding loading history (particularly during the immediately precedent unloading process) are more dominant than the positive effects of becoming more reversible by cyclic loading. It is shown later that, despite the above, the peak strength may not have decreased because of these negative effects of additional damage.

(1) The stiffness before reaching a large-scale yield point at the last step in the ML loading decreasing \( \sigma_h \) is noticeably lower than that during the primary loading at the first step in the ML loading increasing \( \sigma_h \). This unusual trend is likely due to the fact that the negative effects of the additional damage that has taken place during the preceding loading history (particularly during the immediately precedent unloading process) are more dominant than the positive effects of becoming more reversible by cyclic loading. It is shown later that, despite the above, the peak strength may not have decreased because of these negative effects of additional damage.

(2) The stress-strain behaviour after the start of large-scale yielding at the last step in the ML loading decreasing \( \sigma_h \) exhibits a strong trend of strain-softening. Although it is to a lesser extent, a similar trend of post-yielding strain-softening can also be seen at the third and fourth steps in the ML test decreasing \( \sigma_h \) (Fig. 14(b) and (c)). Furthermore, the maximum stress (\( q_{\text{max}} \)) at the last step in the ML loading decreasing \( \sigma_h \) is noticeably lower than the peak strength (\( q_{\text{peak}} \)) that can be inferred from the \( q_{\text{max}} \) value observed at the first step in the ML loading increasing \( \sigma_h \). This comparison is relevant, because, in Fig. 14(b) (\( \sigma_h = 0.25 \text{ MPa} \)), the \( q_{\text{max}} \) value at the fourth step in the ML loading decreasing \( \sigma_h \) is very similar as the \( q_{\text{peak}} \) value that can be inferred from the \( q_{\text{max}} \) value observed at the second step in the ML loading increasing \( \sigma_h \). From these facts, it is obvious that, at the start of the large-scale yielding at the last step of the ML test decreasing \( \sigma_h \), the peak stress state has already been passed during the precedent TC loading steps. Therefore, the \( q_{\text{max}} \) value at the last step at \( \sigma_h = 0.02 \text{ MPa} \) in the ML loading decreasing \( \sigma_h \) is noticeably lower than the \( q_{\text{peak}} \) value obtained by the corresponding SL test.

(3) For the reason described above, the \( q_{\text{peak}} \) value can be approached very closely at some intermediate steps in the ML test decreasing \( \sigma_h \), unlike the ML test increasing \( \sigma_h \).

(4) At the first step of the ML test increasing \( \sigma_h \) (Fig. 14(a)), the \( q_{\text{peak}} \) value in the SL test is noticeably lower than the \( q_{\text{max}} \) value. This is due likely to a variance among the tests.

A similar comparison among different TC test types, as presented above, for the ML tests with partial unloading of...
Fig. 15. Comparison of stress–strain behaviour from SL test and first step, or second and subsequent steps in ML tests without full unloading at each confining pressure: (a) 0.02 MPa, (b) 0.25 MPa, (c) 0.50 MPa, (d) 0.75 MPa and (e) 1.00 MPa.

$q$ is made in Fig. 15(a)–(e). In Fig. 15(e), at the fifth and last step at $s_0 = 1.00\text{MPa}$ in the ML test increasing $s_0$, the post-yielding stress–strain behaviour still exhibits a noticeable trend of strain-hardening. Yet, the $q_{max}$ value ($=4.29\text{MPa}$) is close to the $q_{peak}$ value ($=4.1\text{MPa}$) from the SL test. Also in Fig. 15(d), the $q_{max}$ value at the fourth step at $s_0 = 0.75\text{MPa}$ in the ML test increasing $s_0$ is nearly the same as the $q_{peak}$ value that can be inferred from the $q_{max}$ value at the second step in the ML test decreasing $s_0$. On the other hand, in Fig. 15(a), at the last step at $s_0 = 0.02\text{MPa}$ in the ML test decreasing $s_0$, the post-large scale yielding stress–strain behaviour exhibits a very strong trend of strain-softening with a strong trend of dilatancy (Fig. 11(c)). Furthermore, the $q_{max}$ value in this test is noticeably lower than the $q_{peak}$ value that can be inferred from the $q_{max}$ value observed in the ML test increasing $s_0$. Although, in Fig. 15(b) ($s_0 = 0.25\text{MPa}$), the $q_{max}$ value at the fourth step in the ML test decreasing $s_0$ is noticeably higher than the $q_{peak}$ value that can be inferred from the $q_{max}$ value at the second step in the ML test increasing $s_0$. These trends are consistent with those observed in the ML tests with full-unloading of $q$ (Fig. 14) (i.e., at the last step at $s_0 = 0.02\text{MPa}$ in the ML test decreasing $s_0$, large-scale yielding starts already in the strain-softening regime in the corresponding SL test). These trends also confirm that the effects of the full-unload/reload cycles of $q$, applied between consecutive steps of TC loading at different $s_0$, on the post-large scale yielding stress–strain behaviour, are negligible. This point is reconfirmed by the results from the TC tests of Series 3 (shown later).

Failure envelopes: to examine whether the failure envelope obtained by the ML tests is comparable to the one obtained from the conventional SL tests, the maximum stress states observed at all the TC loading steps in the ML tests, together with the peak stress states from the SL tests, are summarised in Fig. 16. The values of the peak strength ($q_{peak}$) from the TC tests in which the peak stress state was reached when the axial strain became larger than the measurement range of the axial LDTs were obtained from the relationships between $q$ and the externally measured axial strain. These values are listed in Table 1 and plotted in the figure. The following trends may be seen:

1. Other than the following two groups of data, denoted by a circle and two arrows in Fig. 16, all the data from the ML and the SL tests are located in a narrow zone between two broken lines. Their parameters for the Mohr–Coulomb failure criterion, which are shown in this figure, are not largely different. This result indicates that, if we can exclude data for relevant reasons that apparently underestimate the peak strength, we can reasonably infer the failure envelope under the SL condition from results of such ML tests. (a) The two data points in a circle are those obtained at the first step in the two ML tests decreasing $s_0$. They are obviously obtained far before the peak stress state during strain-hardening. This trend is more clearly seen from Figs. 17 and 18, which compare the maximum stress points obtained from the two ML tests increasing and decreasing $s_0$ with full unloading (w/U) (Fig. 17) and without full unloading (w/oU) (Fig. 18). It can also be seen from these figures, as well as from Fig. 20, that the maximum stress levels at the second step in these ML tests decreasing $s_0$ also slightly underestimate the peak stress state during strain-hardening.

Fig. 16. Comparison of maximum stresses or peak strengths from all the SL and ML tests (w/U and w/oU denote ML tests with and without full unloading of deviator stress: $s_1 - \sigma_h$ and $s_3 - \sigma_h$).
strength obtained by SL tests under otherwise the same conditions.

(b) The data points denoted by two arrows in Fig. 16 are the maximum stress states at the last step (at $\sigma_0 = 0.02$ MPa) in the two ML tests decreasing $\sigma_h$. As stated before, these two maximum stresses noticeably underestimate the peak strength in the SL tests. This trend can be clearly seen from zoomed-upped figures (Figs. 17 and 18). This is due apparently to the start of large-scale yielding already in the post-peak strain-softening regime.

2. The width of the zone between two broken lines in Fig. 16 is small, but noticeable. Furthermore, the data points from some SL tests are located slightly lower than the average of the data located in this zone. These trends are likely due to (a) an inevitable scatter in the compacted dry density ($\rho_d$) and water content at compaction ($w_i$) among the different specimens (Table 1) and (b) the effects of the differences in shape and size between the rectangular prismatic and the cylindrical specimens. The contributions of factor (b) are not yet well understood. Another factor for this scatter is that, as seen from Figs. 17 and 18, the maximum stress levels measured in the strain-hardening regime at the first and second steps in the ML tests increasing $\sigma_h$ slightly underestimates the peak strength, which is due to the cessation of loading early before the peak stress state during strain-hardening.

3. The broken lines depicted in Figs. 17 and 18 are the upper bound of a respective set of data from a pair of ML tests increasing and decreasing $\sigma_h$. As seen from these figures, the Mohr–Coulomb criteria representing the upper bounds for the peak strength from respective pairs of ML tests increasing and decreasing $\sigma_h$ are very similar. This trend indicates that the effects of over-consolidation on the peak strength in the ML tests decreasing $\sigma_h$ are very small, if any. This trend is likely due to the fact that the specimens were well-compacted. We can consider that this upper-bound is representative of the failure envelope under SL conditions.

4. Figs. 19 and 20 compare the maximum stress levels measured in ML tests with and without full unloading of $q$ when increasing $\sigma_h$ (Fig. 19) and when decreasing $\sigma_h$ (Fig. 20). The effects of full-unloading on the strengths at each step are negligible. This issue is examined later by the results from the TC tests of Series 3.

In summary, to evaluate the failure envelope under primary loading conditions of a material like CMG, used in the present study, the following ML test procedure can be recommended based on the test results shown above:

(1) If only a single ML test is to be performed, the first priority is a ML test increasing $\sigma_h$. Yet, the peak
strength is underestimated if loading is reversed before the peak stress state is reached during strain-hardening. On the other hand, in a ML test decreasing $\sigma_0^h$, the peak strength may be underestimated to a higher extent both at early steps for the same reason cited above and at later steps due mainly to the start of large scale yielding in a post-peak strain-softening regime.

(a) At least with CMG, used in the present study, depending on the convenience of testing in a given case, the deviator stress could be either fully unloaded or partially unloaded between consecutive TC steps. The method in which the confining pressure is increased while locking the axial loading device is one of the simplest testing procedures in this respect. Moreover, this method could exclude the possible negative effects of additional damage that may take place during the full unloading of $q$, as explained in details in the next section.

(b) The total axial strain that can be applied to a single specimen in a ML test should be subjected to some limitations. In the present study, the TC loading was reversed at each step so that the total axial strain at the last (i.e., fifth) step would be within the measurement range of axial LDTs (about 2.5%). With an increase in $\sigma_0^h$, the specimen exhibits more ductile behaviour. Then, in a ML test increasing $\sigma_0^h$, the TC loading at all steps can be maintained within a strain-hardening regime.

(2) If feasible, it is recommended that an additional ML test decreasing $\sigma_0^h$ be performed using as similar a sample as possible to that used in the ML test increasing $\sigma_0^h$. In this ML test, it is very likely that the peak stress state is approached very closely at the intermediate steps. When the effects of data scatter can be properly evaluated, the upper envelope of a data set from a pair of ML tests increasing and decreasing $\sigma_0^h$ (as shown in Figs. 17 and 18) may be obtained, which should be more representative of the true failure envelope than the one obtained by a single ML test increasing $\sigma_0^h$.

4.2. Effects of unload/reload cycles with large stress amplitudes (Series 3)

It is examined in this section whether the effects of cyclic loading with a relatively large stress amplitude, as applied in the ML tests with full-unloading of $q$, on the stress–strain behaviour (including the peak strength) at subsequent loading steps are significant or not. Fig. 21 shows the whole stress–strain relations from the TC tests at different fixed $\sigma_0^h$s in which deviator stress $q$ was decreased from different fixed deviator stress levels, followed by full reloading with increasing the deviator stress amplitude with cycles (Series 3, Fig. 9). These relations are compared with those from a SL test at the same $\sigma_0^h$. After the start of large-scale yielding during the last reloading toward ultimate failure, the stress–strain behaviour tends to rejoin the one obtained by continuous primary ML performed under otherwise the same conditions. In Fig. 22, the peak strengths from these TC tests (hollow circles) have been added to those presented in Fig. 16. It may be seen that, despite some scatter in the data, the peak strength does not decrease by applying several unload/reload cycles with relatively large stress amplitudes. These results reconfirm that, in the ML tests, effects on the stress–strain behaviour when exhibiting large-scale yielding and the peak strength of additional damage that takes place during intermediate loading histories, other than the one that takes place during primary loading in the SL tests, are negligible, if any.

Fig. 23 illustrates the proportional rule that is often employed to describe the hysteretic stress–strain ($y - x$)
relations during drained cyclic loading of unbound granular material (e.g., Masuda et al., 1999; Tatsuoka et al., 2003). According to this rule, the reloading relation from any point, where the coordinate is \((y_i, x_i)\) (such as Points 2, 3 and 4), is obtained as

\[
y - y_i = f\left(\frac{x - x_i}{n}\right)
\]  

(1)

where \(y = f(x)\) is the primary stress–strain relation and \(n\) is the coefficient of proportionality. All reloading curves are obtained by parallel shifting of the same full-reloading curve starting at Point a, where the coordinate is \((y_{ia}, x_{ia})\), described as

\[
y - y_{ia} = f\left(\frac{x - x_{ia}}{n}\right)
\]  

(2)

If the primary loading curves in the opposite directions of shearing are perfectly symmetric, \(n\) becomes two. A reloading curve that follows Eq. (1) suddenly rejoins the primary loading curve at Point \(r\) (Fig. 23). The analysis based on Eqs. (1) or (2) shown below is to highlight different behaviours during cyclic loading between unbound and bonded materials, not to discuss the applicability of any elasto-plastic theory.

Fig. 24(a1)–(e3) shows the zoom-ups of the stress–strain relations during unload/reload cycles presented in Fig. 21. The stress–strain behaviour during reloading becomes softer after the start of large-scale yielding. A reloading curve that follows Eq. (1) suddenly rejoins the primary loading curve at Point \(r\) (Fig. 23). The analysis based on Eqs. (1) or (2) shown below is to highlight different behaviours during cyclic loading between unbound and bonded materials, not to discuss the applicability of any elasto-plastic theory.

Fig. 24(a1)–(e3) shows the zoom-ups of the stress–strain relations during unload/reload cycles presented in Fig. 21. The stress–strain behaviour during reloading becomes softer after the start of large-scale yielding. A reloading curve that follows Eq. (1) suddenly rejoins the primary loading curve at Point \(r\) (Fig. 23). The analysis based on Eqs. (1) or (2) shown below is to highlight different behaviours during cyclic loading between unbound and bonded materials, not to discuss the applicability of any elasto-plastic theory.

(1) the cyclic strain-hardening effect, by which the peak-to-peak secant modulus of a hysteresis loop for a fixed stress amplitude, or a fixed strain amplitude, increases with cyclic loading,

(2) the viscous effect, by which the tangent modulus when approaching the maximum stress in the respective hysteresis loops decreases with time and

(3) the effect of damage that has taken place during the immediately preceding unloading, by which the tangent modulus at a given state during reloading decreases.

It is obvious from Fig. 24 that the tangent modulus at a given deviator during reloading decreases with an increase in the unloaded increment of \(q\). In particular, when unloaded to a triaxial extension (TE) stress state (Fig. 24(e)), the stiffness during reloading is very low. Mainly for this reason, and partially because of the creep deformation that takes place due to the viscous property, the axial strain becomes significantly larger than the value when the unloading started when the reloading curve reaches the maximum stress level. This difference increases with an increase in the unloaded increment of \(q\).

The broken curves presented in Fig. 24(a1)–(e3) are the reloading curves starting from respective stress reversal points obtained by assuming that the reloading curves follow Eqs. (1) or (2) with \(n = 2\). It may be seen that the measured reloading curves tend to follow this rule only under the following conditions: (i) when cyclic loading was made with a small stress amplitude, (ii) at low strain levels during reloading and (iii) when reloading was made during otherwise TC loading at low \(a'_{h}\) (i.e., Fig. 24(a-1) and (b-1)). In the other cases, the reloading curves generally do not follow this rule. This may be due partly to the fact that the stress–strain relations in the TC and TE tests at the same constant \(a'_{h}\) are substantially un-symmetric and partly to that significant effects of the following three factors are not taken into account by this proportional rule:

It is obvious from Fig. 24 that the tangent modulus at a given deviator during reloading decreases with an increase in the unloaded increment of \(q\) during the immediately preceding unloading process. This trend of softening, described above, should be due to the dominant effects of factor (3) above. In the framework of the elasto-plastic theory, this trend of softening is described by the shrinkage of the yield locus in the pre-peak regime. It is also obvious from Fig. 24 that this trend of softening is stronger at the earlier stages of reloading while it becomes weaker with an increase in \(q\) and has no noticeable effects on the stress–strain behaviour after the start of large-scale yielding.

To highlight the trend of softening during the reloading described above, the reloading curves presented in Fig. 24 have been replotted in Fig. 25 based on the following concept. That is, Fig. 26(a) shows schematically the stress–strain (\(y - x\)) relations during primary loading and those during unload/reload cycles with the common maximum stress in which the stress amplitude increases with cyclic loading, according to Eq. (1) with a constant \(n\). In this case, \(n\) is considered not necessarily equal to two. In this analysis, \(n\) is kept constant at respective unload/reload
cycles. It is to be noted that the effects of the three factors, (1)–(3), are not involved in the relations presented in Fig. 26(a). When the reloading curves presented in Fig. 26(a) are shifted to start from the common origin, Eq. (1) is changed to Eq. (3) and the relations presented in Fig. 26(a) are replotted, as shown in Fig. 26(b):

$$\frac{\Delta y}{n} = f\left(\frac{\Delta x}{n}\right)$$

(3)

Eq. (3) can be refined as

$$\Delta y = f^\text{*}(\Delta x)$$

(4)

The actual reloading relations shifted to start from the common origin presented in Fig. 25 are generally not like those plotted in Fig. 26(b), but they are like those plotted in Fig. 26(c). Representing the first reloading curve by Eq. (4), it is assumed that the second and subsequent reloading curves can be represented by

$$\Delta y = f^\text{*}\left(\frac{\Delta x}{m}\right)$$

(5)

where $m$ may increase from 1.0 with cyclic loading. The values of $m$ shown in Fig. 25 are those obtained by fitting Eq. (5) to respective reloading relations. That is, each obtained $m$ value is the ratio of axial strain between a given reloading curve and the first reloading curve at a fixed $\Delta q$ value (i.e., 50 kPa). Then, the trend of softening during reloading with cyclic loading can be represented by an increase in $m$ from 1.0 with cyclic loading. It seems that the effect of factor (1) “cyclic strain-hardening effect” is not significant when compared with that of factor (3) “additional damage effect”. In this analysis, it was taken into account that the effect of creep deformation (i.e., factor 2) becomes more significant when approaching the maximum stress point from which unloading has started and this trend becomes stronger as the maximum stress becomes closer to the peak stress observed by continuous monotonic loading. The segments of the stress–strain curve where the tangent stiffness becomes noticeably low due to creep deformation are depicted by dotted curves in Fig. 25 and ignored in the discussion below.

Fig. 24. Zoom-ups of stress–strain relations during several unload/reload cycles by increasing stress amplitude (broken curves denote reloading relations when following proportional rule): (a-1) and (a-2) Test no. CMG-D08 (SC), $\sigma'_0 = 0.02$ MPa, (b-1) and (b-2) Test CMG-D07 (SC), $\sigma'_0 = 0.02$ MPa, (c-1) and (c-2) Test CMG-D01, $\sigma'_0 = 0.05$ MPa, (d-1) and (d-2) Test CMG-D02, $\sigma'_0 = 1.0$ MPa and (e-1), (e-2), and (e-3) Test CMG-D14 (SC), $\sigma'_0 = 0.50$ MPa (SC: solid cylindrical specimens).
Then, the following trends of behaviour may be seen from these figures:

1. Eq. (4) is valid only with the data presented in Fig. 25(a1) and (c1), in which the negative axial strain increment during the respective immediately preceding unloading processes is rather small.

2. In the other cases, the reloading stiffness decreases with an increase in the negative axial strain increment during the respective immediately preceding unloading processes. In particular, the stress–strain behaviour becomes very soft when the reloading starts from zero deviator stress following unloading from a higher stress level (Fig. 25(b2), (c2) and (d2)). This trend becomes even stronger when the reloading starts from a triaxial extension stress state after unloading from a higher stress level (Fig. 25(e2) and (e3)).

3. The trend of softening during reloading is well represented by an increase in the $m$ value in Eq. (5). The largest $m$ value is obtained at the last reloading stage in Fig. 25(e-3).

These results indicate that the CMG is damaged by negative irreversible shear strain increments that take place during previous unloading processes (particularly the immediately preceding one), and the negative effects on the reloading stress–strain behaviour increases with an increase in this negative irreversible strain increment.

It may also be seen from Fig. 25 that, even with a reloading stress–strain curve that exhibits largely reduced stiffness at the initial stage, the tangent modulus tends to increase with an increase in the deviator stress, showing that this negative effect gradually decreases with an increase in the positive irreversible axial strain (i.e., with an increase in the positive irreversible shear strain). This trend makes the reloading curve less nonlinear (Fig. 24). In these tests, it appears that the effect of this additional damage fully disappears after the start of large scale yielding and the peak strength is not affected by this additional damage. Yet, it is not known whether effects of this additional damage on the stress–strain behaviour after the start of large-scale yielding could become noticeable or significant if the negative irreversible shear strain increment during unloading becomes larger and if similar tests are performed on a material that is more prone to this type of damage effect.

In summary, if only the peak strengths and the M–C failure criterion are of concern, it is better to perform ML tests without the unloading of $q$ before the restart of the TC loading at the next step at different confining pressure levels to exclude the possible effects of this kind of damage effect.
on the measured peak strength values. More detailed analyses on the stress–strain behaviour during the reloading in the tests presented above, as well as the one during reloading in the ML tests, are beyond the scope of this study and will be reported in the near future.

5. Conclusions

The following conclusions can be drawn from the test results and their analysis presented above:

1. Shear strengths at different levels of confining pressure ($\sigma_0^h$) and an associated Mohr–Coulomb failure criterion of compacted cement-mixed gravelly soil (CMG) as obtained by single step loading TC tests (SL TC tests) at different confining pressures using multiple similar specimens can be estimated rather accurately by a relevant multiple-step loading triaxial compression test (ML TC test) using a single specimen.

2. The peak strengths measured by SL TC tests may be underestimated by a ML TC test:
   a. if TC loading is ceased too early before the peak stress state during strain-hardening at a given step, which is more likely at the first and second steps
   b. if TC reloading is started after the peak stress state has been passed during previous loading steps, which is more likely at later steps where $\sigma_0^h$ is relatively low in the ML test decreasing $\sigma_0^h$.

3. Unlike ordinary unbound granular materials, the reloading stress–strain becomes softer as additional damage associated with the negative irreversible shear strain increment that has taken place during previous unloading processes (in particular, the immediately preceding one) becomes larger. With the CMG tested in the present study, this damage effect gradually decreases with an increase in the irreversible shear strain during reloading, and it disappears after the start of large-scale yielding and the peak strength is not affected by this damage. If only peak strengths and the Mohr–Coulomb failure criterion are to be evaluated, it is recommended that a ML test without full unloading (or with partial unloading) of $q$ be performed before the start of the TC loading at the next step.

4. For reasons a) and b), cited in Term 2, it is of first priority to perform a ML test increasing $\sigma_0^h$ in order to obtain a reasonable result. If a similar second sample is available, it is recommended that a ML test decreasing $\sigma_0^h$ be

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**Fig. 25.** Replot of reloading stress–strain relations: values of $\alpha$ and $m$ are relative values obtained by applying proportional rule: (a-1) and (a-2) Test no. CMG-D08 (SC), $\sigma_0^h = 0.02$ MPa, (b-1) and (b-2) Test CMG-D07 (SC), $\sigma_0^h = 0.02$ MPa, (c-1) and (c-2) Test CMG-D01, $\sigma_0^h = 0.5$ MPa, (d-1) and (d-2) Test CMG-D02, $\sigma_0^h = 1.0$ MPa and (e-1), (e-2) and (e-3) Test CMG-D14 (SC), $\sigma_0^h = 0.50$ MPa (SC: solid cylindrical specimens).
additionally performed. This is because, unlike a ML test increasing \( \sigma'_{f} \), it is more likely that the peak stress state is very closely approached at some intermediate steps in a ML test decreasing \( \sigma'_{f} \). If the effects of data scatter can be properly taken into account, the upper envelope of data from a pair of ML tests increasing and decreasing \( \sigma'_{f} \) could be more representative of the true failure envelope than the one by a single ML test increasing \( \sigma'_{f} \).

5. The proportional rule, often employed to describe the hysteretic stress–strain relations of unbound granular materials, was modified to express the negative effects of damage that have taken place during the immediately preceding unloading branch on the reloading stress–strain behaviour. Based on the above, if only the peak strengths and the M–C failure criterion are of concern, it is recommended that ML tests without unloading of \( q \) be performed before the restart of the TC loading at the next step at different confining pressure levels to exclude the possible effects of this kind of damage on the measured peak strength values.

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**References**


