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Pre-service elementary mathematics teachers' use of strategies in mathematical problem solving

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Abstract

The purpose of this study was to examine pre-service elementary mathematics teachers' strategies used in mathematical problem solving. To collect data about pre-service elementary mathematics teachers' problem solving strategies, survey study design was used. Problem solving test which was developed by Arslan (2002) was used as a measuring instrument. There were ten open ended items in the test and each item check whether specific problem solving strategies were used or not. Participants' performances were graded dichotomously and the strategies that students used to solve problem correctly were determined. The test was administered to 93 pre-service elementary mathematics teachers studying at Aksaray University in elementary mathematics education department during 2010-2011 fall semester. In sample choice, convenience sampling method was used. In this study, descriptive statistics was used. The demographic information was analyzed by using frequencies and percentages. The results of the study revealed that pre-service elementary mathematics teachers have capability to use problem solving strategies and to solve problems; however the use of different strategies is rather limited.

Keywords: Problem solving; problem solving strategies; pre-service elementary mathematics teachers

1. Introduction

1.1. What is a problem?

Heddens and Speer (1997) state that problems are perceived as exercises that need basic computational skills to solve in math courses. Whereas, problems are not limited with mathematics courses. Dewey (1933) defines problem as everything that gets someone confused, creates challenging situation and makes beliefs uncertain. Also, problem is defined as a situation that one faces with some blockage while solving the problem. That is, a task can be a problem if it involves a point that problem solver does not know how to proceed (Kroll & Miller, 1993). Another definition of problem is a situation that one cannot find any ready solution for it (Henderson & Pingry, 1953). Whether a situation is a problem or not changes from person to person depending on the individual's reaction to it. Besides, in order for a situation to be a problem, a person should be aware of the situation and be interested in solving it but s/he should be unable to proceed to find the solution (Lester, 1980). Today a problem for a person may not be a problem another day (Henderson & Pingry, 1953). When definitions are analyzed, in order for a situation to be a problem; there should be a challenge, the situation confronted should be new, the person facing a problem should be perplexed and willing to find a solution to that situation.

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Problems are divided into two according to various approaches; first one is routine problems and the second one is non-routine problems. According to Polya (1957), routine problems are formed by adding different data to already solved problems and routine problems are solved by without adding new things but only applying a known algorithm step by step. Altun (2002) states that, in developing computational skills, solving routine problems plays an important role. Routine problems can be solved by using an algorithm and they can be solved in one, two or more steps (Holmes, 1995). Thus, routine problem occurs when a problem solver knows the way of finding the correct answer and knows that the way is suitable for that problem (Mayer & Hegarity, 1996). Although routine problems can be solved by using only computational skills and by applying formulas, non-routine problems require organizing given data, classifying, making relationship in addition to computational skills (Jurdak, 2005). Besides, a non-routine problem exists when a problem solver does not know how to solve the problem and not able to see the solution because it is not obvious (Mayer & Hegarity, 1996).

1.2. What is problem solving?

Polya (1962) defines problem solving as trying to find a suitable action to reach a desired point but being unable to reach expected end. According to Branca (1980), problem solving is a primary aim that should be developed and according to Charles, Lester and O'Daffer (1987), problem solving is a scientific research method. Besides, problem solving is a teaching method that requires thinking mathematically (Baroody, 1993). NCTM (1989) emphasizes the importance of problem solving in mathematics education so much that, it defines mathematics as problem solving. According to Branca (1980), problem solving is not only a method or a strategy to give meaning to a situation but also a kind of thinking that is used to solve non-algorithmic situations. Since problem solving includes coordination of knowledge, intuitional and critical thinking, it is not reaching a solution by only applying procedures or rules but it means far more complex process (Charles et al., 1987). According to Schoenfeld (1992), problem solving is not related with what is known but it is related with how and when this knowledge is used. Altun (2005) states that, problem solving is making a research to reach a target that is obvious but not easy to reach. If mathematics is problem solving, then problem solving can be defined as eliminating the problem situation by using critical reasoning processes and required knowledge. Problem solving can be generally defined as getting involved in a task for which there is no immediate answer (NCTM, 2000).

There are different approaches in teaching mathematical problem solving. The most well known distinction between these approaches is made by Hatfield (1978). According to Hatfield (1978), there are three basic approaches for problem solving instruction: *teaching via problem solving*, *teaching for problem solving* and *teaching about problem solving*. Later, Schroeder and Lester reemphasized these three approaches in 1989. In *teaching via problem solving*, mathematics topics are introduced with a problem. That is, problems are vehicles to introduce and study on a mathematical task. (Manuel, 1998). In *teaching via problem solving*, problems are valued as primary means of doing mathematics. In *teaching for problem solving*, to solve problems students apply the knowledge that is learned in mathematics lessons. In other words, mathematics is taught in order to teach problem solving. Students are expected to solve both routine and non-routine problems during the learning of mathematics. In *teaching about problem solving*, the strategies and process of problem solving are taught. The teacher who teaches about problem solving underlines the set of four independent phases that are used to solve problems in Polya's problem solving model. These phases are "understanding problem", "devising a problem", "carrying out the plan" and "checking solution". Besides, "heuristics" or "strategies" used in devising a plan phase are taught in teaching about problem solving (Schroeder & Lester, 1989).

1.3. Problem solving strategies

In order to apply problem solving steps, the strategies should be used during problem solving. Hatfield, Edwards, Bitter and Morrow(2007) emphasize that, the strategies help students make progress in solving more challenging and hard problems. They also advise teachers to learn and use the strategy during problem solving. These strategies are logical reasoning, intelligent guessing and testing, extreme cases, accounting all possibilities, adopting a different point of view, visual representation and organizing data. *Logical reasoning* is a thinking process and it helps in doing proofs. Without doing algebraic operation students use their reasoning to find the answer and they do

not waste time in doing operations. *Intelligent guessing and testing* is guessing and trying processes to check the probable conditions. *Extreme cases strategy* is trying maximum and minimum conditions by making one variable constant then problem solver realizes the results of each case. *Accounting for all possibilities* refers to considering all conditions or instances to look for the most suitable one. In solving probability problems, it helps students to see all possible events. *Adapting a different point of view* is thinking of a problem from different perspective. *Visual representation strategy* (making a drawing) is drawing figures or geometric shapes to see the related connections in the problem easily. It is very useful in solving set problems. *Organizing data* is making a list of given data to make the problem clearer (Charles & Lester, 1984).

In addition to these strategies, there are other strategies namely, *working backwards*, *finding a pattern* and *solving simpler analogous problem*. Problem solver begins to *work backwards* when the goal is unique but there are many possible starting points. *Finding a pattern* includes determining a pattern or extending it to discover the answer to the question. A pattern is systematic and predictable repetition of numeric, visual or behavioural data. *Solving simpler analogous problem* changing given problem into one that may be easier to solve and gain the insight needed to solve the original problem (Posamentier & Krulik, 1998).

Bingham (1998) emphasizes the importance of problem solving strategies since a problem can be solved in different ways. Beside knowing the problem strategies, knowing how and when to use these strategies is also important (Polya, 1957). Chapman (2005), states that problem solving has an important role on doing, learning and teaching mathematics. From this perspective, when we think that problem solving should be taught to the students, it should also be taught to pre-service teachers. If we accept problem solving as a basis of teaching mathematics, pre-service mathematics teachers should understand problem solving from a pedagogical perspective. Since teachers are important components of problem solving process, knowing teachers' understanding of problem solving is important and investigation of pre-service elementary mathematics teachers' strategies that are used in solving mathematical problems is an important issue. Exploring this will help future developments of mathematical problem solving in teacher education program. Besides, learning pre-service elementary mathematics teachers understanding and using problem solving strategies will help educators in developing future training programs for pre-service and in-service elementary mathematics teachers.

1.4. Statement of the problem

This study aimed to determine the strategies used by pre-service elementary mathematics teachers in solving mathematical problems. Depending on this aim, the research problem was determined as "What are the strategies used by pre-service elementary mathematics teachers in solving mathematical problems?"

2. Methods

2.1. Research design

Since survey studies collect data from a group of people in order to describe some aspects or characteristics (such as abilities, opinions, attitudes, beliefs or knowledge) of the population of which that group is a part (Fraenkel &Wallen, 2005), this research was carried out by using survey method.

2.2. Sample

The sample of the study consists of first year and second year pre-service elementary mathematics teachers studying at Aksaray University during 2010-2011 fall semester. A convenience sample is defined as a group of individuals who are available for the study (Fraenkel & Wallen, 2005). Therefore, in this study convenience sampling method was used.

2.3. Instrument

In order to collect data for the research problem "What are the problem solving strategies that used by pre-service elementary mathematics teachers in solving mathematical problems", a problem solving test developed by Arslan (2002) was used. The reliability of the instrument was tested by its own developer and Cronbach alpha coefficient was found to be 0,77.

2.4. Data analysis

Descriptive statistics was used in the analysis of the quantitative data. Data analysis primarily consisted of frequencies and percentages. They were calculated using the SPSS 15.0 statistics programme and presented in a table. The instrument consists of 10 items however; in this study 5 of them will be examined. When scoring the items, "1" is used for each correct answer and "0" is used for each incorrect answer or for items that were left empty that is, items on the instrument were scored dichotomously. Since problem solving strategies used to solve mathematical problem were considered, only correct answers were evaluated to see students' use of different strategies

3. Findings

Analysis of students' correct responses showed seven distinct solution strategies: making a drawing, accounting for all possibilities, adapting a different point of view, finding a pattern, organizing data, logical reasoning and working backwards.

Item 1was a general problem that can be solved by using several problem solving strategies. In Item 1, students were asked to respond to "In a room with ten people, everyone shakes hands with everybody else exactly once. How many handshakes are there?" When correct solutions evaluated (68%), it was seen that students used visual representation (8%), accounting for all possibilities (28%) and adopting a different point of view (7%) strategies. Besides, a few students applied combination formula in order to solve this problem. Finally, 32% of the students couldn't solve the problem correctly. These values show that students mostly used accounting for all possibilities in the solution of Item 1. Besides, the use of different strategies was rather limited.

Item 2 mainly intended to determine students' use of finding a pattern strategy. The pattern was introduced without a word problem context. By the figures that were composed of specific series of number students were asked to find the number of triangles needed to form the 20^{th} figure. Students who solved problem correctly (59%) used finding a pattern strategy and 41% of students could not solve the problem. This shows that nearly half of the students were not able to use finding a pattern strategy.

Item 3 was a specific example for making a drawing problem. Making a drawing was the key point for this problem since this problem was much easier to solve via using drawings. In Item 3 students were asked to respond to "There is a frog at the bottom of 10 meters well. This frog can climb 4 meters however it slips back 1 meter after each jump. So, how many times should the frog jump to reach the top of the well?" The results showed that students used making a drawing (18%), organizing data (15%), logical reasoning (5%) and working backwards strategy (3%). However, 59% of the students were not able to solve this item correctly. Students' common mistakes showed that students firstly operated as 4-1=3 and thought that the frog climbed 3 meters for each jump then found the answer as 4. In fact the answer was 3 since the frog reached the top of the well and escaped from the well at its third jump. Therefore, it can be pointed out that more than half of the students missed the key point of the problem.

Item 4 was also a general problem that can be solved by using several problem solving strategies. In Item 4 students were asked to respond to "In a farm the number of the chickens triples for each month and 3 months later number of chickens will be 189, what is the number of the chickens in the farm at the beginning?" The results showed that 8% of students used working backwards strategy. 39% of the students solve this problem by writing an equation and finding the solution directly by solving this equation. 53% of the students were not able to solve the problem correctly. Students made incorrect solutions since they wrote equations in a wrong way. Thus, problem solving strategies were almost not used during the solution of this item.

Item 5 was another general problem that can be solved by using several problem solving strategies. Students were asked to respond to "There are eight iron bars with a length of 2,5 meters each. How many 1 meter iron bars can be obtained from these eight iron bars?" The results showed that students used organizing data (34%) and making a drawing strategy (3%) in the solution of this item. However, 63% of the students were not able to solve the problem correctly. This reveals that more than half of the students were not able to use problem solving strategies.

4. Conclusions and Recommendations

The purpose of this study was to examine pre-service elementary mathematics teachers' strategies used in mathematical problem solving. The results of the study revealed that pre-service elementary mathematics teachers

have capability to use problem solving strategies and to solve problems; however the use of different strategies is rather limited. Analysis of students' correct responses showed they used seven distinct solution strategies: making a drawing, accounting for all possibilities, adapting a different point of view, finding a pattern, organizing data, logical reasoning and working backwards. Despite, most of the students were not able to solve problems correctly. It is thought that the low achievement in problem solving may be attributed to not knowing to use problem solving strategies effectively. In order to achieve objectives or goals, pre-service elementary mathematics teachers should be provided to graduate with well developed problem solving skills.

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