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Geometrical Method for Increasing Precision of Machine Building Parts

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Abstract

Calculation of complex curves with high accuracy is challenging. The curve is approximated by segments of straight lines and carried out smoothing. A new method of analytic geometry to calculate the trajectories of mechatronic systems and CAD/CAM is offered. The theoretical basis of the method is symmetries in the Euclidean plane. The method can accurately calculate the trajectory for the centrally symmetric conic sections and, in some cases, arbitrary differentiable planar curves. The method gives an accurate analytical solution without using radicals. This allows you to find the formulaic dependencies for further calculations. The existence of Lissajous figures allows assuming the presence of a universal method of calculation of complex trajectories based on planar differentiable curves. Some examples of the design of the kinematic mechanisms are presents additionally.

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Keywords: mechatronic systems; calculation of trajectory; symmetries; arbitrary curves.

1. Introduction

Precision manufacturing of engineering products is one of the important tasks. Calculation of complex curves with high accuracy is challenging. The curve is approximated by segments of straight lines and carried out smoothing. Curves and surfaces are sleek by splines methods adequately, but the process is not exact by definition. The solution is offered in the field of analytical geometry. Thus we solve the problem of the uniquely transfer of geometric models between CAD/CAM systems and enterprises additionally. Superposition of simple kinematic links gives rise to a complex mechatronic system. The problem of path accuracy is manifested in the design of metal-cutting equipment is particularly acute. Even flat trajectory of the mechanism in the Euclidean plane is a complex curve [1,2,3]. Jordan curve parameters in the analytic representation are difficult to find. To do this, you must solve the characteristic equation , where – transformation matrix, – scalars, – vector.

Equation can be solved by modern mathematical methods using projective transformations in spaces with lots of measurements [4]. Research is focus in the field of topology, so use them for engineering calculations is difficult. Convex geometry is close to analytic geometry greatest. Important practical results are obtained from its use. She is formulated by adding the ZF-axioms of set theory. The main objective of the reported studies getting sensible results in mechatronic systems and CAD/CAM. Codd's relational algebra as a practical part of ZF-theory has been applied additionally. Let's look at the theory of motion of a curves point in the Euclidean plane initially.

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2. Extended table of Diuedonne's symmetries

Hilbert formulating axioms for Euclidean plane suggested that must be considered construction of linguistic rules. We obtain the Euclidean plane as text by analogy with semiotic analyze of drawings according Leibniz's method of similarity. Levels of study of the text are internal relations in plane. As basic postulates were used: permutation, mirror and with unitary matrix symmetries by Dieudonne [5]; table automorphisms and transfer symmetry by H. Weyl; definition of symmetry by M. Born and binary automorphisms by F. Bachman.

Main attention of research was devoted on permutation symmetry [6, 7]. The Euclidean plane is relation table. The proof is easy, because relation algebra may be working with finite and indefinite table. The conduct of symmetry was considered by relation algebra and semiotic analyze. Application of the method to the Dieudonne automorphisms show that binary symmetry belongs to two mathematical disciplines: set theory and universal algebra.

Extended table of Dieudonne symmetries was built on the base of knowledge symmetry and relational algebra:

- Existing of set ($A \neq \varnothing$ Zermelo)
- Existing of relation ($a_1 R a_2$ Codd)
- Membership element of set ($a \in A$ Fraenkel)
- Universal relation ($f : \Omega \to \Omega'$ implication)
- Linguistic description of the set
- Linguistic presentation of the relation
- Saving cardinality ($\mathbf{m}(A) = const$ Lagrange)
- Saving power relations ($n = const$ in $C_1 x^n + C_2 y^n + C_3 x^{n-1} y^{n-1} + \dots + C_{k-1} x + C_k y + C_0$ Klein)
- Linguistic order ($X \neq Y$ Descartes)
- Mathematical order ($a_i \prec a_{i+1}$, where $a_i, a_{i+1} \in \mathbf{R}$ Kantor).
- Permutation ($a_i \leftrightarrow a_j$)
- Mirror ($a_i \bullet -1 = -a_i$)

Since the connection between automorphisms 10 and 12 first opened Gilbert, the symmetry of knowledge may be named in his honor. Symmetry with numbers 1 and 2 is combined $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Symmetries are grouped in three structured stages: symmetries existence; properties of automorphisms; numeric symmetry. Floor of existence consists the levels the antinomies and the main properties of the object. Stage of attributes include levels of representation in linguistic systems and cardinalities. Numeric floor consists of a order description and the classical symmetries. Gilbert opened the structural properties of symmetry, and Euler's formula $e^{i\pi} = -1$ to determine the relationship between them in universal algebra. Symmetries are joined of the two sciences, so there are exist two methods for solving the characteristic equation.

3. Direct method of linear transformation for central symmetric conic sections

Elliptic and hyperbolic trajectory can be calculated using not projective transformations by the classical method [8] or direct analytical method [6]. Not projective solutions for Jordan curves are absent.

Let there be an arbitrary figure Φ – Jordan curve in the Euclidean plane $\mathbf{R}^2$ in a Cartesian coordinate system defined by the parametric equation

$$\begin{cases} x = f_x(t) \\ y = f_y(t) \end{cases}, \tag{1}$$

where $x, y, t \in \mathbf{R}$. Functions $f_x(t)$ and $f_y(t)$ are piecewise continuous. If the equation's figure is defined $y = f(x)$, it is always possible to write $\begin{cases} x = t \\ y = f(t) \end{cases}$. Class of shapes defined only by the implicit function $F(x, y) = 0$ will not consider in this research. We carry out any transformation of the figure Φ defined by the matrix $\begin{pmatrix} a & h \\ g & b \end{pmatrix}$, where $a, b, h, g \in \mathbf{R}$. It is necessary to obtain the parameters of the transformed figure (to solve the characteristic equation).

Let consider the solution of the characteristic equation for the centrally symmetric conic sections. Only equation of own angle α take of the classic method [8]:

$$\tan 2\alpha = \frac{2\left(bh + ag\right)}{\left(a^2 + h^2\right) - \left(b^2 + g^2\right)}. \tag{2}$$

Parameters semiaxes considered difficult in the classical method, since they represent a radical dependence.

A new direct analytical method for the linear transformation was proposed earlier. He is free from radicals, so it is more simple and accessible for further mathematical derivations. The method is based on the permutation symmetry and other symmetries [7].

We calculate the angle β which is symmetrical own corner α for system (1)

$$\tan 2\beta = \frac{2(gb+ha)}{a^2-h^2-b^2+g^2} \tag{3}$$

in the first step. The angle α is determined from the two equations

$$\tan\alpha_1 = \frac{a\tan\beta - h}{b - g\tan\beta} \tag{4}$$

and

$$\tan\alpha_2 = \frac{b\tan\beta + g}{a + h\tan\beta}\,. \tag{5}$$

Angles are equal if the calculation and transformation is correct. The coefficients $\lambda_1 = c,\ \lambda_2 = d$ are equal

$$c_1 = \frac{a\cos\beta + h\sin\beta}{\cos\alpha} \tag{6}$$

and

$$c_2 = \frac{b\sin\beta + g\cos\beta}{\sin\alpha}\quad , \tag{7}$$

$$d_1 = \frac{a\sin\beta - h\cos\beta}{\sin\alpha}\,, \tag{8}$$

and

$$d_2 = \frac{b\cos\beta - g\sin\beta}{\cos\alpha}\,. \tag{9}$$

The initial system will be: $\begin{cases} x = c_1\cos(t+\alpha) \\ y = d_1\sin(t+\alpha) \end{cases}$ and $\begin{cases} x = c_2\cos(t+\alpha) \\ y = d_2\sin(t+\alpha) \end{cases}$. Both solutions are valid and the choice is not necessary. Semiaxes are equivalent $c_1 = c_2$ and $d_1 = d_2$. The decision does not have square roots in contrast to the classical, so it can easily be used in further calculations.

4. Application for Jordan curves

General solution for Jordan curve could not get unfortunately. If the result of the calculations obtained systems

$$\begin{cases} x = mf_x + nf_y \\ y = mf_x - nf_y \end{cases}, \tag{10}$$

$$\begin{cases} x = kmf_x + nf_y \\ y = -knf_x + mf_y \end{cases}, \tag{11}$$

$$\begin{cases} x = mf_x + knf_y \\ y = -nf_x + kmf_y \end{cases}, \tag{12}$$

than solution may be find.

For example, the system $\begin{cases} x = \cos t - 2\sin 2t \\ y = \cos t + 2\sin 2t \end{cases}$ for the sake of $m = 2$ and $n = 2$ in equation (9) can be reduced to the form $\begin{cases} x = c\cos(t+\alpha) \\ y = d\sin 2(t+\alpha) \end{cases}$.

Parameters for converting epicycloid modulo 2 with parametric system of equations $\begin{cases} x = 3R\cos t - R\cos 3t \\ y = 3R\sin t - R\sin 3t \end{cases}$ (Fig. 1) for the geometric modeling were leaded to incorrect results unfortunately. The same parameters for the Astroid (hypocycloid modulo 4) and ellipse are match exactly. Astroid is described by two identical systems of parametric equations

$\begin{cases} x = \frac{3}{4}R\cos\frac{t}{4} + \frac{1}{4}R\cos\frac{3t}{4} \\ y = \frac{3}{4}R\sin\frac{t}{4} - \frac{1}{4}R\sin\frac{3t}{4} \end{cases}$ and $\begin{cases} x = R\cos^3\frac{t}{4} \\ y = R\sin^3\frac{t}{4} \end{cases}$ [11]. The test was performed for transforming. $\begin{pmatrix} 1 & 1.2 \\ 1 & -1.2 \end{pmatrix}$

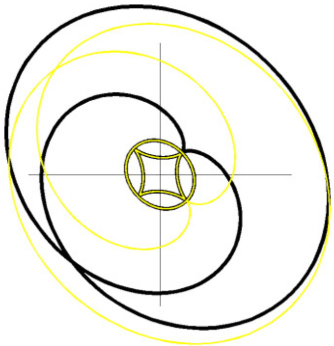

Fig. 1. Error in transformation of epicycloid.

The program algorithm is quite simple:
1. The requested transformation parameters;
2. We output the desired asteroid, epicycloids and ellipse will transform every part of the line of it. Output is black;
3. Forward transformation parameters;
4. Output a new curves with parameters obtained in yellow.

Please note all the curves have the same parameters after the same transformation.

Remember a few mathematical facts from the theory of plane curves before proceeding directly to the review of the calculation of trajectories. One of the most important types of Jordan curves are Lissajous figures. They are defined by the parametric equations: $\begin{cases} x = a_1 \sin(\omega_1 t + \phi_1) \\ y = a_2 \sin(\omega_2 t + \phi_2) \end{cases}$, where $a_i, t, \phi_i \in \mathbf{R}$, $\omega_i = n_i / m_i$, $n_i, m_i \in \mathbf{Z}$. Coefficients determine the form of the curve belong to integers. Studies systems (9) - (11) were also performed for transformation $\begin{pmatrix} a & h \\ g & b \end{pmatrix}$, where $a, b, h, g \in \mathbf{Z}$ [7]. Therefore it is necessary to solve two problems: to check whether the parameters of the matrix to be real numbers and deal with the "wrong" calculations.

We will carry out the research method of geometric modeling with minimal appeals to the mathematical theory. Consider the three curves astroid, epicycloid modulo 2 and the ellipse. The last curve to use checks is not contradictory method of calculation. We will investigate the direct analytical method as a black box, and the results are compared with the mathematical theory.

Factor k in the transformation matrix $\begin{pmatrix} km & n \\ -kn & m \end{pmatrix}$ is only valid in the coordinate x, so it defines a tension/compression along the axis x. Coefficient determined by the matrix $\mathbf{C_x} = \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$. Matrix multiplication is not commutative in general. It is easy to prove that in this case the product is commutative: $\mathbf{GRC_x} = \mathbf{RGC_x} = \mathbf{C_x GR}$, etc. This is symmetry in definition by M. Born.

Solution of the characteristic equation $\mathbf{T}\vec{v} = \lambda\vec{v}$ with linear transformations (9-11) can be different. Vector $\vec{v}$ does not change after the transformation of the ellipse. Vector belongs to the set $\vec{v} \in \left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} y \\ x \end{pmatrix}, \begin{pmatrix} -y \\ -x \end{pmatrix} \right\}$ after the transformation of the Jordan curve. Parameters symmetric transformations find possible and to simplify the system of parametric equations for the complex movements as a result. One system of parametric equations $\begin{cases} x = -mf_x + nf_y \\ y = mf_x + nf_y \end{cases}$ was discovered as additional to those three found earlier. The decision depends on the value of angle β. It is more significant to find a result than the classical own corner α.

Symmetries has an important influence on the trajectory of this is obvious. Let anticipated changes in trajectory when applying an arbitrary linearly independent transformation. The system of parametric equations is divided into two or four portions of movement: $\begin{cases} x = c_1 f_x(t) \\ y = d_1 f_y(t) \end{cases}$, $\begin{cases} x = c_1 f_y(t) \\ y = d_1 f_x(t) \end{cases}$, $\begin{cases} x = -c_1 f_x(t) \\ y = -d_1 f_y(t) \end{cases}$, $\begin{cases} x = -c_1 f_y(t) \\ y = -d_1 f_x(t) \end{cases}$. This hypothesis is tested in research now. Unfortunately, the amount of experiments is very large, and the authors are not ready to publish the results.

5. Application for the design of kinematic mechanisms

The circle and the ellipse are the basis for many of the kinematic mechanisms, such as a flat crank, crank, etc. Payment mechanism causes the trajectory of its motion [8]. The solution of differential equations can be obtained in an analytical form if the trajectory is given of the exact analytical formula.

Rational calculation of the trajectory of motion actuators automatic machines is important for efficient production [9]. The calculations of the motion trajectory of the kinematic mechanisms are often yields a system of parametric equations of a circle or ellipse $\begin{cases} x = a\cos t + h\sin t \\ y = g\cos t + b\sin t \end{cases}$, where $a, b, h, g, t \in \mathbf{R}$.

Experiments were carried out on the basis of the developed stand to check the adequacy of the developed mathematical model. The design of the stand was assembled from aluminum profiles, linear modules and fasteners RK Rose + Krieger. Simple and cheap robot to move along the path of the executive body of the ellipse and the circle may develop on the basis of the proposed mathematical model [10]. Experiments have shown a difference between the theoretical and actual trajectory less than 5%. This robot can be carved with milling cutter details in the form of an ellipse and a circle made of different materials, such as glass, metal, ceramics and wood. The robot can with laser burn in different materials is not necessarily circular holes. Researches are carried out for the production of complex mechanisms based on Jordan curves now.

6. Conclusion

Thus the analytical solution for calculating opened for the complex trajectories of the mechatronic system. Precision of designing is defined within a computer data storage and precision of machining equipment. People design symmetrical mechatronic systems as a rule. Therefore, most of mechatronic systems can be calculated using the method proposed in this paper. Only the first steps are made for the calculation of Jordan curves, but it is obvious that the possibility of calculation of plane and space curves with a complex formula will give new opportunities for engineering and manufacturing.

1. The table of Euclidean plane symmetries is formulated on the basis of semiotic analysis and relational algebra. Table integrates the classic presentation of Dieudonné and Weyl automorphisms. Symmetries belong to set theory and universal algebra.
2. A new method of non projective linear transformations for the centrally symmetric conic sections is presented briefly.
3. Application of the method for analytical formulas calculating for the flat differentiable curves is shown.
4. The hypothesis of a universal method formulated for Jordan curves. The use of other symmetries can give him.
5. Examples of the method in the mechatronic systems are shown.

Thus the analytical solution for calculating opened for the complex trajectories of the mechatronic system. Precision of designing is defined within a computer data storage and precision of machining equipment. People design symmetrical mechatronic systems as a rule. Therefore, most of mechatronic systems can be calculated using the method proposed in this paper.

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