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MODELING THE DYNAMICS OF SOCIAL SYSTEMS

R. Bronson
Fairleigh Dickinson University, Teaneck, NJ 07666, U.S.A.

C. Jacobsen
Technion, Haifa, Israel

Abstract—A procedure is demonstrated for constructing mathematical models of social systems and then simulating the behavior of such systems on a computer. First, sociological concepts are transformed into well-defined variables, and then system dynamics techniques are employed to construct a set of differential and algebraic equations that describe the dynamics of those variables. These dynamics are then simulated and used to establish the empirical adequacy of the theory being modeled. An example is provided for the specific social theory dealing with norm systems.

INTRODUCTION

The availability of small, powerful microcomputers along with simulation software expressly designed for such machines gives mathematical modelers the opportunity to simulate and analyze complex social systems. These simulations can provide a basis for comparing conflicting theories of social behavior and also a laboratory environment for subjecting such theories to systematic empirical testing. None of this existed or was possible until very recently.

Computer simulation is a two-phase process: a mathematical model is first constructed, and then the model is coded and run on a computer for output and analysis. Both stages of the simulation process, modeling and analyzing, provide insight into the social dynamics under study. James S. Coleman, dean of mathematical sociologists, pointed out the advantages of the first phase: "One of the values of mathematics as a language for science is its tendency to bring together research and theory. It forces the theorist to examine just what he does mean, and to set it down in unambiguous form [1]." The utility of the second phase was noted by Donald Knuth: "We often fail to realize how little we know about a thing until we attempt to simulate it on a computer [2]".

The modeling process is much easier to accomplish in the natural sciences than in the social sciences because the "hard sciences" have well-established physical laws that can be expressed in mathematical equations. These equations are the model. The situation is different in the social sciences because the theories are neither so generally accepted nor as precisely expressed as in the natural sciences. Coleman expressed the matter succinctly when he noted that: "... in sociology a set of concepts has never been developed which has some [such] correspondence to a simple yet powerful mathematical language, and at the same time, to actual social behavior." [3]

The first successful simulations of social and managerial systems were the highly publicized works of Forrester [4–6] between 1961 and 1971. His modeling strategy, known as system dynamics, provided a framework for developing mathematical equations to express the dynamic behavior of such systems. Initially, the computing power and software required to run such models limited their use, but today anyone with an IBM-compatible PC or a later version Apple can run very complex systems. The necessary software is commercially available at a modest price. The Achilles' heel of this approach (and the basis for much of the criticism of system dynamics) is the conversion of verbal concepts into mathematically well-defined variables. Such transformations are difficult but not impossible.

In this paper we shall show how to develop a mathematical model for a social theory dealing with the dynamics of change in a norm system. A segment of the theory is briefly described and its concepts are converted into well-defined variables. The system dynamics approach is then used to explore the causal relationships between these variables, which then must be quantified into mathematical equations. The technique should be readily transferable to other social theories.

We coded our model in DYNAMO, because the transposition into computer code is straightforward and the language does not assume more mathematical expertise than that possessed by most sociologists. Furthermore, DYNAMO code is available for IBM-compatibles and Apple computers. However, any good continuous simulation language could be used.

FROM CONCEPTS TO VARIABLES

Many concepts in the social sciences are seductively simple because we use them in everyday discourse. For example, who has not heard of "status", "power" or "deviance", and who does not have intuitive notions about what these concepts mean? Meanings, however, reside not in the words themselves, but rather in the people who use them, and therefore they tend to vary from one person to another. Thus, we must first formulate an acceptable nominal definition for each concept.

A nominal definition should be specific enough to permit only one unequivocal interpretation of the concept no matter by whom it is used, but also transferable, so it can be applied to all relevant situations. To mathematicians, this is nothing more than having a term be "well-defined". For sociological concepts the idea of a well-defined variable involves two distinct processes. First, one must formulate an unambiguous and generally applicable nominal definition, and then one must specify the empirical indicator by which it may be measured.

To illustrate, let us take a section from the model we have developed of a theory about the dynamics of change of informal social norms in industrialized modern societies. The theory states among other matters that if the legitimacy of violating such a norm increases, then informal social control decreases, and, as a result, the number of compliers with the social norm will also decrease. This in turn will cause a further increase in the legitimacy of the violation, and thus the process continues until behavior that had previously been considered a norm violation becomes institutionalized as a new norm (see Fig. 1).

This partial theory involves four concepts: informal social norms, norm compliers, informal social control and legitimacy of violations. Each of these had first to be nominally defined and then transformed into a variable with a numerical range.

For the concept of social norm, we take Morris's [7] standard nominal definition: "*social norms* are generally accepted, sanctioned prescriptions for, or prohibitions against others' behaviors, beliefs, or feelings (i.e. what others ought to do, believe, feel—or else)." We then define an *informal social norm* as a social norm which has not been put in writing or otherwise formally proclaimed. A *norm complier* is someone who behaves according to the norm in question. Informal social control is generally taken to mean the spontaneous pressures that others exert on people to conform with the norm. We define *informal social control* as those individuals who are deterred by family,

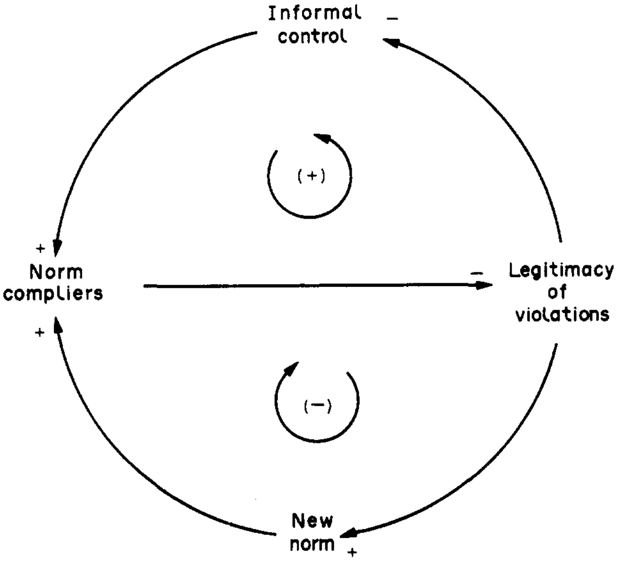

Fig. 1. Causal-loop diagram.

neighbors, friends or others with influence from violating the norm. By specifying the observable results of the social pressures, namely deterrence, our definition brings us one step closer to numerical quantifications. There is no contradiction between the two definitions; ours is just more specific.

Legitimacy of violations is a trickier concept, and it will serve to illustrate some of the theoretical analysis that can be and frequently is involved in arriving at nominal definitions. The concept of legitimacy implies agreement among people that its object is justified and reasonable. Thus, the legitimacy of a norm violation implies general agreement that it is justified and reasonable to violate the norm, or conversely, that the norm in question is unreasonable or unjustified. The more widespread that agreement, the more legitimate the norm violation. Therefore (to facilitate the necessary subsequent quantification), we define *legitimacy of violation* as those who consider the norm unreasonable.

Once the concepts have been given more precise nominal definitions, they must be transformed into variables with measurable units which are realistic, reliable and have face validity [8]. To be *realistic*, the units of measurement must correspond to observable entities in the real world. Thus "level of compliance" is not a realistic variable, though it may have intuitive meaning for many people, while "the population percentage who comply" is. Measurement units are *reliable* when different users, or the same user at different times, will assign the same unit value under the same conditions. It is a necessary condition for consistent measurement. To have *face validity* a measurement unit must adequately convey the connotation of the concept that is being measured. Since many sociological concepts are abstract ideas about social realities, sociologists who need to measure them can only use indirect indicators, that is, readily measurable entities which are closely associated with the concepts to be measured. The essential precondition is, of course, that the unit does indeed measure the intended meaning of the concept.

For the purposes of our model, we defined four variables with the following measurement units:

Legitimacy of violations—the percentage of the population who find the norm in question unreasonable.

Norm compliers—the percentage of the population who comply with the norm.

A new informal norm—a norm violation which is considered reasonable by 65% or more of the population.

Informal control—the percentage of the population who feel restricted by the norm but who are deterred from violating it because of pressures exerted by role partners.

RATES OF CHANGE

Once the variables have been appropriately defined, equations relating these variables to each other can be developed using system dynamics techniques [9]. The first step is to construct a causal-loop diagram—a network of oriented curves that signifiy direct cause and effect relationships between connected variables in the directions indicated by the arrowheads. Figure 1 is a causal-loop diagram for the segment of the theory described in the previous section.

The next step in the system dynamics modeling process is to construct a rate-level diagram (Fig. 2). Rectangles denote state-variables in the system (model) in which units accumulate. System dynamicists call these *levels*. For our model we define two levels: norm compliers and norm violators. The solid lines that connect the two levels denote material flows in the system—in our case, movement of people from the category of compliers to that of violators, and vice versa. The valves on the flow lines represent the actual flows between the levels and they are called *rates*. Circles denote *auxiliary variables*: quantities in the system that enter into the calculation of other quantities (usually the rates), and the dotted lines indicate information flows from one variable to another. Computations may be simplified and the model be refined by adding new variables at this stage, provided they are realistic, reliable, and have face validity. The partial rate-level diagram in Fig. 2 was derived from the causal-loop diagram of Fig. 1, with the acronyms defined in Table 1.

There are two levels in Fig. 2, norm compliers (Comply) and norm violaters (Violat). TryVio and QuitVi are the rates of flows between the two levels, denoting the percentage of the population

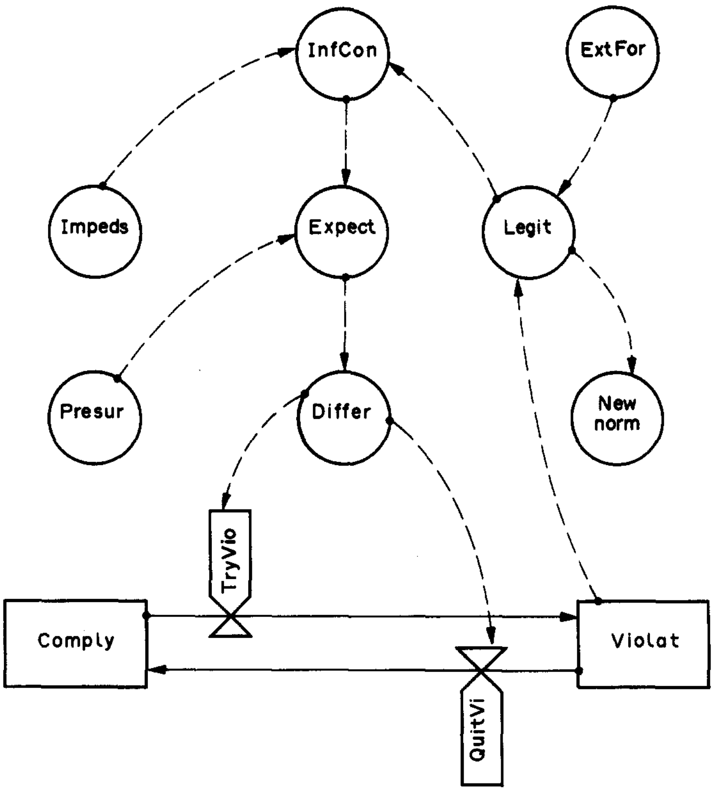

Fig. 2. Rate-level diagram.

who either become violators or quit violations over one time period. If we quantify each of these rates with positive values, then the differential equations governing the model are:

$$\text{d(Violat)/d}t = \text{TryVio} - \text{QuitVi} \tag{1}$$

and

$$\text{d(Comply)/d}t = \text{QuitVi} - \text{TryVio}. \tag{2}$$

Observe that Violat + Comply = 100% of the population, so one of the differential equations is redundant. The remaining problem is to quantify the rates, ultimately in terms of the levels.

DEFINING FUNCTIONAL RELATIONSHIPS

Mathematical modeling is part art and part science, and when dealing with sociological systems the art frequently outweighs the science because we do not know the laws that govern such systems. To develop appropriate functions between variables modelers often rely on the insights of professional sociologists and whatever scraps of quantitative knowledge that are available. The

Table 1

Acronym	Description
Comply	Percentage of the population who comply with the norm
Differ	Arithmetic difference between the expected and the actual percentage of norm violators
Expect	Expected percentage of norm violators in the population
ExtFor	Exogenous forces that create strain between the norm and social reality
Impeds	Structural factors that impose limits on the application of informal control
InfCon	Percentage of those feeling restricted by the norm who are deterred from violation by peer pressure
Legit	Percentage of the population who find the norm unreasonable
Presur	Percentage of the population who feel restricted by the norm
QuitVi	Rate at which people move from Violat to Comply
TryVio	Rate at which people move from Comply to Violat
Violat	Percentage of the population who violate the norm

ultimate test of the correctness of these functions is the extent to which the model can replicate different sets of real data.

The movement of people from compliers to violators and vice versa is influenced by two factors, pressure to violate (Presur) and deterrence, measured here by informal social control (InfCon). Although some people may violate a norm inadvertently, we assume their numbers are marginal and ignore them, since the vast majority of norm violators do so deliberately. Such individuals violate a norm because they perceive the norm to be in conflict with their wants.

We define *Presur* as the percentage of the population who have something to gain by violating the norm, and they are found in both population categories. Individuals who feel pressure to violate and are not deterred from doing so are violators; compliers are either people who have nothing to gain by violating the norm or individuals who do feel pressure to violate but are deterred from doing so.

For simplicity, we let Presur be a constant which must be estimated according to the social situation being studied. There may be cases in which Presur varies, but in the data sets we have studied we have not encountered such a case. Therefore,

$$\text{Presur} = \text{constant}. \tag{3}$$

Informal social control (InfCon) is the percentage of Presur which is deterred from violating a norm because others pressure them into compliance. Thus, 100 − InfCon is the percentage of Presur which is not deterred. The proportion is given by (100 − InfCon)/100. The expected percentage of violators (Expect) is then

$$\text{Expect} = \text{Presur} * (100 - \text{InfCon})/100. \tag{4}$$

Expect is the expected percentage of violators based on the current amounts of Presur and InfCon. The actual percentage of violators is given by the level Violat. Thus, the difference between the expected value and the actual value is

$$\text{Differ} = \text{Expect} - \text{Violat}. \tag{5}$$

Differ is the driving force behind the rates TryVio and QuitVi. If Differ is positive, we expect movement from Comply to Violat; if Differ is negative, we expect movement in the opposite direction. If the other variables do not fluctuate, Differ must eventually be zero, but since it often takes time to commit a violation, there may be a delay before this happens.

If we consider the differential equation

$$\text{d(Violat)/d}t = k * \text{Differ},$$

when Differ is positive and k denotes a positive constant, we can show that the average delay in moving from Comply to Violat equals $1/k$ time units. If we call this delay DelToV, then

$$\text{TryVio} = \begin{cases} (1/\text{DelToV}) * \text{Differ}, & \text{if Differ} > 0, \\ 0 & , \text{ otherwise.} \end{cases} \tag{6}$$

Similarly, if we denote the average delay in moving from Violat to Comply as DelToC, we have

$$\text{QuitVi} = \begin{cases} -(1/\text{DelToC}) * \text{Differ}, & \text{if Differ} < 0, \\ 0 & , \text{ otherwise.} \end{cases} \tag{7}$$

Informal Control (InfCon) is affected by legitimacy of violations (Legit). As violations gain legitimacy, informal control must weaken, so InfCon must be a monotonically decreasing function of Legit. The only question is the shape of the curve. Hamblin *et al.* [10], have demonstrated quite convincingly that a great many diffusion processes follow S-shaped curves, so it seems reasonable to use the same type of model here. The precise shape of the curve may be deduced from available data sets, but whatever S-shaped curve is ultimately chosen, it must remain invariant from one data set to the next. Since a theory should explain the behavior of all instances of the process it proposes to explain, any model representing the theory must remain structurally invariant simulating those processes.

In modern industrialized societies there are structural factors that impose limits to the application of informal social control. The more important of these are anonymity, transience and cultural heterogeneity. We operationalize such impediments as a constant percentage of the population who would not be deterred by their peers from violating the norm even if everyone else considered the norm legitimate. Thus,

$$\text{Impeds} = \text{constant} \tag{8}$$

and this constant has to be estimated according to the social situation being studied. Then 100 – Impeds represents the percentage of Presur who are deterred by InfCon when Legit is zero. Thus

$$\text{InfCon} = f(\text{Legit}), \tag{9}$$

where f is a monotonically decreasing, S-shaped function beginning at (100 – Impeds) when Legit is zero and tending towards zero as Legit approaches 100% (see Fig. 3).

The legitimacy of norm violations varies not only with the percentage of violators in the population but also with external forces (ExtFor) which are exogenous to the particular norm system under investigation. Technological or demographic changes occurring in the society are examples of such exogenous forces. To take these into account, we initialize Legit at run time zero as InLeg (for initial legitimacy), set

$$\text{InLeg} = \text{constant} \tag{10}$$

and estimate this initial value. Then, if there are *known* exogenous time-dependent forces impinging on the legitimacy of violations, we define the base value of Legit (BasLeg) as

$$\text{Basleg} = \text{InLeg} + \text{ExtFor} \tag{11}$$

and enter *the data that document these forces* into the model as a function of time.

Legitimacy of violations will grow as the percentage of violators increases. The maximum value of Legit is 100%, so the maximum increase at any point in time is (100 – BasLeg). We hypothesize that the fractional increase in Legit due to violators (Fract) will also follow an S-shaped curve. That is, Fract is an S-shaped curve that begins at zero and increases to 1.0 as the percentage of violators increases to 100. Thus

$$\text{Legit} = \text{BasLeg} + (100 - \text{Basleg}) * \text{Fract}. \tag{12}$$

EMPIRICAL TESTING

Equations (1)–(12) constitute a mathematical model of dynamic change in social norms, as conceptualized in the theory. To test the model by applying it to any particular social norm, it must be initialized with appropriate constants and delay times which are norm dependent and reflect the

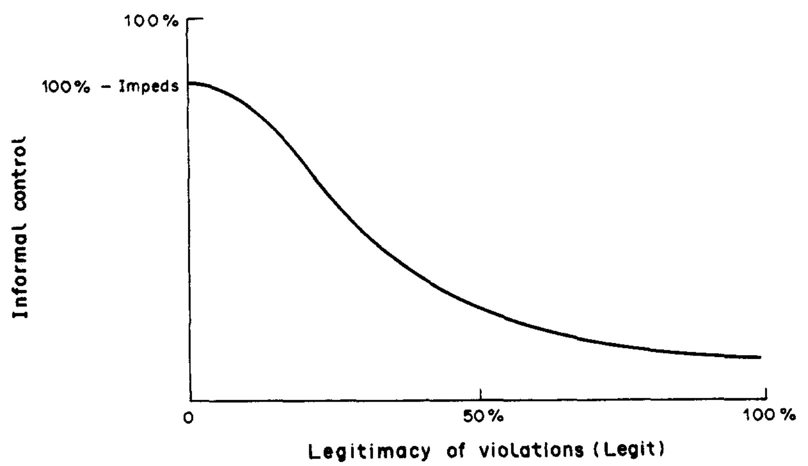

Fig. 3

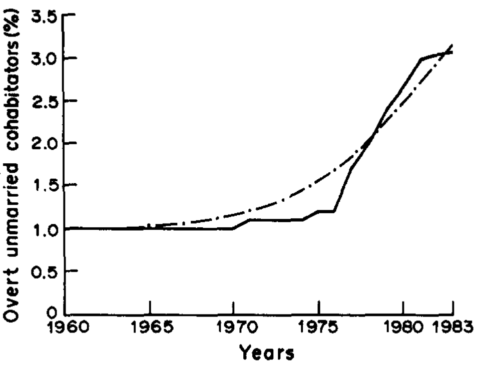

Fig. 4. Model output and data—unmarried cohabitation.

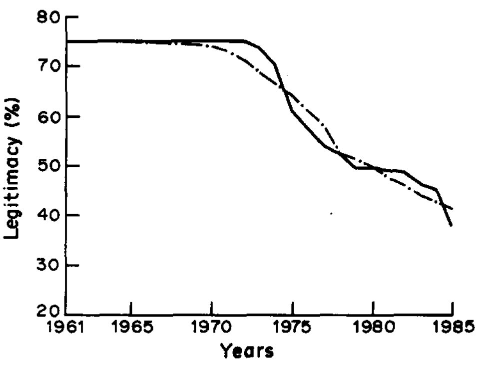

Fig. 5. Model output and data—smoking in public.

social situation at simulated time zero. The model is then run, and the output compared for reproducibility with a time series of empirical data points. This procedure is repeated with a number of different data sets over different time periods, reflecting changes in different norms. For each data set, the model as defined by the differential and algebraic equations remains the same; only the initial conditions at simulated time zero are changed from one run to the next. To the extent that the model can replicate the dynamic behavior of a variety of data sets, it will gain in credibility, and with it the underlying theory gains in empirical adequacy.

We have presented here only a small section of our model to demonstrate the feasibility of modeling dynamic change in social systems. The complete model [11], consisting of some 45 variables and constants, has so far successfully reproduced data sets on unmarried cohabitation in the U.S., 1960–1983; emigration from Israel, 1949–1982; violation of factory safety regulations in the U.K., 1907–1937; violations of factory safety regulations in Israel, 1949–1982; the growth of a norm proscribing smoking in the company of others in the U.S., 1960–1985 and illegal immigration into the U.S., 1960–1985 (cf. Refs [11–13]. Output from three of these simulations are shown in Figs 4–6. In each, the data are marked with a solid curve and the computer output for the corresponding model output is shown with a broken curve. The close fit between the two is obvious and indicates that the theory is empirically adequate.

Experimentation with the model can take many forms. We have asked, for example, what would be the effect of replacing each of the hypothesized nonlinear curves such as Fig. 1 with straight lines. The answer is a marked deterioration in the model's performance [14]. We have also routinely sought confidence limits on the range of values one might take for the initial conditions without adversely affecting the performance of the model, which is a sensitivity analysis of the model parameters. Finally, we hope to use the model to test alternative hypotheses for system behavior. To date, we have produced one data set which allowed for such an investigation [12].

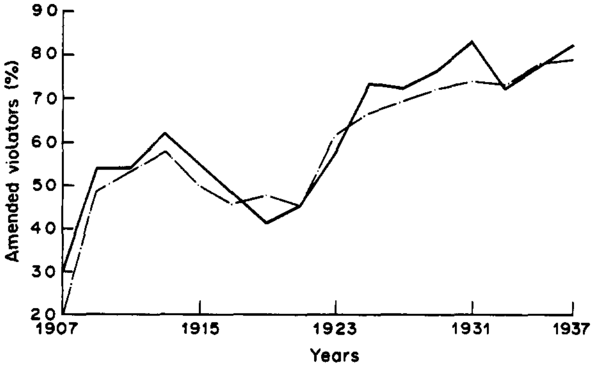

Fig. 6

CONCLUSIONS

If society is to gain confidence in social theories and use such theories in formulating public policy, those theories must be tested and proven, at least empirically. One approach is to develop mathematical models for theories, and then test the theories by testing the models. In this paper, we have indicated how such an approach can be implemented. The advent of microcomputers and the wide availability of simulation software for these machines gives allows anyone with access to the equipment and the requisite knowledge of sociology the ability to carry on such studies.

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