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Optimal multinational capital budgeting under uncertainty

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ABSTRACT

This paper discusses the multinational capital budgeting problem — when there are some candidate foreign projects, which project(s) should the investor choose? In the paper, special cash flows and value sources of foreign projects are introduced. Regarding project parameters such as construction costs, annual net operating cash flows, terminal values of the projects as well as the foreign exchange rates as uncertain variables, the paper proposes one new uncertain zero–one integer model for optimal multinational project selection. To solve the problem, a hybrid intelligent algorithm integrating the 99 Methods and genetic algorithm is provided. As an illustration, an application example is also presented.

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1. Introduction

The original capital budgeting problem is concerned with the allocation of financial resources among candidate projects to obtain a maximum investment return. Since most of the projects are indivisible, the capital budgeting problem usually equals to the project selection problem. In the past, much work has been done on how to select optimal projects in a domestic country. A major contribution to the theoretical formulation and treatment of the problem was made by Weingartner [1]. Regarding the investment parameters such as cash inflows, cash outflows and available investment capital as deterministic numbers, he proposed an integer programming model with the aim of maximizing the net present value profit of the selected projects under budget constraint. Later on, the basic model has been substantially extended to increase its relevance and applicability to the real world situations. Some prime research topics include the choice of a suitable objective function [2,3], the integration of the investors' capital structure into the problem [4], the handling of differences between borrowing rates and lending rates [5], and the integration of project portfolio selection, staff assignment and learning in a holistic model [6], etc.

With the development of economic globalization, more and more corporations make project investment abroad, which means that the investors need to select projects among candidate projects located in different foreign countries. Multinational capital budgeting is much more complex than domestic capital budgeting. One great difference is that the foreign country and the home country may take different policies on the project located in the foreign country, which may result in special cash flows of the international project and finally a different project NPV value from the home country perspective or from the project itself (i.e., the foreign country perspective). In addition, the multinational capital budgeting problem has to handle the foreign exchange translation problem because the cash flows of the foreign project are denominated in the foreign currency and should be converted into home country currency. In the research of multinational capital budgeting, studies mainly focused on analysing the special cash flows encountered in foreign capital expenditures and extending the single-country one project NPV evaluation method to multinational environment from the perspective of the home country. For example, Booth [7] discussed the multinational capital budgeting frameworks. Lessard [8] proposed a multinational Adjusted Present Value (APV) method which is an extension of NPV technique used in analysing domestic

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capital expenditures. Other discussions can be found in the literature such as Shapiro [9], Holland [10], and Eun and Resnick [11] etc.

In both these domestic and multinational capital budgeting studies, parameters such as project cash flows and foreign exchange rates were all assumed to be deterministic numbers. However, in reality, these parameters are usually uncertain. In domestic capital budgeting studies, scholars have tried several ways to handle uncertainty in project parameters. For example, when project parameters were obtained as random variables, stochastic chance-constrained programming models were introduced to produce optimal capital budgeting plans [12–14]. Monte Carlo simulation was used [15] to evaluate the expected return and the risk of a project, and an evolutionary method was proposed to handle multiple stochastic objectives for the project selection problem [16]. When parameters were treated as fuzzy numbers, Carlsson et al. [17] proposed a fuzzy mixed integer programming selection model for R & D projects. Wei et al. [18] used fuzzy set theory to resolve the ambiguities involved in assessing supply chain management alternatives and aggregating the linguistic evaluations. Bas and Kahraman [19] developed a primal–dual pair based on t-norm/conorm relation for the constraints and objective function for a fully fuzzified pure capital rationing problem. Based on credibility measure, Huang [20,21] extended chance-constrained programming idea to solve fuzzy capital budgeting problems. In addition, Huang [22] proposed a mean-variance model for the fuzzy capital budgeting problem in which the expected net present value was used to quantify fuzzy project returns and variance to measure the risk of the project not being able to achieve the investment return.

However, as far as the authors know, there is little research on multinational project selection with uncertain parameters. This motivates the authors to research the problem. As it is more difficult to obtain enough information for international projects than for domestic projects, the international project parameters will more rely on experts' evaluation, which means that project parameters will contain much subjective uncertainty. As foreign exchange rates are influenced not only by economic factors but also heavily by political factors and people's psychological expectation, the evaluation of foreign exchange rates will rely on experts' judgement in many cases too, and contains much subjective uncertainty. Therefore, in this paper, we will discuss the way to select multinational projects with uncertain foreign exchange rates and project parameters containing subjective uncertainty.

The rest of the paper is organized as follows. In Section 2, we will first discuss why we choose uncertainty measure to gauge the uncertain event in multinational project selection. Next, in Section 3, we will introduce the special cash flow terms of international projects for a good understanding of a typical multinational project selection problem. After that, we will propose an uncertain zero–one integer model for multinational project selection with uncertain parameters in Section 4, and will give a hybrid intelligent algorithm for solving the uncertain model in Section 5. In Section 6, we will present an application example. Finally, in Section 7, we will give some conclusion remarks. In addition, for better understanding of our paper, we will review some fundamentals of uncertainty theory in the Appendix.

2. Why uncertain measure?

As discussed in Introduction, the parameters of projects are usually uncertain. If historical data about the project parameters can be obtained and these data can fairly well reflect the future of the project parameters, probability theory is a powerful tool to handle the uncertainty of the project parameters. However, for international projects, it is usually more difficult to obtain enough information for each candidate project and the prediction about the project parameters may have to rely heavily on experts' knowledge and evaluation. Thus the uncertainty of parameters is of subjective uncertainty rather than randomness. Many scholars argued that in this situation we should find another way to describe the project uncertainty, and some of them have tried using fuzzy set theory or credibility theory to solve the different domestic capital budgeting problems in this situation, e.g., Carlsson et al. [17], Huang [22], and Bas and Kahraman [19] etc. However, with the deeper research on the problem, we find that paradoxes will appear if we use fuzzy variable to describe the subjective estimation of project parameters. For example, if a project cost is regarded as a fuzzy number, then we have a membership function to characterize it. Suppose it is a triangular fuzzy variable $\xi = (100, 120, 140)$ million dollars. Based on the membership function, it is known from possibility theory (or credibility theory) that *the cost is exactly* 120 million dollars *with belief degree* 1 *in possibility measure* (*or* 0.5 *in credibility measure*). However, this conclusion is unacceptable because the belief degree of *exactly* 120 million dollars should be almost zero. In addition, the cost being *exactly* 120 and *not exactly 120* million dollars have the same belief degree in either possibility measure or credibility measure, which implies that the cost being *exactly* 120 and *not exactly* 120 million dollars will occur equally likely. This conclusion is quite astonishing and hard to accept. Recently, Liu [23] proposed an uncertain measure which can be used to measure the subjective imprecision quantities. When we use the uncertain measure to gauge the subjective uncertain quantities, the above mentioned paradoxes will disappear immediately. Based on the uncertain measure, much research work, e.g., [24–29], etc., has been done on the properties of uncertain variable and the development of uncertainty theory and the related theoretical work. In the area of capital budgeting, Bhattacharyya et al. [30] proposed an uncertain theory based multi-objective optimization technique for R & D project portfolio selection. Therefore, in this paper, we will also use uncertain measure to gauge the imprecise subjective evaluation and make use of uncertainty theory to select multinational projects with parameters containing subjective uncertainty.

Furthermore, to illustrate our consideration of choosing uncertain measure instead of probability measure to gauge the subjective imprecision quantities, let us study two examples. Suppose we have 10 independent candidate projects, and the real construction cost of each project is 130 million dollars. If the 10 projects are all selected, then the real total construction cost should be 1300 million dollars. However, before the projects have been constructed, the real cost is unknown. After we have acquired information from experts, the construction cost of each project is believed to be between 100 and 140 million dollars, and uniform random variable is used to describe the construction cost between 100 and 140 million dollars. It can be calculated that the probability of the total cost equal to or less than 1290 million dollars is over 99%, which means that the event that the total cost is equal to or over 1290 will not happen almost surely. It is clear that if the decision is made based on this result, the decision will be risky because the total cost will exceed the budget. If we use uniform uncertain variable that has the following uncertainty distribution to describe each project cost

$$\Phi(t) = \begin{cases} 0, & \text{if } t < 100 \\ 0.5, & \text{if } 100 \le t \le 140 \\ 1, & \text{if } t > 140, \end{cases} \tag{1}$$

then the total cost is between 1000 and 1400 million dollars, and the chance (gauged by uncertain measure) of the total cost equal to or over 1290 is 0.5. It is clear that the result is reasonable and the decision made based on this result will be safer.

As another example, suppose we have another 10 independent candidate projects with expected value of each project cost being 120 and variance 100. Suppose the real cost of each project is still 130 million dollars. Before construction, after having acquired information from experts, we use normal random variable $\mathcal{N}(120, 10^2)$ to describe the project cost. It can be calculated that if the 10 projected are selected, the probability of the total cost equal to or less than 1270 million dollars is over 99%, which means that the cost equal to or more than 1270 million dollars will not happen almost surely. Again, the decision made based on this result will be risky because the total cost will exceed the budget. If we use normal uncertain variable $\mathcal{N}(120, 10^2)$ to describe each project cost, it can be calculated that if the 10 projected are selected, the chance (gauged by uncertain measure) of the total cost equal to or less than 1270 million dollars is 78%, which means that there is 22% chance of the total cost equal to or more than 1270 million dollars. It is seen that the result is more reasonable and the decision made based on this result will be safer. For necessary knowledge about the uncertain measure, normal uncertain variable and uncertainty distribution, etc., please refer to the Appendix.

3. Multinational capital budgeting problem

To understand the complex cash flows and value sources of international projects, let us consider a corporation which has k numbers of independent candidate foreign projects for investment. The products produced by the foreign projects are sold locally, which means that the profits earned by the foreign projects are denominated in foreign currencies. The objective is to select an appropriate portfolio of plants such that the profit of the projects in the present value sense can be maximized. The constraint is the budget limitation. The corporation must make a decision at the beginning of the investment year. Suppose it will take one year to construct each candidate project, and each project begins to make money in the same year. There are no lost sales brought by the new plants and there is no capital remittance restriction or other capital restrictions by the foreign countries, either. For description convenience, notations used are described as follows:

d_{tj}: the net operating cash flow of the j-th foreign project denominated in the j-th country currency at the end of the t-th year;
T: the life of projects;
u_j: the terminal value of project j denominated in the j-th country currency;
i_{ud}: the unlevered domestic capital (i.e., the equity capital, or say the corporation's own capital) cost rate;
i_d: the normal domestic borrowing rate;
S_{tj}: the exchange rate of home currency to the j-th foreign country currency (in the form of home currency equivalent per unit of the j-th foreign country currency) in the t-th year;
r_h: corporation tax rate in the home country;
r_j: corporation tax rate in the j-th foreign country;
D_{tj}: depreciation value of project j denominated in the j-th country currency in the t-th year;
c_j: concessionary loan granted by the j-th foreign country denominated in the j-th country currency at the beginning of the investment year;
I_{tj}: interest of loan for project j denominated in the j-th country currency in the t-th year;
b_j: benefit from the concessionary loan granted by the j-th foreign country;
a_j: the initial construction cost of project j denominated in the j-th country currency;
a: the available equity capital of the corporation (i.e., the company's own capital) denominated in the home currency;
x_j: decision variable which is defined by

$$x_j = \begin{cases} 1, & \text{if project } j \text{ is selected}, \\ 0, & \text{otherwise}, \end{cases} \quad j = 1, 2, \ldots, k.$$

Typically, the cash flows and the value sources of foreign projects should include the following terms [8]:

The first term is the after tax net operating cash flow of the projects that can be remitted back to the home country. When employing the corporation tax rate, the higher rate of the home country or the foreign country should be adopted because the tax authority in the home country will give a foreign tax credit for foreign taxes paid up to the amount of the tax liability in the home country. In other words, if the corporation tax rate in the home country is higher, additional taxes are due in the home country which equals the difference between the domestic tax liability and the foreign tax credit. On the other hand, if the foreign tax rate is higher, no additional taxes are due in the home country because the foreign tax credit offsets more than the domestic tax liability. Since each cash flow is considered as a source of value individually, each cash flow is discounted at a rate consistent with the risk inherent in that cash flow. Therefore, from the home country perspective, when we convert the net operating cash flow into the present value form, the home unlevered domestic rate should be used because the operating cash flows will be realized regardless of whether the corporation is levered or unlevered (i.e., the operating cash flows will be realized regardless of whether the corporation borrows or does not borrow money when making investment). Note that the foreign cash flows should be converted into the home country currency. Thus, the present values of the net operating cash flows from the k numbers of candidate projects are calculated as follows:

$$F_1 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{S_{tj} d_{tj} x_j (1 - r_h \vee r_j)}{(1 + i_{ud})^t} \tag{2}$$

where $\vee$ denotes the maximal operator.

The second value source comes from depreciation tax shields since no tax will be paid on the depreciation. The present values of depreciation tax shields are calculated as follows:

$$F_2 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{S_{tj} D_{tj} x_j (r_h \vee r_j)}{(1 + i_d)^t}. \tag{3}$$

When straight line depreciation is adopted, we have

$$D_{tj} = a_j / T.$$

Thus, the present value of depreciation tax shields become

$$F_2 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{S_{tj} a_j x_j (r_h \vee r_j)}{T(1 + i_d)^t}. \tag{4}$$

The third value source is from interest tax shields because no tax will be paid on the interest. The present values of interest tax shields are calculated as follows:

$$F_3 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{S_{tj} I_{tj} x_j (r_h \vee r_j)}{(1 + i_d)^t}. \tag{5}$$

Please note that since the tax savings from both depreciation shields and interest shields are relatively less risky than net operating cash flows, the levered borrowing rate is adopted. Since the foreign cash flows should be converted into the home country currency, the domestic rate is used.

Another value source comes from terminal values of the projects. The present values can be calculated as follows:

$$F_4 = \sum_{j=1}^{k} \frac{S_{Tj} u_j x_j}{(1 + i_{ud})^T}. \tag{6}$$

Usually, the foreign countries will provide concessionary loans (below-market-rate loans) to the foreign direct investment projects in order to attract foreign capital to create employment for their citizens. If that is the case, the corporation can gain benefits from the concessionary loans, which is another source of value. The value of the benefits is the difference between the face amount of the concessionary loan and the present value of the same amount of liability as the concessionary loan (principal and interest included) discounted at the higher normal market rate. Thus, the benefits from the selected project portfolio are calculated as follows:

$$F_5 = \sum_{j=1}^{k} b_j x_j. \tag{7}$$

Let

$$F_6 = \sum_{j=1}^{k} S_{0j} a_j x_j, \tag{8}$$

where S_{0j} is the spot foreign exchange rate of the home country currency to the j-th country currency at the beginning of the investment year (in the form of home currency equivalent of one unit of j-th country currency). It is clear that the project values in the sense of present form can be expressed as follows:

$$F_1 + F_2 + F_3 + F_4 + F_5 - F_6. \tag{9}$$

Thus, to pursue maximum project portfolio value in the present form within the constraint of budget limitation, the projects should be selected based on the following model:

$$\begin{cases} \max F_1 + F_2 + F_3 + F_4 + F_5 - F_6 \\ \text{subject to:} \\ \qquad \sum_{j=1}^{k} S_{0j}c_jx_j + a \geq \sum_{j=1}^{k} S_{0j}a_jx_j \\ \qquad x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, k. \end{cases} \tag{10}$$

4. Uncertain multinational capital budgeting model

In Model (10), all parameters are assumed to be deterministic. Since it is usually difficult to obtain enough information for each foreign project, the project parameters are usually given by experts' evaluation, and thus many parameter values are usually subjective imprecision quantities. As discussed in Section 2, we will use uncertain variables to describe these parameters. Let $\tilde{d}_{tj}$ denote the uncertain net operating cash flows, $\tilde{a}_j$ the uncertain construction cost, $\tilde{b}_j$ the uncertain benefits gained from concessionary loans, and $\tilde{S}_{tj}$ and $\tilde{S}_{Tj}$ the uncertain exchange rates. We have

$$\tilde{F}_1 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{\tilde{S}_{tj}\tilde{d}_{tj}x_j(1 - r_h \vee r_j)}{(1 + i_{ud})^t} \tag{11}$$

$$\tilde{F}_2 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{\tilde{S}_{tj}\tilde{a}_jx_j(r_h \vee r_j)}{T(1 + i_d)^t}. \tag{12}$$

$$\tilde{F}_3 = \sum_{j=1}^{k} \sum_{t=1}^{T} \frac{\tilde{S}_{tj}I_{tj}x_j(r_h \vee r_j)}{(1 + i_d)^t}. \tag{13}$$

$$\tilde{F}_4 = \sum_{j=1}^{k} \frac{\tilde{S}_{Tj}\tilde{u}_jx_j}{(1 + i_{ud})^T}. \tag{14}$$

$$\tilde{F}_5 = \sum_{j=1}^{k} \tilde{b}_jx_j \tag{15}$$

$$\tilde{F}_6 = \sum_{j=1}^{k} S_{0j}\tilde{a}_jx_j. \tag{16}$$

Since the objective is uncertain now, we cannot maximize the objective directly because it is meaningless to maximize a variable. The uncertain constraint does not define any crisp feasible sets either. In this case, we can use expected value as the representative value of the objective and try to maximize it. To ensure that the event of the total expenditure not exceeding the available capital will be very likely to occur, the investors can require that the chance of the total cost of the selected projects not exceeding the available capital should be high enough to a preset safety confidence level. To express the idea mathematically, we have the model as follows:

$$\begin{cases} \max E[\tilde{F}_1 + \tilde{F}_2 + \tilde{F}_3 + \tilde{F}_4 + F_5 - \tilde{F}_6] \\ \text{subject to:} \\ \qquad \mathcal{M}\left\{ \sum_{j=1}^{k} S_{0j}c_jx_j + a \geq \sum_{j=1}^{k} S_{0j}\tilde{a}_jx_j \right\} \geq \alpha \\ \qquad x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, k \end{cases} \tag{17}$$

where E is the expected value operator of the uncertain variable, $\mathcal{M}$ the uncertain measure, and α the preset high confidence level. For safety reason, the α is usually set higher than 0.9.

5. Hybrid intelligent algorithm

Since Model (17) is complex, it is difficult to use traditional method to solve it. We have successfully integrate simulation and genetic algorithm (GA) to produce a hybrid intelligent algorithm for solving domestic capital budgeting problems with fuzzy parameters [21,22]. In book [31], Liu introduced a 99 Method for calculating the expected value of an uncertain variable. Now we will use the 99 Method to calculate the uncertain measure and expected value of the investment return, and then integrate the calculation results into the GA to produce a hybrid intelligent algorithm to solve the uncertain multinational capital budgeting problem. Since the GA in the paper is similar to the GA in paper [22], we just introduce the 99 Method in detail and tell the representation structure and initialization process, and then summarize the algorithm.

5.1. Calculation of uncertain measure and expected value via the 99 Method

It is seen that in the multinational capital budgeting problem, the uncertain variables $\tilde{d}_{tj}, \tilde{a}_j, \tilde{u}_j, \tilde{b}_j, \tilde{S}_{tj}$ and $\tilde{S}_{Tj}$ are all nonnegative uncertain variables. For description convenience, let $x_j, j = 1, 2, \dots, k$ be decision variables, ξ_i be nonnegative uncertain variables with distributions $\Phi_i,\ i = 1, 2, \dots, n$, respectively, and $f(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_n)$ be strictly increasing function with respect to $y_1, y_2, \dots, y_m$, and strictly decreasing function with respect to $y_{m+1}, y_{m+2}, \dots, y_n$. Then it is easy to see that in order to solve the uncertain multinational capital budgeting model (17), we need to handle the following two types of uncertain functions, i.e.,

$$U_1 : x_1, x_2, \dots, x_k \to \mathcal{M}\{f(x_1, x_2, \dots, x_k, \xi_1, \xi_2, \dots, \xi_m, \xi_{m+1}, \dots, \xi_n) \le 0\}$$

and

$$U_2 : x_1, x_2, \dots, x_k \to E[f(x_1, x_2, \dots, x_k, \xi_1, \xi_2, \dots, \xi_m, \xi_{m+1}, \dots, \xi_n)].$$

In order to compute the uncertain function $U_1(x_1, x_2, \dots, x_k)$, we can use the following 99 Method.

First, represent the uncertain variable ξ_i on a computer according to the corresponding uncertainty distributions Φ_i, $i = 1, 2, \dots, n$, as follows:

α_i	0.01	0.02	0.03	$\cdots$	0.99
$\Phi_1^{-1}(\alpha_i)$	$t_{1/1}$	$t_{1/2}$	$t_{1/3}$	$\cdots$	$t_{1/99}$
$\Phi_2^{-1}(\alpha_i)$	$t_{2/1}$	$t_{2/2}$	$t_{2/3}$	$\cdots$	$t_{2/99}$
$\Phi_3^{-1}(\alpha_i)$	$t_{3/1}$	$t_{3/2}$	$t_{3/3}$	$\cdots$	$t_{3/99}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\Phi_n^{-1}(\alpha_i)$	$t_{n/1}$	$t_{n/2}$	$t_{n/3}$	$\cdots$	$t_{n/99}$

(18)

Then, according to Theorem 1 in the Appendix, the uncertainty distribution Ψ of the uncertain variable

$$f(x_1, x_2, \dots, x_k, \xi_1, \xi_2, \dots, \xi_m, \xi_{m+1}, \dots, \xi_n)$$

can be represented on a computer via

α_1	0.01
$\Psi^{-1}(\alpha_1)$	$f(x_1, x_2, \dots, x_k, t_{1/1}, t_{2/1}, \dots, t_{m/1}, t_{m+1/99}, \dots, t_{n/99})$
α_2	0.02
$\Psi^{-1}(\alpha_2)$	$f(x_1, x_2, \dots, x_k, t_{1/2}, t_{2/2}, \dots, t_{m/2}, t_{m+1/98}, \dots, t_{n/98})$
α_3	0.03
$\Psi^{-1}(\alpha_3)$	$f(x_1, x_2, \dots, x_k, t_{1/3}, t_{2/3}, \dots, t_{m/3}, t_{m+1/97}, \dots, t_{n/97})$
$\cdots$	$\cdots$
α_{99}	0.99
$\Psi^{-1}(\alpha_{99})$	$f(x_1, x_2, \dots, x_k, t_{1/99}, t_{2/99}, \dots, t_{m/99}, t_{m+1/1}, \dots, t_{n/1})$

(19)

Thus, it is seen from (19) that we can design the process for computing the uncertain measure

$$\mathcal{M}\{f(x_1, x_2, \dots, x_k, \xi_1, \xi_2, \dots, \xi_m, \xi_{m+1}, \dots, \xi_n) \le 0\}$$

as follows:

99 *Method* 1:

Step 1. Let $i = 1$.
Step 2. Set $y_i = g(x_1, x_2, \dots, x_k, t_{1/i}, t_{2/i}, \dots, t_{m/i}, t_{m+1/100-i}, \dots, t_{n/100-i})$.
Step 3. If $y_i \le 0$, let $i = i + 1$, and then turn back to Step 2.
Step 4. Return $\gamma = 0.01i$.

This γ value is the approximation of the uncertain measure we need.

In order to compute the expected value $U_2(x_1, x_2, \dots, x_k)$, according to Theorem 2, the expected value of ξ can be approximately calculated by

$$E[\xi] = \frac{y_1 + y_2 + \cdots + y_{99}}{99}$$

where $y_i = f(x_1, x_2, \dots, x_k, t_{1/i}, t_{2/i}, \dots, t_{m/i}, t_{m+1/100-i}, \dots, t_{n/100-i}),\ i = 1, 2, \dots, 99$, which are given in (19).

Thus, we design the process for computing the expected value

$$E[f(x_1, x_2, \ldots, x_k, \xi_1, \xi_2, \ldots, \xi_m, \xi_{m+1}, \ldots, \xi_n)]$$

as follows:

99 *Method* 2:

Step 1. Let $i = 1$.
Step 2. Set $y_i = f(x_1, x_2, \ldots, x_k, t_{1/i}, t_{2/i}, \ldots, t_{m/i}, t_{m+1/100-i}, \ldots, t_{n/100-i})$.
Step 3. If $i < 99$, let $i = i + 1$. Then turn back to Step 2.
Step 4. Return $e = \frac{y_1+y_2+\cdots+y_{99}}{99}$.

Please note that a 99 Method can also be a 999 Method or 9999 Method according to the precision requirement of the researcher.

5.2. Solution representation

A solution $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ is represented by the chromosome $\mathbf{C} = (c_1, c_2, \ldots, c_k)$, where the genes $c_i \in \{0, 1\}$, $i = 1, 2, \ldots, k$. The mapping between a solution and a chromosome is $\mathbf{x} = \mathbf{C}$.

5.3. Initialization

Step 1. Determine the number of population size *pop_size*.

Step 2. Randomly generate an integer vector $\mathbf{C}(c_1, c_2, \ldots, c_k)$ from the integer set $\{0, 1\}$.

Step 3. Use 99 Method 1 to calculate the constraint. Then check the feasibility of the chromosome as follows:

$$\textit{If } \mathcal{M}\left\{\sum_{j=1}^{k} S_{0j}c_jx_j + a \geq \sum_{j=1}^{k} S_{0j}\tilde{a}_jx_j\right\} < \alpha, \ \textit{return } 0;$$
*return*1;

in which 1 means feasible, and 0 non-feasible.

Step 4. Take the chromosome as one feasible initial chromosome if it is checked to be feasible. Otherwise, repeat the steps 2 and 3 until a feasible chromosome is obtained.

Step 5. Repeat the above round of steps 2, 3 and 4 *pop_size* times. Then *pop_size* feasible initial chromosomes are generated.

5.4. Hybrid intelligent algorithm

When *pop_size* feasible initial chromosomes are generated, the chromosomes will undergo the process of selection, crossover and mutation operations to produce a new generation of chromosomes. New rounds of evolution will continue until a given number of cyclic repetitions is met. We summarize the algorithm as follows.

Step 1. Input the parameters of GA: the population size *pop_size*, the probability of crossover P_c, the probability of mutation P_m, the parameter in the rank-based evaluation function ν.

Step 2. Initialize *pop_size* chromosomes, in which 99 Method 1 is used to calculate the constraints and check the feasibility of the chromosomes.

Step 3. Calculate the objective values for all chromosomes by 99 Method 2.

Step 4. Give the rank order of the chromosomes according to the objective values, and compute the rank-based evaluation function for all the chromosomes.

Step 5. Calculate the fitness of each chromosome according to the values of rank-based evaluation function.

Step 6. Select the chromosomes by spinning the roulette wheel.

Step 7. Update the chromosomes by crossover and mutation operators, in which 99 Method 1 is used to calculate the constraints and check the feasibility of the chromosomes.

Step 8. Repeat the third to seventh steps for a given number of cycles.

Step 9. Report the best chromosome as the final plan for capital budgeting.

6. An example

LN is a Chinese multinational high-tech corporation that develops, manufactures and markets desktop and notebook computers, workstations, servers, storage drives, IT management software, and other related products and services. Now the corporation is considering investing in 4 foreign countries. They are Mexico, India, Spain, and Brazil. The investment period for each candidate project is 1 year. The lives of the projects are all 10 years. There are no terminal value for each project. The home country corporation rate for high-tech corporations is 15%. The unlevered capital cost in home country is 10%. The normal interest rate of loan in home country is 6%. Available equity capital for project investment is 40 million CNY. The initial construction cost, the net operating cash flows, and the future foreign exchange rates are evaluated by the

Table 1
Uncertain annual operating cash flows and initial construction cost of 4 foreign projects (million foreign currency).

Country j	Annual operating cash flow $\tilde{d}_{tj}$	Initial construction cost $\tilde{a}_j$
1 (Mexico)	$\mathcal{LOGN}(2.6, 0.25^2)$	$\mathcal{LOGN}(3.4, 0.4^2)$
2 (India)	$\mathcal{LOGN}(2.8, 0.3^2)$	$\mathcal{LOGN}(4.2, 0.45^2)$
3	$\mathcal{LOGN}(0.4, 0.1^2)$	$\mathcal{LOGN}(1.6, 0.16^2)$
4	$\mathcal{LOGN}(0.5, 0.12^2)$	$\mathcal{LOGN}(2.1, 0.2^2)$

Table 2
Future foreign exchange rates (equivalent of home currency per foreign currency).

Country j	Uncertain foreign exchange rate $\tilde{S}_{tj}$
1 (Mexico)	$\mathcal{L}(0.3, 0.6)$
2 (India)	$\mathcal{L}(0.1, 0.3)$
3 (Spain)	$\mathcal{L}(7.8, 9.2)$
4 (Brazil)	$\mathcal{L}(2.6, 4.4)$

Table 3
Spot foreign exchange rates (equivalent of home currency per foreign currency).

Foreign country j	Spot foreign exchange rate S_{0j}
1 (Mexico)	0.47
2 (India)	0.13
3 (Spain)	8.5
4 (Brazil)	3.5

Table 4
Company rates, concessionary loan amounts, concessionary loan rates.

Country j	Company rate r_j (%)	Concessionary loan amount l_j (million host country currency)	Concessionary rate $i_{\text{con}-j}$ (%)
1	15	20	3
2	33.66	40	3
3	35	2	5
4	15	3	4

experts and are regarded to be uncertain variables given in Tables 1 and 2. The spot foreign exchange rates are given in Table 3 and the foreign country corporation rates, the foreign concessionary loan amounts and the concessionary loan rates are provided in Table 4. It is required that the interests be paid annually and the principal of the concessionary loans be paid back at the end of the 10-th year.

The company requires that the chance of investment cost not exceeding the company equity capital plus the loans should not be lower than 0.95. Let $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ represent uncertain operating cash flows, uncertain depreciation tax shield values, uncertain interest tax shield values, the benefits gained from concessionary loans, and uncertain construction costs, respectively. According to the discussion in Section 4, to maximize the expected investment return, we build the model as follows:

$$\begin{cases} \max E[\xi_1 + \xi_2 + \xi_3 + \xi_4 - \xi_5] \\ \text{subject to:} \\ \qquad \mathcal{M}\{0.47 \times 20x_1 + 0.13 \times 40x_2 + 8.5 \times 2x_3 + 3.5 \times 3x_4 + 40 \geq \xi_5\} \geq 0.95 \\ \qquad x_j \in \{0, 1\}, \quad j = 1, 2, 3, 4. \end{cases} \tag{20}$$

We use 999 999 Methods to calculate the objective and constraint values. The parameters in the GA are set as follows: the population size is 30, the probability of crossover $P_c = 0.5$, the probability of mutation $P_m = 0.2$, the parameter ν in the rank-based evaluation function is 0.05. A run of the hybrid intelligent algorithm with 100 generations shows that to maximize the expected return under the constraints, the decision maker should select projects 1 and 2. The maximum expected return is 32.55 million CNY in the present value sense.

7. Conclusions

This paper discussed the multinational capital budgeting problem with uncertain foreign exchange rates and project parameters. Typical cash flows and value sources of foreign projects are introduced, and one new uncertain zero–one integer model is proposed. To solve the proposed model, a hybrid intelligent algorithm integrating 99 Methods and the genetic algorithm is provided. The application example shows that the proposed approach is effective for solving the uncertain multinational capital budgeting problem.

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Appendix. Fundamentals of uncertainty theory

Probability theory is a powerful tool for handling randomness, while uncertainty theory is a useful tool for handling subjective uncertainty. Probability theory is an axiomatic mathematical system with probability the core of it, and probability is a set function that satisfies the normality, nonnegativity and countable additivity axioms. Uncertainty theory is an axiomatic mathematical system with uncertainty measure the core of it, and uncertain measure is defined based on the normality, monotonicity, self-duality and countable subadditivity axioms.

Definition 1. Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is called an event. A set function $\mathcal{M}\{\Lambda\}$ is called an uncertain measure if it satisfies the following four axioms [23]:

(i) (Normality) $\mathcal{M}\{\Gamma\} = 1$.
(ii) (Monotonicity) $\mathcal{M}\{\Lambda_1\} \le \mathcal{M}\{\Lambda_2\}$ whenever $\Lambda_1 \subset \Lambda_2$.
(iii) (Self-Duality) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$.
(iv) (Countable Subadditivity) For every countable sequence of events$\{\Lambda_i\}$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

In order to define product uncertain measure, Liu [27] proposed the fifth axiom as follows:

(v) (Product Measure) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots, n$. The product uncertain measure is $\mathcal{M} = \mathcal{M}_1 \wedge \mathcal{M}_2 \wedge \cdots \wedge \mathcal{M}_n$.

Definition 2 (*Liu [23]*). An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set of B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event. In practice, we usually express the event $\{\xi \in B\}$ by $\{\xi \le t\}$ or $\{\xi \ge t\}$ where t is a real number.

In application, a random variable is usually characterized by a probability density function or probability distribution function. Similarly, an uncertain variable can be characterized by an uncertainty distribution function.

Definition 3 (*Liu [23]*). The uncertainty distribution $\Phi : \Re \to [0, 1]$ of an uncertain variable ξ is defined by

$$\Phi(t) = \mathcal{M}\{\xi \le t\}.$$

For example, by a normal uncertain variable, we mean the variable that has the following normal uncertainty distribution

$$\Phi(t) = \left(1 + \exp\left(\frac{\pi(\mu - t)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad t \in \Re,$$

where μ and σ are real numbers and $\sigma > 0$. For convenience, it is denoted in the paper by $\xi \sim \mathcal{N}(\mu, \sigma^2)$. It can be proven from the below Theorem 1 that if $\xi_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$ are normal uncertain variables, then for $k_i \ge 0,\ i = 1, 2, \ldots, n$, the $\sum_{i=1}^{n} k_i\xi_i$ is also a normal uncertain variable $\mathcal{N}\left(\sum_{i=1}^{n} k_i\mu_i, \left(\sum_{i=1}^{n} k_i\sigma_i\right)^2\right)$.

We call an uncertain variable ξ the lognormal uncertain variable if $\ln \xi$ is a normal uncertain variable $\mathcal{N}(\mu, \sigma^2)$. That is, a lognormal uncertain variable has the following uncertainty distribution:

$$\Phi(t) = \left(1 + \exp\left(\frac{\pi(\mu - \ln t)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad t \ge 0.$$

For convenience, it is denoted in the paper by $\xi \sim \mathcal{LOGN}(\mu, \sigma^2)$. It is noted that $\sigma > 0$.

We call an uncertain variable the linear uncertain variable if it has the following linear uncertainty distribution

$$\Phi(t) = \begin{cases} 0, & \text{if } t < a \\ (t-a)/(b-a), & \text{if } a \le t \le b \\ 1, & \text{if } t > b. \end{cases}$$

For convenience, it is denoted in the paper by $\xi \sim \mathcal{L}(a, b)$ where $a < b$.

Especially, we call an uncertain variable the uniform uncertain variable if it has the following uncertainty distribution

$$\Phi(t) = \begin{cases} 0, & \text{if } t < a \\ 0.5, & \text{if } a \le t \le b \\ 1, & \text{if } t > b. \end{cases}$$

For convenience, it is denoted in the paper by $\xi \sim \mathcal{U}(a, b)$ where $a < b$. It can be proven from the below Theorem 1 that if $\xi_i \sim \mathcal{U}(a_i, b_i)$ are uniform uncertain variables, then for $k_i \ge 0,\ i = 1, 2, \ldots, n$, the $\sum_{i=1}^n k_i\xi_i$ is also a uniform uncertain variable $\mathcal{U}\left(\sum_{i=1}^n k_i a_i, k_i b_i\right)$.

When the uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are represented by uncertainty distributions, the operational law is given by Liu [31] as follows:

Theorem 1 (*Liu [31]*). *Let $\xi_1, \xi_2, \ldots, \xi_m, \xi_{m+1}, \ldots, \xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_m, \Phi_{m+1}, \ldots, \Phi_n$, respectively. Let $f(t_1, t_2, \ldots, t_m, t_{m+1}, \ldots, t_n)$ be strictly increasing with respect to $t_1, t_2, \ldots, t_m$ and strictly decreasing with respect to $t_{m+1}, t_{m+2}, \ldots, t_n$. Then*

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_m, \xi_{m+1}, \ldots, \xi_n)$$

is an uncertain variable with uncertainty distribution

$$\Psi(t) = \sup_{f(t_1,t_2,\ldots,t_n)=t} \left(\min_{1\le i\le m} \Phi_i(t_i) \wedge \min_{m+1\le i\le n} (1 - \Phi_i(t_i))\right), \quad t \in \Re \tag{21}$$

whose inverse function is

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \Phi_{m+2}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)), \quad 0 < \alpha < 1 \tag{22}$$

if $\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)$ are unique for each $\alpha \in (0, 1)$.

To tell the size of an uncertain variable, Liu defined the expected value of uncertain variables.

Definition 4 (*Liu [23]*). Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{\infty} \mathcal{M}\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^0 \mathcal{M}\{\xi \le r\}\mathrm{d}r \tag{23}$$

provided that at least one of the two integrals is finite.

It can be calculated that the expected value of the normal uncertain variable $\xi \sim \mathcal{N}(\mu, \sigma^2)$ is $E[\xi] = \mu$, and the expected value of the linear uncertain variable $\xi \sim \mathcal{L}(a, b)$ is $E[\xi] = (a + b)/2$.

Theorem 2 (*Liu [31]*). *Let ξ be an uncertain variable whose uncertainty distribution is Φ. If its expected value exists, then*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha. \tag{24}$$

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