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Understanding $D_{sJ}(2317)$

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Abstract

We analyze the hadronic and radiative decay modes of the recently observed $D_{sJ}(2317)$ meson, in the hypothesis that it can be identified with the scalar $s_{\ell}^P = \frac{1}{2}^+$ state of $c\bar{s}$ spectrum (D_{s0}). The method is based on heavy quark symmetries and vector meson dominance ansatz. We find that the hadronic isospin violating mode $D_{s0} \rightarrow D_s\pi^0$ is enhanced with respect to the radiative mode $D_{s0} \rightarrow D_s^*\gamma$. The estimated width of the meson is $\Gamma(D_{s0}) \simeq 7$ keV.

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1. Introduction

The BaBar Collaboration has reported the observation of a narrow peak in the $D_s^+\pi^0$ invariant mass distribution, corresponding to a state of mass 2.317 GeV [1]. The observed width is consistent with the resolution of the detector, thus $\Gamma \leq 10$ MeV. In the same analysis no significant signals are found in the $D_s\gamma$ and $D_s\gamma\gamma$ mass distributions. The meson, denoted as $D_{sJ}(2317)$, has been subsequently detected by CLEO [2] and BELLE [3] Collaborations, which have also announced the observation of another narrow state, $D_{sJ}(2460)$ decaying to $D_s^{*+}\pi^0$. The announcements of such observations have immediately prompted different interpretations [4–6].

Possible quantum number assignment to $D_{sJ}(2317)$ is $J^P = 0^+$, as suggested by the angular distribution of the meson decay with respect to its direction in the e^+e^- center of mass frame. This assignment can

identify the meson with the D_{s0} state in the spectrum of the $c\bar{s}$ system. Considering the masses of the other observed states belonging to the same system, $D_{s1}(2536)$ and $D_{sJ}(2573)$, the mass of the scalar D_{s0} meson was expected in the range 2.45–2.5 GeV, therefore ~ 150 MeV higher than the observed 2.317 GeV. A D_{s0} meson with such a large mass would be above the threshold $M_{DK} = 2.359$ GeV to strongly decay by S -wave kaon emission to DK , with a consequent broad width. For a mass below the DK threshold the meson has to decay by different modes, namely the isospin-breaking $D_s\pi^0$ mode observed by BaBar, or radiatively. The $J^P = 0^+$ assignment excludes the final state $D_s\gamma$, due to angular momentum and parity conservation; indeed such a final state has not been observed. On the other hand, for a scalar $c\bar{s}$ meson the decay $D_{s0} \rightarrow D_s^*\gamma$ is allowed. However, no evidence is reported yet of the $D_s\gamma\gamma$ final state resulting from the decay chain $D_{s0} \rightarrow D_s^*\gamma \rightarrow D_s\gamma\gamma$. In order to confirm the identification of $D_{sJ}(2317)$ with the scalar D_{s0} , one has at first to understand whether the decay modes of a scalar particle with mass of 2317 GeV can

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be predicted in agreement with the experimental findings presently available. In particular, the isospin violating decay to $D_s\pi^0$ should proceed at a rate larger than the radiative mode $D_{s0} \rightarrow D_s^*\gamma$, though not exceeding the experimental upper bound $\Gamma \leq 10$ MeV. This Letter is devoted to such an issue.

2. Mode $D_{s0} \rightarrow D_s\pi^0$

In order to analyze the isospin violating transition $D_{s0} \rightarrow D_s\pi^0$ one can use a formalism that accounts for the heavy quark spin–flavour symmetries in hadrons containing a single heavy quark, and the chiral symmetry in the interaction with the octet of light pseudoscalar states.

In the heavy quark limit, the heavy quark spin \vec{s}_Q and the light degrees of freedom total angular momentum \vec{s}_ℓ are separately conserved. This allows to classify hadrons with a single heavy quark Q in terms of s_ℓ by collecting them in doublets the members of which only differ for the relative orientation of \vec{s}_Q and \vec{s}_ℓ .

The doublets with $J^P = (0^-, 1^-)$ and $J^P = (0^+, 1^+)$ (corresponding to $s_\ell^P = \frac{1}{2}^-$ and $s_\ell^P = \frac{1}{2}^+$, respectively) can be described by the effective fields

$$H_a = \frac{(1 + \not{v})}{2} [P_{a\mu}^* \gamma^\mu - P_a \gamma_5], \quad (1)$$

$$S_a = \frac{1 + \not{v}}{2} [P'_{1a}{}^\mu \gamma_\mu \gamma_5 - P_{0a}], \quad (2)$$

where v is the four-velocity of the meson and a is a light quark flavour index. In particular in the charm sector the components of the field H_a are $P_a^{(*)} = D^{(*)0}, D^{(*)+}$ and $D_s^{(*)}$ (for $a = 1, 2, 3$); analogously, the components of S_a are $P_{0a} = D_0^0, D_0^+, D_{s0}$ and $P'_{1a} = D_1^0, D_1^+, D_{s1}^+$.

In terms of these fields it is possible to build up an effective Lagrange density describing the low energy interactions of heavy mesons with the pseudo Goldstone π, K and η bosons [7–10]

$$\begin{aligned} \mathcal{L} = & i \text{Tr} \{ H_b v^\mu D_{\mu ba} \bar{H}_a \} + \frac{f_\pi^2}{8} \text{Tr} \{ \partial^\mu \Sigma \partial_\mu \Sigma^\dagger \} \\ & + \text{Tr} \{ S_b (i v^\mu D_{\mu ba} - \delta_{ba} \Delta) \bar{S}_a \} \\ & + i g \text{Tr} \{ H_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \bar{H}_a \} \\ & + i g' \text{Tr} \{ S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \bar{S}_a \} \\ & + [i h \text{Tr} \{ S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \bar{H}_a \} + \text{h.c.}]. \end{aligned} \quad (3)$$

In (3) \bar{H}_a and \bar{S}_a are defined as $\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0$ and $\bar{S}_a = \gamma^0 S_a^\dagger \gamma^0$; all the heavy field operators contain a factor $\sqrt{M_P}$ and have dimension 3/2. The parameter Δ represents the mass splitting between positive and negative parity states.

The π, K and η pseudo Goldstone bosons are included in the effective Lagrangian (3) through the field $\xi = e^{i\mathcal{M}/f}$ that represents a unitary matrix describing the pseudoscalar octet, with

$$\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \quad (4)$$

and $f \simeq f_\pi$. In Eq. (3) $\Sigma = \xi^2$, while the operators D and \mathcal{A} are given by

$$\begin{aligned} D_{\mu ba} &= \delta_{ba} \partial_\mu + \mathcal{V}_{\mu ba} \\ &= \delta_{ba} \partial_\mu + \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{ba}, \end{aligned} \quad (5)$$

$$\mathcal{A}_{\mu ba} = \frac{1}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)_{ba}. \quad (6)$$

The strong interactions between the heavy H_a and S_a mesons with the light pseudoscalar mesons are thus governed, in the heavy quark limit, by three dimensionless couplings: g, h and g' . In particular, h describes the coupling between a member of the H_a doublet and one of the S_a doublet to a light pseudoscalar meson, and is the one relevant for our discussion.

Isospin violation enters in the low energy Lagrangian of π, K and η mesons through the mass term

$$\mathcal{L}_{\text{mass}} = \frac{\tilde{\mu} f^2}{4} \text{Tr} \{ \xi m_q \xi + \xi^\dagger m_q \xi^\dagger \} \quad (7)$$

with m_q the light quark mass matrix

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \quad (8)$$

In addition to the light meson mass terms, $\mathcal{L}_{\text{mass}}$ contains an interaction term between π^0 ($I = 1$) and η ($I = 0$) mesons: $\mathcal{L}_{\text{mixing}} = \frac{\tilde{\mu}}{2} \frac{m_d - m_u}{\sqrt{3}} \pi^0 \eta$ which vanishes in the limit $m_u = m_d$. As in the case of $D_s^* \rightarrow D_s \pi^0$ studied in [11], the isospin mixing

term can drive the $D_{s0} \rightarrow D_s \pi^0$ transition.¹ The amplitude $A(D_{s0} \rightarrow D_s \pi^0)$ is simply written in terms of $A(D_{s0} \rightarrow D_s \eta)$ obtained from (3), $A(\eta \rightarrow \pi^0)$ from (7) and the η propagator that puts the strange quark mass in the game. The resulting expression for the decay amplitude involves the coupling h and the suppression factor $(m_d - m_u)/(m_s - \frac{m_d+m_u}{2})$ accounting for isospin violation, so that the width $\Gamma(D_{s0} \rightarrow D_s \pi^0)$ reads

$$\begin{aligned} \Gamma(D_{s0} \rightarrow D_s \pi^0) &= \frac{1}{16\pi} \frac{h^2}{f^2} \frac{M_{D_s}}{M_{D_{s0}}} \left(\frac{m_d - m_u}{m_s - \frac{m_d+m_u}{2}} \right)^2 \\ &\times \left(1 + \frac{m_{\pi^0}^2}{|\vec{p}_{\pi^0}|^2} \right) |\vec{p}_{\pi^0}|^3. \end{aligned} \quad (9)$$

As for h , the result of QCD sum rule analyzes of various heavy-light quark current correlators is $|h| = 0.6 \pm 0.2$ [9]. Using the central value, together with the factor $(m_d - m_u)/(m_s - \frac{m_d+m_u}{2}) \simeq \frac{1}{43.7}$ [12] and $f = f_\pi = 132$ MeV we obtain

$$\Gamma(D_{s0} \rightarrow D_s \pi^0) \simeq 6 \text{ keV}. \quad (10)$$

Eq. (9) can receive $SU(3)_F$ corrections: a hint on their size comes from the use of $f = f_\eta = 171$ MeV instead of f_π in (9), which gives $\Gamma(D_{s0} \rightarrow D_s \pi^0) \simeq 4$ keV. On the other hand, we neglect corrections related to the finite charm quark mass.

The analogous calculation for $D_s^* \rightarrow D_s \pi^0$ involves the coupling g in (3). Since h and g have similar sizes ($0.3 \leq g \leq 0.6$), it turns out that the transitions $D_{s0} \rightarrow D_s \pi^0$ is enhanced with respect to $D_s^* \rightarrow D_s \pi^0$ essentially due to kinematics, being $\frac{|\vec{p}_{\pi^0}(D_{s0} \rightarrow D_s \pi^0)|^3}{|\vec{p}_{\pi^0}(D_s^* \rightarrow D_s \pi^0)|^3} \simeq 3 \times 10^2$.

3. Radiative $D_{s0} \rightarrow D_s^* \gamma$ decay

Let us now turn to $D_{s0} \rightarrow D_s^* \gamma$, the amplitude of which has the form

$$\begin{aligned} \mathcal{A}(D_{s0} \rightarrow D_s^* \gamma) &= ed [(\epsilon^* \cdot \eta^*)(p \cdot k) - (\eta^* \cdot p)(\epsilon^* \cdot k)], \end{aligned} \quad (11)$$

¹ Electromagnetic contributions to $D_{s0} \rightarrow D_s \pi^0$ are expected to be suppressed with respect to the strong interaction mechanism considered here.

where p is the D_{s0} momentum, ϵ the D_s^* polarization vector, and k and η the photon momentum and polarization. The corresponding decay rate is

$$\Gamma(D_{s0} \rightarrow D_s^* \gamma) = \alpha |d|^2 |\vec{k}|^3. \quad (12)$$

The parameter d gets contributions from the photon couplings to the light quark part $e_s \bar{s} \gamma_\mu s$ and to the heavy quark part $e_c \bar{c} \gamma_\mu c$ of the electromagnetic current, e_s and e_c being strange and charm quark charges in units of e . Its general structure is

$$d = d^{(h)} + d^{(\ell)} = \frac{e_c}{\Lambda_c} + \frac{e_s}{\Lambda_s}, \quad (13)$$

where Λ_a ($a = c, s$) have dimension of a mass. Such a structure is already known from the constituent quark model. In the case of M1 heavy meson transitions, an analogous structure predicts a relative suppression of the radiative rate of the charged D^* mesons with respect to the neutral one [13–16], suppression that has been experimentally confirmed [17]. From (12), (13) one could expect a small width for the transition $D_{s0} \rightarrow D_s^* \gamma$, to be compared to the hadronic width $D_{s0} \rightarrow D_s \pi^0$ which is suppressed as well.

In order to determine the amplitude of $D_{s0} \rightarrow D_s^* \gamma$ we follow a method based again on the use of heavy quark symmetries, together with the vector meson dominance (VMD) ansatz [14,16]. We first consider the coupling of the photon to the heavy quark part of the e.m. current. The matrix element $\langle D_s^*(v', \epsilon) | \bar{c} \gamma_\mu c | D_{s0}(v) \rangle$ (v, v' meson four-velocities) can be computed in the heavy quark limit, matching the QCD $\bar{c} \gamma_\mu c$ current onto the corresponding HQET expression [18]

$$\begin{aligned} J_\mu^{\text{HQET}} &= \bar{h}_v \left[v_\mu + \frac{i}{2m_Q} (\vec{\partial}_\mu - \vec{\delta}_\mu) \right. \\ &\quad \left. + \frac{i}{2m_Q} \sigma_{\mu\nu} (\vec{\partial}^\nu + \vec{\delta}^\nu) + \dots \right] h_v, \end{aligned} \quad (14)$$

where h_v is the effective field of the heavy quark. For transitions involving D_{s0} and D_s^* , and for $v = v'$ ($v \cdot v' = 1$), the matrix element of J_μ^{HQET} vanishes. The consequence is that $d^{(h)}$ is proportional to the inverse heavy quark mass m_Q and presents a suppression factor since in the physical case $v \cdot v' = (m_{D_{s0}}^2 + m_{D_s^*}^2)/2m_{D_{s0}}m_{D_s^*} = 1.004$. Therefore, we neglect $d^{(h)}$ in (13).

To evaluate the coupling of the photon to the light quark part of the electromagnetic current we invoke

the VMD ansatz and consider the contribution of the intermediate $\phi(1020)$:

$$\begin{aligned} & \langle D_s^*(v', \epsilon) | \bar{s} \gamma_\mu s | D_{s0}(v) \rangle \\ &= \sum_\lambda \langle D_s^*(v', \epsilon) \phi(k, \epsilon_1(\lambda)) | D_{s0}(v) \rangle \frac{i}{k^2 - M_\phi^2} \\ & \quad \times \langle 0 | \bar{s} \gamma_\mu s | \phi(k, \epsilon_1(\lambda)) \rangle \end{aligned} \quad (15)$$

with $k^2 = 0$ and $\langle 0 | \bar{s} \gamma_\mu s | \phi(k, \epsilon_1) \rangle = M_\phi f_\phi \epsilon_{1\mu}$. The experimental value of f_ϕ is $f_\phi = 234$ MeV. The matrix element $\langle D_s^*(v', \epsilon) \phi(k, \epsilon_1) | D_{s0}(v) \rangle$ describes the strong interaction of a light vector meson (ϕ) with two heavy mesons (D_s^* and D_{s0}). It can also be obtained through a low energy Lagrangian in which the heavy fields H_a and S_a are coupled, this time, to the octet of light vector mesons.² The Lagrange density is set up using the hidden gauge symmetry method [8], with the light vector mesons collected in a 3×3 matrix $\hat{\rho}_\mu$ analogous to \mathcal{M} in (4). The Lagrangian³ reads as [19]

$$\mathcal{L}' = i \hat{\mu} \text{Tr} \{ \bar{S}_a H_b \sigma^{\lambda\nu} V_{\lambda\nu}(\rho)_{ba} \} + \text{h.c.}, \quad (16)$$

with $V_{\lambda\nu}(\rho) = \partial_\lambda \rho_\nu - \partial_\nu \rho_\lambda + [\rho_\lambda, \rho_\nu]$ and $\rho_\lambda = i \frac{g_V}{\sqrt{2}} \hat{\rho}_\lambda$, g_V being fixed to $g_V = 5.8$ by the KSRF rule [20]. The coupling $\hat{\mu}$ in (16) is constrained to $\hat{\mu} = -0.13 \pm 0.05 \text{ GeV}^{-1}$ by the analysis of the $D \rightarrow K^*$ semileptonic transitions induced by the axial weak current [19]. The resulting expression for $1/\Lambda_s$ is

$$\frac{1}{\Lambda_s} = -4 \hat{\mu} \frac{g_V}{\sqrt{2}} \sqrt{\frac{M_{D_s^*}}{M_{D_{s0}}} \frac{f_\phi}{M_\phi}}. \quad (17)$$

The parameters are obtained from independent channels; we use their central values.

The numerical result for the radiative width

$$\Gamma(D_{s0} \rightarrow D_s^* \gamma) \simeq 1 \text{ keV} \quad (18)$$

shows that the hadronic $D_{s0} \rightarrow D_s \pi^0$ transition is more probable than the radiative mode $D_{s0} \rightarrow D_s^* \gamma$. In particular, if we assume that the two modes essentially saturate the D_{s0} width, we have

$$\Gamma(D_{s0}) \simeq 7 \text{ keV} \quad (19)$$

² The standard $\omega_8 - \omega_0$ mixing is assumed, resulting in a pure $\bar{s}s$ structure for ϕ .

³ The role of other possible structures in the effective Lagrangian contributing to radiative decays is discussed in [16].

and

$$\begin{aligned} \mathcal{B}(D_{s0} \rightarrow D_s \pi^0) &\simeq 0.85, \\ \mathcal{B}(D_{s0} \rightarrow D_s^* \gamma) &\simeq 0.15 \end{aligned} \quad (20)$$

at odds with the case of the D_s^* meson, where the radiative mode dominates the decay rate.

The same conclusion concerning the hierarchy of $D_{s0} \rightarrow D_s \pi^0$ versus $D_{s0} \rightarrow D_s^* \gamma$ is reached in [5,6] using the quark model. Since our calculation is based on a different method, the $s_\ell^P = \frac{1}{2}^-$ and $s_\ell^P = \frac{1}{2}^+$ doublets being treated as uncorrelated multiplets and the radiative transition being computed without relying on quark model notions, we find the agreement noticeable.

4. Conclusions and perspectives

We have found that the observed narrow width and the enhancement of the $D_s \pi^0$ decay mode are compatible with the identification of $D_{sJ}(2317)$ with the scalar state belonging to the $s_\ell^P = \frac{1}{2}^+$ doublet of the $c\bar{s}$ spectrum. However, this conclusion does not avoid other questions raised by the experimental observations, the first one being the low mass of the state, an issue requiring additional investigations.

The second point to mention is that the radiative mode, although suppressed, is not negligible, and should be observed at a level typically represented by the ratios in (20). Both CLEO and BELLE Collaborations have provided the first measurements of the ratio of the radiative to the hadronic $D_{sJ}(2317)$ decay rates

$$\frac{\mathcal{B}(D_{s0} \rightarrow D_s^* \gamma)}{\mathcal{B}(D_{s0} \rightarrow D_s \pi^0)} \leq \begin{cases} 0.059, & [2], \\ 0.05, & [3], \end{cases} \quad (21)$$

both at 90% CL. Taking into account, on the experimental side, the reduced reconstruction efficiency of the radiative mode and the small number of events, and, on the theoretical side, the uncertainty in the numerical result (18) mainly due to the uncertainty in the parameter $\hat{\mu}$, the bounds in (21) are still compatible with the estimate giving (20). However, if the experimental bound for the ratio in (21) will become more stringent and close to zero, this will represent a strong indication of the necessity of a different,

more complex interpretation than a simple $c\bar{s}$ state for $D_{sJ}(2317)$.⁴

Our final remark is that the confirmation of the quantum number assignment $J^{PC} = 0^{++}$, $s_\ell^P = \frac{1}{2}^+$ to $D_{sJ}(2317)$ has two main and rather straightforward consequences. The first one is the existence of the axial vector partner D'_{s1} , with $J^{PC} = 1^{++}$, belonging to the same heavy quark spin doublet. Even in the case where the hyperfine splittings between positive and negative parity states are similar: $M_{D'_{s1}} - M_{D_{s0}} \simeq M_{D'_s} - M_{D_s}$, this meson is below the D^*K threshold. Therefore, its hadronic decay to $D_s^*\pi^0$, at the rate

$$\begin{aligned} \Gamma(D'_{s1} \rightarrow D_s^*\pi^0) &= \frac{h^2}{48\pi f^2} \frac{M_{D_s^*}}{M_{D'_{s1}}} \left(\frac{m_d - m_u}{m_s - \frac{m_d + m_u}{2}} \right)^2 \\ &\times \left[2 + \frac{(M_{D_s^*}^2 + M_{D'_{s1}}^2 - M_{\pi^0}^2)^2}{4M_{D_s^*}^2 M_{D'_{s1}}^2} \right] \\ &\times \left(1 + \frac{m_{\pi^0}^2}{|\vec{p}_{\pi^0}|^2} \right) |\vec{p}_{\pi^0}|^3 \\ &\simeq \Gamma(D_{s0} \rightarrow D_s\pi^0) \end{aligned} \quad (22)$$

would produce a narrow peak in the $D_s^*\pi^0$ mass distribution. The meson $D_{sJ}(2460)$, observed by CLEO [2] and BELLE [3] and the existence of which is suggested by the analysis of the $D_s\gamma\pi^0$ mass distribution by BaBar [1], is a natural candidate for such a state.

The second consequence concerns the doublet of scalar and axial vector mesons in the $b\bar{s}$ spectrum. Since the mass splitting between B and D states is similar to the corresponding mass splitting between B_s and D_s states, such mesons should be below the BK and B^*K thresholds, thus producing narrow peaks in $B_s\pi^0$ and $B_s^*\pi^0$ mass distributions, with rates resulting from expressions analogous to (9)–(22).

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⁴ The same conclusion is reached in [6].