ELSEVIER

Available online at www.sciencedirect.com
ScienceDirect

Procedia Technology 21 (2015) 540 – 546

Procedia
Technology

SMART GRID Technologies, August 6-8, 2015

Recursive Variational Mode Decomposition Algorithm for Real Time Power Signal Decomposition

Soman K P[a], Prabaharan Poornachandran[b], Athira S[a]*, Harikumar K[a]

[a]*Center for excellence in Computational Engineering and Networking, Amrita Vishwa Vidyapeetham*
[b]*Amrita Centre for Cyber Security systems and Networks, Amrita Vishwa Vidyapeetham*

Abstract

Conventional methods of signal decomposition are observed to fail in power system applications and computationally intensive algorithms like EMD, VMD, EWT are found to give better performance. The heavy computations associated with them restricts their use in real time applications and stream processing. This paper presents a recursive block processing technique for real time signal decomposition. The use of recursive FFT and the clever initializations of the center frequencies in the existing VMD algorithm helps in reducing the computational complexity and hence speeds up the process. This low complexity algorithm was tested on synthetically generated power signals and the results were observed to be consistent with the existing VMD algorithm.

Peer-review under responsibility of Amrita School of Engineering, Amrita Vishwa Vidyapeetham University

*Keywords:*Recursive FFT, VMD, computational complexity, power signal, decomposition.

1. Introduction

The advent of smart grid technology and the modern day electronics has introduced many sensitive devices to our electrical distribution networks and this makes the maintenance of power quality a critical issue. Deviations from the standard power signal parameters results in poor device performance and may even lead to permanent damages in certain situations. Hence, there is the need for efficient real time power quality monitoring systems which can identify even slight variations in the signal parameters and thus take preventive action to avoid damages due to abnormal system behaviour. Power quality is typically a function of power signal parameters and hence a signal processing technique to identify signal variations can effectively identify distortions in the power signal. Signal

* Corresponding author. Tel.09790446239
*E-mail address:*athirasubhash@outlook.com

Peer-review under responsibility of Amrita School of Engineering, Amrita Vishwa Vidyapeetham University
doi:10.1016/j.protcy.2015.10.048

analysis and inference is a widely explored field of signal processing. One of the oldest and simplest methods for the frequency analysis of a stationary signal is the analysis of the signals Fourier spectra, however this method essentially fails in the case of power signals with distortions which are non-stationary in nature. Methods like the short time fourier transform (STFT) analyses the signal over fixed sized windows over which the signal characteristics are assumed to be stationary and hence offers good time resolution but poor frequency resolution (uncertainty principle). The Fast Fourier Transform (FFT) introduced formerly by Gauss came to be used widely after the paper by Cooley and Tukey [9] in 1965 accelerated the heavy computations behind Fourier transform and made the real time implementation of these algorithms imaginable. Then came up the wavelet transform which uses size adjustable windows for analysis and thus offers better resolution as compared to methods like STFT and also offers better denoising methodologies. However the performance heavily relies on the choice of wavelet and the number of decomposition levels chosen for analysis. [2], [3] discusses methods for implementation of algorithms using wavelet transforms in real time. All these methods decomposes the signal into a set of fixed basis and hence proves to be the wrong choice of decomposition technique in many instances. Further, they may lead to misleading inferences in many cases due to their poor adaptability to the signal nature. Basis functions which could adapt to signal nature would thus be a better choice for representation of all types of signals. It was with this idea that the Empirical Mode Decomposition was proposed by Huang *et.al* [11], and gave satisfactory results in certain cases where the conventional methods failed; but there was no strong mathematical reasoning behind the framework. [4], [5] and many more works tried to develop a mathematical reasoning for the same and arrived at similar techniques with slight variations and different interpretations. One such work by Daubechies [10] resulted in the synchro-squeezed wavelet transform which proved to be a good decomposition technique but is a heavily time consuming task and hence a real time realization of the same is a challenge. Then came up methods like Variational Mode Decomposition (VMD), Empirical Wavelet Transform (EWT) [6], [1] which also take up an adaptive basis for signal representation. Letter [8] discusses the use of these methods for power signal analysis and proves that they are good choice for the study of power signal distortions. Significant energy is present in only a very small number of frequency components in power signals and this facilitates the easy decomposition of power signal into Intrinsic Mode Function (IMF) as in VMD. Further the prior knowledge about the possible frequency components of the power signals allows us to fix the modes in decomposition of these signals and thus facilitates easy identification of presence of noise and other signal distortions and the exact time of onset of these distortions. Further, the components responsible for distortion can be captured and hence analysed for identification of the sources for these distortions. One major factor which prevents the real time implementation of these methodologies is the heavy computations involved behind these decompositions. This paper proposes a modification to the VMD algorithm by introducing the method of recursive FFT to estimate the Fourier transform as part of the VMD algorithm. In the real time implementation of the algorithm one new sample is introduced to the frame in each iteration to replace the oldest sample in that frame. Implementation of the recursive FFT for the Fourier transform calculation in each stage saves a lot of calculation overheads and hence results in an algorithm implementable in a real time system.

2. Algorithm

Variational Mode Decomposition decomposes a signal into a set of Intrinsic Mode Functions (IMF) such that each IMF has a compact frequency support around a central frequency and the sum of these components exactly gives the original signal. The problem of identifying these IMFs is formulated as an optimization problem and solved using the Alternating Direction Method of Multipliers (ADMM) [12]. The optimization routine tries to minimize the sum of bandwidths of 'K' IMFs subject to the condition that the sum of the components is equal to the original signal. 'K', the number of components to be involved in the decomposition is chosen prior to the optimization routine. The detailed formulation and the algorithm are given in [1]. The corresponding optimization problem is given by

$$\min_{u_k, w_k} \left\{ \sum_k \left\| \partial t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\} \quad (1)$$
$$s.t. \quad \sum_k u_k = x$$

where $u_k(t)$ represents the k^{th} IMF with centre frequency at ω_k and x is the original signal. The corresponding ADMM update functions as discussed in [1] is as follows:

$$\hat{u}_k^{n+1} \leftarrow \frac{\hat{x} - \sum_{i<k} \hat{u}_i^{n+1} - \sum_{i>k} \hat{u}_i^{n} + \dfrac{\hat{\lambda}^n}{2}}{1 + 2\alpha(\omega - \omega_k^n)^2} \quad (2)$$

$$\omega_k^{n+1} \leftarrow \frac{\int_0^\infty \omega \left| \hat{u}_k^{n+1}(\omega) \right|^2 d\omega}{\int_0^\infty \left| \hat{u}_k^{n+1}(\omega) \right|^2 d\omega} \quad (3)$$

$$\hat{\lambda}^{n+1} \leftarrow \hat{\lambda}^n + \tau(\hat{x} - \sum_k \hat{u}_k^{n+1}) \quad (4)$$

where $\hat{u}_k, \hat{x}, \hat{\lambda}$ are the fourier transforms of the components, original signal and the Lagrangian multiplier (part of the optimization routine) respectively.

In the case of real time analysis of power signals, the signal can be sampled at a fixed rate and analyzed in blocks of fixed size. The energy in the VMD components of each block can be separately analyzed to identify variations in signal behaviour and hence identify distortions. The computation of the VMD algorithm for each block is quite computation intensive and hence direct application of the real time scenario is a difficult task. As mentioned earlier in (1), the VMD algorithm requires the computation of the fourier transform of each block of signal and here we propose to compute this transform using recursive FFT to save computations [7] . Let $x \in R^N$ represent the samples corresponding to one block of signal taken for analysis using VMD. In each block a new sample is introduced into x to replace the oldest sample so that the sequence remains unchanged except for a single shift and change in value at one location. Once the Fourier transform of the initial sequence is computed then the transforms of successive blocks can be estimated without explicitly computing all the coefficients by FFT algorithm. This formulation can be arrived at by making use of the properties of Fourier transform. The initial set of N samples can be represented as $x_{initial} = [x(0)\ x(1) \ldots\ldots x(N-1)]$ and let $X(k)$ represent the corresponding transform coefficients obtained by $X = W_N x_{initial}$ where

$$W_N = \begin{bmatrix} W_N^{n=0,k=0} & W_N^{n=0,k=1} & \cdots & W_N^{n=0,k=N-1} \\ W_N^{n=1,k=0} & W_N^{n=1,k=1} & \cdots & W_N^{n=1,k=N-1} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^{n=N-1,k=0} & W_N^{n=N-1,k=1} & \cdots & W_N^{n=N-1,k=N-1} \end{bmatrix}; W_N^{nk} = e^{\frac{-j2\pi nk}{N}}, k = \{0,1,...,N-1\} \quad (5)$$

Then in the next instant the block for processing would be $x_{next} = [x(1)\ x(2) \ldots\ldots x(N-1)\ x(N)]$ which can in fact be visualized as the sum of two sequences x_1 , $x_2 \in R^N$ given by $x_1 = [x(N-1)\ x(1) \ldots\ldots x(0)]$ and

$x_2 = [0\ 0 \ldots\ldots\ldots(x(N) - x(0))]$ where x_1 is the original signal shifted by 1 sample and x_2 is the difference between $x_{initial}$ and x_{next}. Now, by the linearity property of Fourier transforms the transform coefficients of x_{next} is the sum of the corresponding coefficients of x_1 and x_2. The transform coefficients of x_1 is given by the shifting property of Fourier transform expressed as and the coefficients corresponding to x_2 is given by multiplying $(x(N) - x(0))$ with the last column of W_N. Thus in each stage the Fourier transform of the present block can be calculated as

$$x((n - n_0))_N \xleftrightarrow{DFT} X(k)e^{-j2\pi kn_0/N} \tag{6}$$

$$X_{next}(k) = (X_{initial}(k) + (x(N) - x(0)))e^{\frac{j2\pi k}{N}} \tag{7}$$

where $a \in R^N$ is a vector of all ones. In the algorithm $R(.)$ represents the FFT computed using this method. This step essentially avoids the computation intensive FFT and replaces it by far lesser number of additions and multiplications. The computation of an N point FFT requires $N/2 \log_2 N$ complex multiplications and $N \log_2 N$ complex additions whereas in the proposed approach we require only N multiplications and N additions. Another computation intensive section of the algorithm is the ADMM portion which continuously iterates until convergence at the solution for the optimization routine. This rate of convergence depends on the initial values of u_k, ω_k and λ and can be enhanced if the initializations are more close to the original solution. After the computations for the first block are done, these values of $\hat{u}_k, w_k$ can provide a rough estimate of the corresponding values for the upcoming blocks and can then be used as initializers to ensure faster convergences.

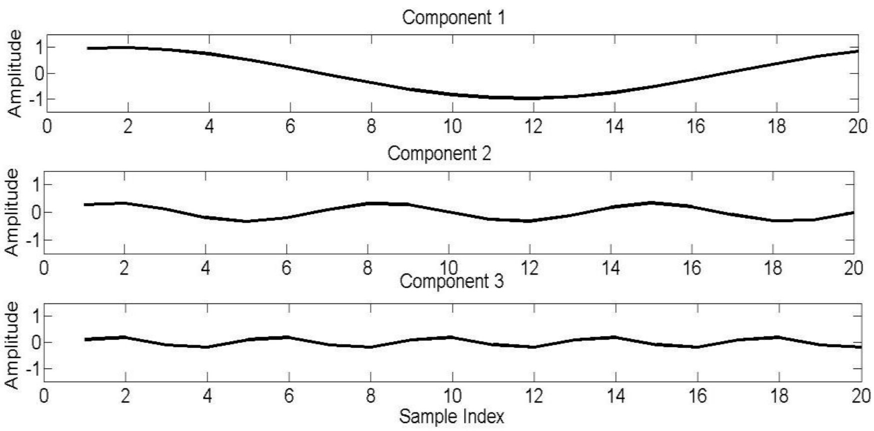

Fig.1. Decomposition Components

The corresponding algorithm is given below:

$b \leftarrow 0$

initialize $\hat{u}^1_{k,b},\ \omega^1_{k,b},\ \hat{\lambda}^1_b,\ n \leftarrow 0, a = [1\,1 \ldots 1]_{1\times N}$

$\hat{x} = fft(x)$

repeat

$b \leftarrow b+1$

set $\hat{u}^1_{k,b} = \hat{u}^n_{k,b-1},\ \omega^1_{k,b} = \omega^n_{k,b-1},\ \hat{\lambda}^1_b = \hat{\lambda}^n_{b-1},\ n \leftarrow 0$

while $\sum_k \left\|\hat{u}^{n+1}_{k,b} - \hat{u}^n_{k,b}\right\|_2^2 \Big/ \left\|\hat{u}^n_{k,b}\right\|_2^2 > \varepsilon$

$n \leftarrow n+1$

for $k = 1:K$ do

Update $\hat{u}_{k,b}$ for all $\omega \geq 0$

$$\hat{u}^{n+1}_{k,b} \leftarrow \frac{R(\hat{x}_b) - \sum_{i<k} \hat{u}^{n+1}_{i,b} - \sum_{i>k} \hat{u}^n_{i,b} + \frac{\hat{\lambda}^n_b}{2}}{1 + 2\alpha(\omega - \omega^n_{k,b})^2}$$

Update $\omega_{k,b}$:

$$\omega^{n+1}_{k,b} \leftarrow \frac{\int_0^\infty \omega \left|\hat{u}^{n+1}_{k,b}(\omega)\right|^2 d\omega}{\int_0^\infty \left|\hat{u}^{n+1}_{k,b}(\omega)\right|^2 d\omega}$$

end for

Dual ascent for all $\omega \geq 0$:

$$\hat{\lambda}^{n+1}_b \leftarrow \hat{\lambda}^n_b + \tau(\hat{x}_b - \sum_k \hat{u}^{n+1}_{k,b})$$

end while

3. Discussion

The performance of the algorithm was tested on synthetically generated signals of the form

$$x(t) = \cos\left(\frac{2\pi f_f t}{f_s}\right) + \frac{1}{3}\cos\left(\frac{2\pi 3 f_f t}{f_s}\right) + \frac{1}{5}\cos\left(\frac{2\pi 5 f_f t}{f_s}\right) \tag{8}$$

where f_f representing the fundamental frequency was taken to be 50Hz , f_s the sampling frequency was taken as 1024Hz and t corresponds to the sample index. The number of IMF's 'K' was set to be 3 for the analysis and the energy of all the components after decomposition of each block was taken as an indicator for signal distortions. Further the block size was set to be 20 for analysis. The block size typically decides how much short lived distortions will be evident in the analysis and hence can be varied for different applications in different contexts. The corresponding components observed for a typical block is given in Fig. 1.

It was observed that the algorithm could perform as efficiently as usual VMD in terms of signal decomposition and at a better rate in terms of time taken for computations. Further, the algorithm was also tested on signals with voltage sag and swell existing for a duration of 0.01 seconds and it was found that the decomposition was proper even in regions of signal transitions and the corresponding plot of energy of blocks clearly reveals the presence of distortion. Further, the effect of signal distortion will be evident in the energy plot as soon as the first effected sample is introduced into the analysis block. Thus by observing the variation trend of the energy signal we can identify the presence of abnormal signal variations and use this as an indicator for various precautionary actions. The corresponding energy plot for a signal with voltage sag and swell are respectively given in Fig. 2 and Fig. 3 .

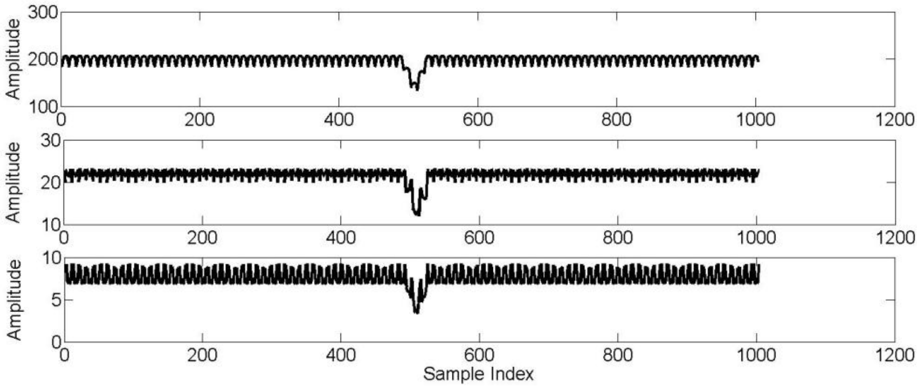

Fig. 2. Energy plot for sag signal components

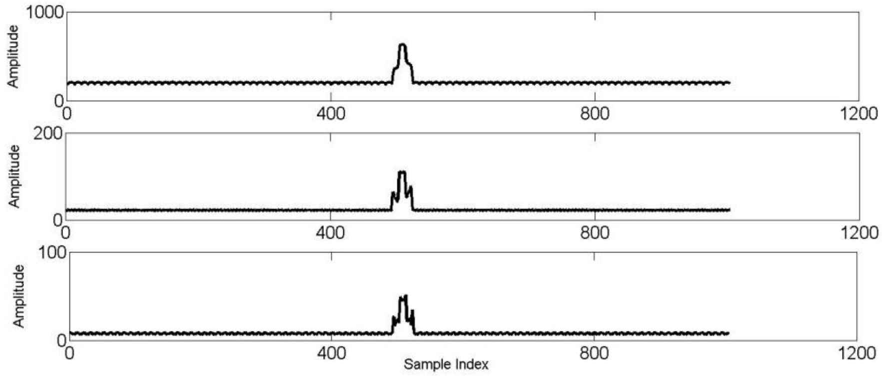

Fig. 3. Energy plot for swell signal components

4. Conclusion

This algorithm addresses the issue of real time power signal decomposition. The performance of the proposed algorithm was compared with the existing VMD algorithm and was found to give consistent results with considerable speedup owing to the reduced computational complexity and modified initialization procedure. The algorithm also offers $(1-(2/\log(n)))\times 100\ \%$ decrease in the number of complex multiplications and $(1-(1/\log(n)))\times 100\ \%$ decrease in complex additions. Sensitivity to short lived distortions depends on the block size and could be varied according to the application. This idea could be extended with slight variations to speed up the decomposition process in other contexts. This allows the algorithm to be used in real time power quality analysis, to detect and analyze voltage/current swells and sags and so on.

Acknowledgement

The authors would like to thank all at the Center for Excellence in Computational Engineering and Networking for their direct and indirect support for this work.

References

[1] Dragomiretskiy, Konstantin, and Dominique Zosso., Variational mode decomposition., Signal Processing, IEEE Transactions on 62.3 (2014): p.531-544.

[2] Daubechies, I., The wavelet transform, time-frequency localization and signal analysis, Information Theory, IEEE Transactions on , vol.36, no.5,p.961,1005,Sep1990 doi: 10.1109/18.57199

[3] Shensa, M., The discrete wavelet transform: wedding the a trous and Mallat algorithms, Signal Processing, IEEE Transactions on , vol.40, no.10,p.2464,2482,Oct1992
doi: 10.1109/78.157290

[4] Deléchelle, Eric, Jacques Lemoine, and Oumar Niang. , Empirical mode decomposition: an analytical approach for sifting process, Signal Processing Letters, IEEE 12.11 (2005): p.764-767.

[5] Flandrin, Patrick, Gabriel Rilling, and Paulo Goncalves. Empirical mode decomposition as a filter bank., Signal Processing Letters, IEEE 11.2 (2004): p.112-114.

[6] Gilles, Jerome. , Empirical wavelet transform. Signal Processing, IEEE Transactions on 61.16 (2013): p. 3999-4010.

[7] Ribeiro, Paulo Fernando, et al. Power Systems Signal Processing for Smart Grids. John Wiley & Sons, 2013.

[8] Aneesh, C., et al. ,Performance Comparison of Variational Mode Decomposition over Empirical Wavelet Transform for the Classification of Power Quality Disturbances Using Support Vector Machine., Procedia Computer Science 46 (2015):p. 372-380.

[9] Cooley, James W., and John W. Tukey. , An algorithm for the machine calculation of complex Fourier series., Mathematics of computation 19.90 (1965):p. 297-301.

[10] Daubechies, Ingrid, Jianfeng Lu, and Hau-Tieng Wu. , Synchrosqueezed wavelet transforms: an empirical mode decomposition-like tool, Applied and computational harmonic analysis 30.2 (2011): p. 243-261.

[11] Huang, Norden E., et al. , The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis., Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. Vol. 454. No. 1971. The Royal Society, 1998.

[12] Boyd, Stephen, et al. , Distributed optimization and statistical learning via the alternating direction method of multipliers. Foundations and Trends® in Machine Learning 3.1 (2011):p. 1-122.