Available online at www.sciencedirect.com
ScienceDirect
Transportation Research Procedia 7 (2015) 598 – 614
Transportation Research Procedia
www.elsevier.com/locate/procedia

21st International Symposium on Transportation and Traffic Theory, ISTTT21 2015, 5-7 August 2015, Kobe, Japan

Information provision strategies eliminating deluded equilibrium caused by travellers' misperception

Takahiko Kusakabe [a]*, Yuta Nakano [a]

[a] *Tokyo Institute of Technology, 2-12-1-M1-20, O-okayama, Meguro, Tokyo, 152-8552, Japan*

Abstract

Providing travel time information may be effective at reducing travel costs. However, this information does not always match the actual travel time that travellers will experience. Furthermore, the information is often asymmetrically provided within the network, owing to the limitations of observation devices, prediction model calibration, and uncertainty about road conditions. The purpose of this study is to investigate the effects of predictive travel time information that is asymmetrically provided to travellers. This study formulated a dynamic traffic assignment model in origin-destination (OD) pair with two parallel routes, while considering travellers' learning processes and within-day and day-to-day dynamics. In this study, it is assumed that different information will be provided to each traveller, according to within-day traffic dynamics. Furthermore, the information is provided for only one of two possible routes, because of observation limitations. The effects of information accuracy are also discussed in this study. The results of numerical analysis indicated that information provisions possibly reduced the negative effects of deluded equilibrium state, even when the information was only provided for one of the routes. Different effects of the travel time information and its variation were illustrated according to the allocation of the bottleneck capacities of two routes.

Selection and peer-review under responsibility of Kobe University

Keywords:deluded euilibrium; bounded rational choice; advanced travel information systems; dynamic assignement

1. Introduction

Providing travel time information may be effective at reducing travel costs. By using this information, travellers can attempt to change their routes according to current conditions. It is apparent that accurate, real-time information is

---

* Corresponding author. Tel.: +81-3-5734-2575; fax: +81-3-5734-2575.
E-mail address: t.kusakabe@plan.cv.titech.ac.jp

Selection and peer-review under responsibility of Kobe University
doi:10.1016/j.trpro.2015.06.031

required to choose the most efficient routes. Many conventional studies have attempted to improve the accuracy of travel time information, and to evaluate the benefits of an advanced travel information systems (ATIS). Ettema & Timmermans (2006) classified the travel time information in three categories: Retrospective, Descriptive and Predictive information. "Retrospective" information is obtained from the historical data. The second category "Descriptive" describes a current situation. The third category "Predictive" describes a situation after starting a trip.

When the predictive travel time information is very accurate, travellers seem to be able to choose appropriate routes to minimise their travel costs. However, the travel time information does not always match the revealed travel time that travellers will experience. Furthermore, the information is often asymmetrically provided within the network, owing to conditions of observation and information provision. For example, information accuracy varies from one road section to the next; occasionally, information is not available for some sections because of observation device limitations, model calibration, and road condition uncertainty. Therefore, travellers may learn possible travel times through their daily commuting experiences, and incorporate that knowledge with the travel time information; based on these observations, they might adjust their route choice behaviour. Thus, as investigated in Chen & Mahmassani (2004 and 2009), mutual dependence of the learning process and road network performance is significant to know day-to-day evolution of traffic flow. Also as shown in Chorus et al. (2006), interactions between perception of travel time and information service's reliability is significant factor to know the effect of the information services.

To understand the role of the information, the effects on bounded rational choices and deluded equilibrium that are caused by false travel time perception must be considered, because of their significance. Nakayama et al., 1999 indicated the deluded equilibrium as a state where "drivers are locked in delusions and do not believe their perceived travel times can be improved by changing routes." In this state, travellers may be still capable of improving perceived travel time of unchosen routes. This state may stabilise the system before converging to an actual equilibrium state. Several studies have investigated the properties of bounded rational assignment in transportation systems (e.g., Mahmassani & Chang, 1987, Nakayama et al., 1999). Many studies that used a behavioural survey and a laboratory experimental approach were conducted to reveal relationships between travel time information provision and travel choice behaviour (Iida et al., 1994, Khattak et al., 1996, Mahmassani & Liu, 1999, and Avineri et al., 2006). Simulation approaches, including day-to-day evolution, were used to show the relationships between travel time information and network-wide performance (Emmertink et al., 1995; Jha et al., 1998). The information may help or hinder travellers' memories, as well as their trial-and-error based route choice behaviour processes. The reliability of the real-time information is a significant variable that influences commuters' pre-trip departure time and route-switching decisions (Mahmassani & Liu, 1999). Notably, when some elements of the network are disrupted or newly added, the role of the information may be more significant, because travellers do not have sufficient experience on the changed network (Guo & liu, 2011).

Travellers adjust their route choice behaviour based on daily experiences when the information is not perfectly accurate. Many conventional studies analysed dynamical traffic evolution. These studies mainly focused on flow evolution (e.g. Smith, 1984, Friesz et al., 1994) and network equilibrium stability (e.g. Horowitz, 1984, Watling, 1999, Bie & Lo, 2010). These dynamical traffic evolution studies assumed that traffic flow systems were static. Within-day dynamics are also an important factor in evaluating the effects of information accuracy. This is because the effects caused by information (such as congestion information) can only affect travellers who enter the network later. To describe within-day dynamics, the dynamic flow model is required in place of the static flow model; in the latter, travel costs affect travellers homogeneously.

The purpose of this study is to investigate the effects of predictive traffic time information that is asymmetrically provided to travellers. In this study, we focus on the interactions between the day-to-day learning, information, and dynamic traffic flow system. We investigate the deluded equilibrium state in the proposed framework to examine the effects of the information. This study assumes that predictive travel time information, according to the evolution of within-day traffic, will be provided to each traveller. Furthermore, the information is provided for only one of two available routes, because of observation limitations. In addition, the effects of information accuracy are discussed. In order to analyse the information's effects on the day-to-day dynamics of route choices, a day-to-day dynamic traffic assignment model with a dynamic traffic flow system is proposed in Section 2. Section 3 describes the analysis of the proposed model, using a Monte Carlo simulation. Section 4 concludes this paper.

2. Model

We assume that the travellers' perceptions of travel time and information accuracy are obtained from learning processes that occur during their daily travel experiences. The model describes a traveller's perception of travel time, their learning processes, and their day-to-day dynamic route choices under a dynamic traffic flow system. In this study, travellers can partially acquire travel time information about their available routes before their departure. It is assumed that the travellers know the statistical property of the travel time information accuracy that describes gaps between the predictive information and actual travel time.

Figure 1 shows the structure of the proposed model. The model consists of three major components that describe travel behaviour, traffic flow, and an information system. In this study, in order to investigate the fundamental properties of the interactions based on dynamic traffic flow system which is more complicated system than the static one, a simple model of which behaviour could be easily anticipated is applied to each element. The travel behaviour model describes travellers' perceptions, learning processes, and route choice behaviours. The learning model is for learning the travel time of each route. After their trips, the learning model updates travellers' perceived travel time, based on their experiences.

The expected travel time is obtained with Bayesian updating that integrates two travel time distributions, derived from experience and travel time information. In this integration process, variations in the perceived travel time distribution and travel time information are regarded as the travellers' confidence in the respective distributions. The travellers deterministically choose their route, according to the expected travel time.

This study considers a single origin-destination (OD) pair with two parallel routes. The dynamic traffic flow system is described by a point queue model satisfying first-in, first-out (FIFO). In order to quantitatively describe the error of the prediction, this study represent the predicted travel time by the accurate predicted travel time and error terms. This is because the model will be used for discussing how the information and its quality influence the system. By using the current queue length and bottleneck capacity of every link, travel times are perfectly predictable immediately before each traveller's departure from the origin, especially in a two-link network with one origin-destination pair. Based on the accurate prediction, decreases in information accuracy caused by observation errors can be evaluated by using artificially given random errors.

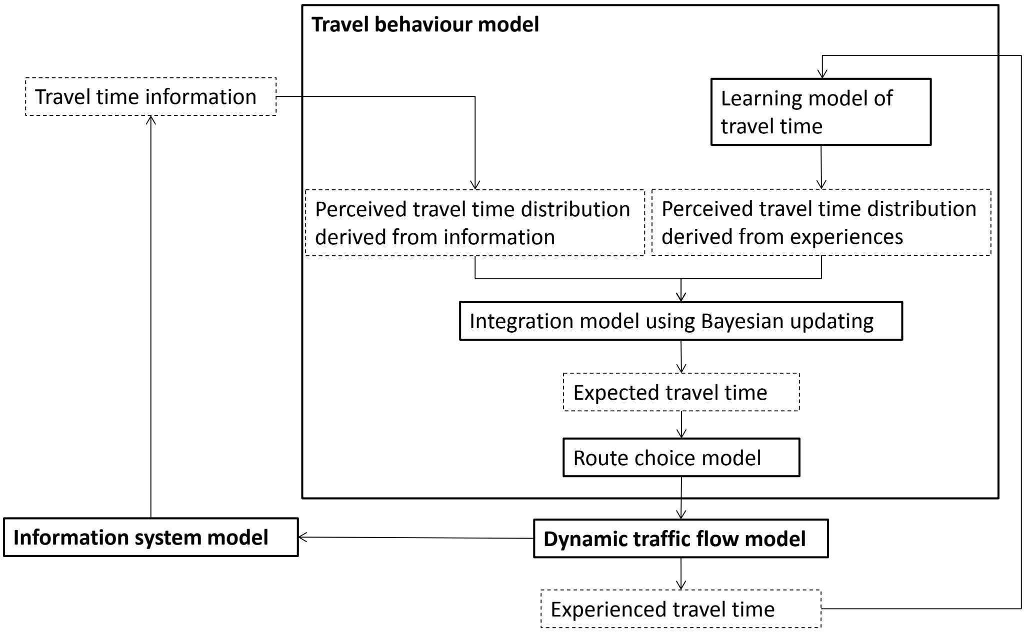


Fig.1 Model framework

2.1. Travel cost and dynamic traffic model

The within-day dynamic traffic flow system is described using a point queue model satisfying FIFO. A two-link network with one OD pair is employed to simplify travellers' route choice behaviours and the information provision systems. The travel cost of each link is related to queuing caused when demand exceeds bottleneck capacities. This study assumes that only one vehicle can depart from the origin during each discrete time step, in order to normalise the number of vehicles. Travel costs experienced by a traveller departing from the n th time step ($n = 1,2,...,N$) on the k th day are described by

$$w_{n,r}^{(k)} = \begin{cases} \dfrac{\delta_{n,r}^{(k)}}{\mu_r} + w_{n-1,r}^{(k)} - 1 & if\left(w_{n-1,r}^{(k)} - 1\right) \geq 0 \\ \dfrac{\delta_{n,r}^{(k)}}{\mu_r} & otherwise \end{cases}, \tag{1}$$

where μ_r is the capacity of Route $r = \{1,2\}$, and $\delta_{n,r}^{(k)}$ is the discrete variable describing the n th travellers' route choice. When the traveller chooses Route r, $\delta_{n,r}^{(k)}$ is set to 1, otherwise it is set to 0. In the two-link network, $\delta_{n,2}^{(k)}$ is represented by $1-\delta_{n,1}^{(k)}$. Note that a queue condition may arise which affects travellers departing after the n th time step, is defined by $0 < \mu_r < 1$.

2.2. Information provision

This study assumes that predicted travel time information is provided to the travellers. As described in Eq. (1), the precise travel time that is expected to be experienced by the n th traveller can be derived after the $n-1$ th traveller has departed. The travel information can be generated based on this travel time. In order to describe the observation errors resulting from observation limitations, random errors are artificially added to the information. The information provided for Route r for the n th traveller on the k th day is described as

$$\tau_{n,r}^{(k)} = \begin{cases} \dfrac{1}{\mu_r} + w_{n-1,r}^{(k)} - 1 + \sigma\varepsilon_{n,r}^{(k)} & if\left(w_{n-1,r}^{(k)} - 1\right) \geq 0 \\ \dfrac{1}{\mu_r} + \sigma\varepsilon_{n,r}^{(k)} & otherwise \end{cases}, \tag{2}$$

where $\varepsilon_{n,r}^{(k)}$ is the random variable that follows the normal distribution $N(0,1)$, and σ describes the standard deviation of the information. The standard deviation σ is known by travellers as a statistical property of prediction accuracy of the travel time information.

2.3. Perception updating and route choice

Travellers' perception of travel time distribution is updated based on their daily experiences. This study assumes the perceived travel time distribution follows the normal distribution where the parameters are mean $m_{n,r}^{(k)}$ and standard deviation $s_{n,r}^{(k)}$. Each traveller updates both $m_{n,r}^{(k)}$ and $s_{n,r}^{(k)}$ after the trip, only the chosen route. The perception update occurring after the $k-1$ day's trip of the n th traveller is respectively described as

$$m_{n,r}^{(k)} = \alpha\delta_{n,r}^{(k-1)} w_{n,r}^{(k-1)} + \left(1 - \delta_{n,r}^{(k-1)}\alpha\right) m_{n,r}^{(k-1)} \tag{3}$$

$$s_{n,r}^{(k)} = \alpha\delta_{n,r}^{(k-1)} \left| w_{n,r}^{(k-1)} - m_{n,r}^{(k-1)} \right| + \left(1 - \delta_{n,r}^{(k-1)}\alpha\right) s_{n,r}^{(k-1)} \tag{4}$$

where α is the weight for updating perceived parameters with the newly experienced travel time ($0 < \alpha \le 1$). If the route is not chosen by a traveller on day $k-1$, these equations equal to $m_{n,r}^{(k)} = m_{n,r}^{(k-1)}$ and $s_{n,r}^{(k)} = s_{n,r}^{(k-1)}$ respectively.

Before a traveller chooses the route, the travel time experienced by the individual traveller and the information acquired from the within-day road network system is combined, if the information is available for Route r. Bayesian updating is used for updating the distribution of perceived travel time with the provided information. One of the characteristics of Bayesian updating is that the posterior distribution can include both the deviation of information and perceived travel time distributions. Integrated travel time distribution is obtained by combining perceived travel time distribution and traffic information distribution, as shown in Fig. 2. According to the Bayesian theorem, the mean and standard deviation of the updated perceived distribution becomes

$$m_{n,r}'^{(k)} = \begin{cases} \dfrac{\sigma^2 m_{n,r}^{(k)} + s_{n,r}^{(k)2} \tau_{n,r}^{(k)}}{\sigma^2 + s_{n,r}^{(k)2}} & if \quad s_{n,r}^{(k)} > 0 \\ m_{n,r}^{(k)} & if \quad \sigma = 0, s_{n,r}^{(k)} = 0 \end{cases}, \tag{5}$$

$$\text{and } s_{n,r}'^{(k)} = \begin{cases} \dfrac{\sigma^2 s_{n,r}^{(k)2}}{\sqrt{\sigma^2 + s_{n,r}^{(k)2}}} & if \quad s_{n,r}^{(k)} > 0 \\ 0 & if \quad \sigma = 0, s_{n,r}^{(k)} = 0 \end{cases}. \tag{6}$$

When the information is not provided for Route r, they are simply described by a prior distribution having a mean and standard deviation of $m_{n,r}'^{(k)} = m_{n,r}^{(k)}$ and $s_{n,r}'^{(k)} = s_{n,r}^{(k)}$ respectively.

Travellers choose the route with the shortest expected travel time. This process is described as a deterministic process. The route of the n th traveller on the k th day is described as

$$\left(\delta_{n,1}^{(k)}, \delta_{n,2}^{(k)}\right) = \begin{cases} (1,0) & if \quad m_{n,1}'^{(k)} < m_{n,2}'^{(k)} \\ (0,1) & otherwise \end{cases} \tag{7}$$

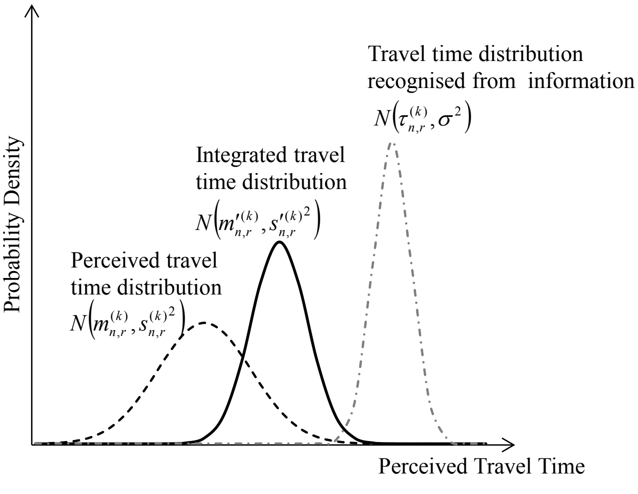


Fig. 2 Relation between information and perceived distributions

2.4. Deluded equilibrium

Nakayama et al, 1999 defined the deluded equilibrium, in which travellers are locked into delusions and do not believe their perceived travel times can be improved by changing routes. In this situation, the system may be stabilised. In the system described in this study, the deluded equilibrium is defined as

$$\delta_{n,1}^{(k)}\left(m_{n,1}^{(k)}-m_{n,2}^{(k)}\right)+\left(1-\delta_{n,1}^{(k)}\right)\left(m_{n,2}^{(k)}-m_{n,1}^{(k)}\right)\leq 0 \quad for \quad n=\{1,2,...,N\}$$

$$\text{and } \left|\delta_{n,r}^{(k)}s_{n,r}^{(k)}\right|<\varepsilon \quad for \quad n=\{1,2,...,N\}, r=\{1,2\}, \forall\varepsilon>0. \tag{8}$$

This equation shows that the standard deviation of the chosen route's perceived travel time is zero. In this case, every traveller believed that his or her chosen route is the least travel time. According to Eq. (4), this situation occurred when the mean of the chosen route's perceived travel time converged toward the actual travel time. When no information is provided for either route, or accurate information ($\sigma=0$) is provided for either or both routes, the state satisfying Eq. (8) is the fixed point of the proposed system. Although the systems are deterministic in these cases, the fixed point of the proposed system does not always equal the actual equilibrium described by

$$\delta_{n,1}^{(k)}\left(w_{n,1}^{(k)}-w_{n,2}^{(k)}\right)+\left(1-\delta_{n,1}^{(k)}\right)\left(w_{n,2}^{(k)}-w_{n,1}^{(k)}\right)\leq 0 \quad for \quad n=\{1,2,...,N\}. \tag{9}$$

However, this state is one of the possible deluded equilibrium states.

2.5. Initial states

In the proposed system, the initial states of $m_{n,r}^{(0)}$, $s_{n,r}^{(0)}$, and $w_{0,r}^{(k)}$ influence the convergence point. This study assumes that $m_{n,r}^{(0)}$ and $s_{n,r}^{(0)}$ are stochastically distributed around the travel time satisfying the static equilibrium state. This situation assumes that travellers have travel time information estimated by road administrators, and are not considering within-day evolutions. The travel time of initial traveller $w_{0,r}^{(k)}$ is given to coincide with the equilibrium state, to consider the situation where the demand is uniformly distributed around the network's capacity. In this situation, initial values for the means of the perceived travel times are determined as:

$$m_{n,r}^{(0)}\sim \mathrm{N}\left(\max\left(\frac{1}{\mu_1},\frac{1}{\mu_2}\right),{s_0}^2\right). \tag{10}$$

where s_0 is a pre-given parameter of the initial variance of the perceived travel time mean.

Initial values of Std for the perceived travel times are determined as

$$s_{n,r}^{(0)}=\left|\max\left(\frac{1}{\mu_1},\frac{1}{\mu_2}\right)-m_{n,r}^{(0)}\right|. \tag{11}$$

The initial travel time state is established to coincide with the travel time of the equilibrium state. This is described as

$$w_{0,1}^{(k)}=\begin{cases}\dfrac{\mu_2}{\mu_1}+\mu_1 & if \quad \mu_1>\mu_2\\ 0 & otherwise\end{cases},$$

$$w_{0,2}^{(k)} = \begin{cases} \dfrac{\mu_1}{\mu_2} + \mu_2 & \mathit{if} \quad \mu_1 < \mu_2 \\ 0 & \mathit{otherwise} \end{cases}. \tag{12}$$

2.6. Discussion of dynamics of perceived travel time of individual travellers

The proposed system is a discrete dynamical system with discrete within-day and day-to-day systems. The cost function described by the within-day system in Eq. (1) causes asymmetrical effects on travellers. That is, travellers' costs are affected by route choices made by travellers who departed earlier. As a result, the perceived travel times of travellers who departed earlier may converge earlier than travellers who depart later. This is because the possible convergence point of $m_{n,r}^{(k)}$ is changing, owing to the route choices of travellers who departed earlier.

In order to discuss the dynamics of the individual day-to-day perception updating, a description of the perception updating difference is required. These are described as:

$$\Delta m_{n,r}^{(k)} = m_{n,r}^{(k)} - m_{n,r}^{(k-1)} = \begin{cases} \alpha\left(w_{n,r}^{(k-1)} - m_{n,r}^{(k-1)}\right) & \mathit{if} \quad m_{n,r}'^{(k-1)} < m_{n,r'}'^{(k-1)} \\ 0 & \mathit{otherwise} \end{cases} \quad \mathit{for} \quad r' \in \{1,2\}, \quad r \neq r' \tag{13}$$

$$\Delta s_{n,r}^{(k)} = s_{n,r}^{(k)} - s_{n,r}^{(k-1)} = \begin{cases} \alpha\left(\left|w_{n,r}^{(k-1)} - m_{n,r}^{(k-1)}\right| - s_{n,r}^{(k-1)}\right) & \mathit{if} \quad m_{n,r}'^{(k-1)} < m_{n,r'}'^{(k-1)} \\ 0 & \mathit{otherwise} \end{cases} \quad \mathit{for} \quad r' \in \{1,2\}, \quad r \neq r' \tag{14}$$

If $m_{n,r}^{(k)}$ converges to $w_{n,r}^{(k-1)}$, the difference described by Eq. (13) becomes zero. After $m_{n,r}^{(k)}$ is converged, $\Delta s_{n,r}^{(k)}$ also gradually converges to zero. These difference equations are asymmetric between routes, because the perception parameters of an unchosen route are not updated. The chosen route is determined by $m_{n,r}'^{(k-1)}$, and $m_{n,r}'^{(k-1)}$ is affected by the initial points, experienced travel time, and travel information. This represents the state space of these parameters divided into several parts, depending on actual travel time and provided information.

To simplify the discussion, this study focused on two situations: in the first situation, no information is provided for either route; in the second situation, accurate information ($\sigma = 0$) is provided for Route 1. Fig. 3 shows a partition map of the dynamics of individual travellers when no information is provided. Let k_{n-1}^{*} be the day when the travellers who departed earlier than the n th traveller converged to a fixed point, and $w_{n,1}'^{\left(k_{n-1}^{*}\right)}$ be the travel time if the n th traveller chooses Route r.

$$w_{n,r}'^{\left(k_{n-1}^{*}\right)} = \frac{1}{\mu_r} + w_{n-1,r}^{\left(k_{n-1}^{*}\right)} - 1 \tag{15}$$

In the figure, $w_{n,1}'^{\left(k_{n-1}^{*}\right)} \leq w_{n,2}'^{\left(k_{n-1}^{*}\right)}$ is assumed to simplify the discussion. The case satisfying $w_{n,1}'^{\left(k_{n-1}^{*}\right)} > w_{n,2}'^{\left(k_{n-1}^{*}\right)}$ can be considered by mutually replacing Route 1 with Route 2. A traveller will choose Route 1 when $m_{n,2}^{(k)}$ is larger than the line for $m_{n,2}^{(k)} = m_{n,1}^{(k)}$. Below this line, the traveller chooses Route 2. The actual travel time of each route is represented by $w_{n,1}'^{\left(k_{n-1}^{*}\right)}$ and $w_{n,2}'^{\left(k_{n-1}^{*}\right)}$ respectively. The area in Fig.3 is divided into four parts corresponding to sign conditions of $\Delta m_{n,1}^{(k)}$ and $\Delta m_{n,2}^{(k)}$. These areas can be described as

$$\Delta m_{n,1}^{(k)} > 0, \Delta m_{n,2}^{(k)} = 0 \quad \mathit{if} \quad m_{n,1}^{(k-1)} \leq m_{n,2}^{(k-1)} \cap m_{n,1}^{(k-1)} < w_{n,1}'^{\left(k_{n-1}^{*}\right)}, \tag{16a}$$

$$\Delta m_{n,1}^{(k)} \le 0, \Delta m_{n,2}^{(k)} = 0 \quad if \quad m_{n,1}^{(k-1)} \le m_{n,2}^{(k-1)} \cap m_{n,1}^{(k-1)} \ge w'^{\left(k_{n-1}^{*}\right)}_{n,1}, \tag{16b}$$

$$\Delta m_{n,1}^{(k)} = 0, \Delta m_{n,2}^{(k)} > 0 \quad if \quad m_{n,1}^{(k-1)} > m_{n,2}^{(k-1)} \cap m_{n,2}^{(k-1)} < w'^{\left(k_{n-1}^{*}\right)}_{n,2}, \tag{16c}$$

$$\text{and } \Delta m_{n,1}^{(k)} = 0, \Delta m_{n,2}^{(k)} \le 0 \quad if \quad m_{n,1}^{(k-1)} > m_{n,2}^{(k-1)} \cap m_{n,2}^{(k-1)} \ge w'^{\left(k_{n-1}^{*}\right)}_{n,2}. \tag{16d}$$

Eqs.(16a)-(16d) respectively correspond to the area (i)-(iv) in Fig.3. According to the equation, when $m_{n,1}^{(k-1)} < w'^{\left(k_{n-1}^{*}\right)}_{n,1} \cap m_{n,2}^{(k-1)} < w'^{\left(k_{n-1}^{*}\right)}_{n,1}$, travellers repeatedly change their route until converging to $m_{n,1}^{*} = w'^{\left(k_{n-1}^{*}\right)}_{n,1}$. When $m_{n,1}^{(k-1)} > m_{n,2}^{(k-1)} \cap w'^{\left(k_{n-1}^{*}\right)}_{n,1} < m_{n,1}^{(k-1)} < w'^{\left(k_{n-1}^{*}\right)}_{n,2}$, a traveller changes the route once.

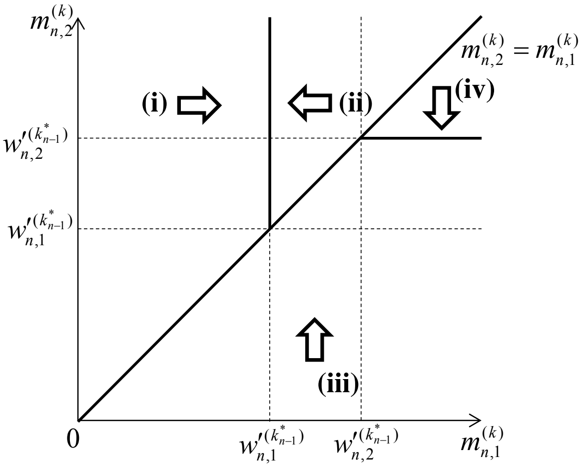
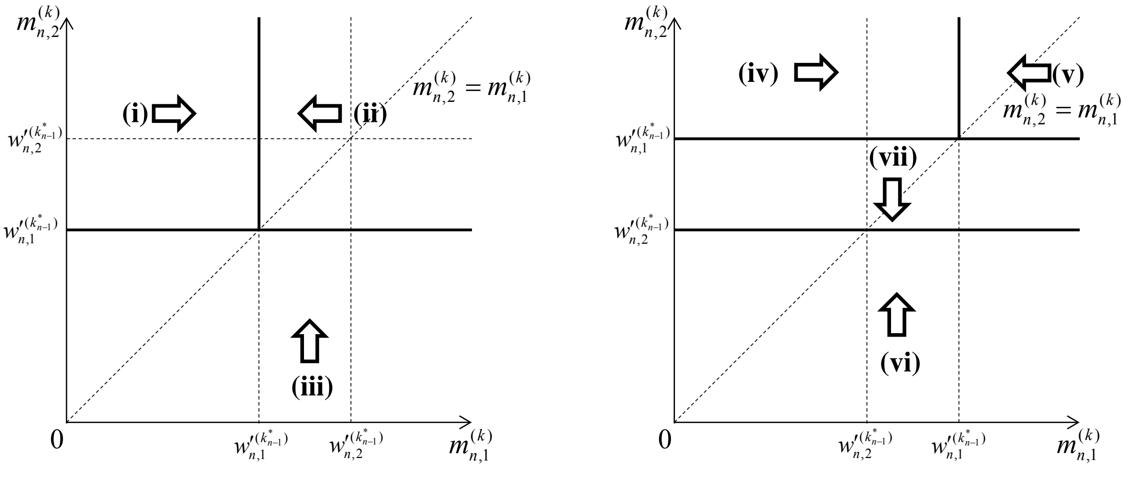


Fig. 3 Partition map of mean of perceived travel time when information is not provided



(a) $w'^{(k_{n-1}^{*})}_{n,1} \le w'^{(k_{n-1}^{*})}_{n,2}$

(b) $w'^{(k_{n-1}^{*})}_{n,1} > w'^{(k_{n-1}^{*})}_{n,2}$

Fig. 4 Partition map of mean of perceived travel time when information is provided

When $w'^{\left(k^*_{n-1}\right)}_{n,1} \le w'^{\left(k^*_{n-1}\right)}_{n,2}$ is satisfied, the n th traveller's convergence point is represented by

$$\left(m^*_{n,1}, m^*_{n,2}\right) = \begin{cases} \left(w^{\left(k^*_{n-1}\right)}_{n,1}, a\right) & if \quad m^{\left(k^*_{n-1}\right)}_{n,1} \le m^{\left(k^*_{n-1}\right)}_{n,2} \bigcup m^{\left(k^*_{n-1}\right)}_{n,1} < w'^{\left(k^*_{n-1}\right)}_{n,2} \\ \left(b, w^{\left(k^*_{n-1}\right)}_{n,2}\right) & if \quad m^{\left(k^*_{n-1}\right)}_{n,1} > m^{\left(k^*_{n-1}\right)}_{n,2} \bigcap m^{\left(k^*_{n-1}\right)}_{n,1} \ge w'^{\left(k^*_{n-1}\right)}_{n,2} \end{cases} \tag{17}$$

where a and b are values satisfying $a > w^{\left(k^*_{n-1}\right)}_{n,1}$ and $b > w^{\left(k^*_{n-1}\right)}_{n,2}$, respectively. When $w'^{\left(k^*_{n-1}\right)}_{n,1} > w'^{\left(k^*_{n-1}\right)}_{n,2}$, the convergence point is described by Eq. (17).

Fig. 4 shows a partition map of the dynamics of an individual traveller when exact information is provided for Route 1. When the information is provided, the partition map is different between $w'^{\left(k^*_{n-1}\right)}_{n,1} \le w'^{\left(k^*_{n-1}\right)}_{n,2}$ and $w'^{\left(k^*_{n-1}\right)}_{n,1} > w'^{\left(k^*_{n-1}\right)}_{n,2}$.

When the actual travel time is $w'^{\left(k^*_{n-1}\right)}_{n,1} \le w'^{\left(k^*_{n-1}\right)}_{n,2}$, the difference equation is described as

$$\Delta m^{(k)}_{n,1} > 0, \Delta m^{(k)}_{n,2} = 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} \le m^{(k-1)}_{n,2} \bigcap m^{(k-1)}_{n,1} < w'^{\left(k^*_{n-1}\right)}_{n,1}, \tag{18a}$$

$$\Delta m^{(k)}_{n,1} \le 0, \Delta m^{(k)}_{n,2} = 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} \le m^{(k-1)}_{n,2} \bigcap m^{(k-1)}_{n,1} \ge w'^{\left(k^*_{n-1}\right)}_{n,1}, \tag{18b}$$

$$\text{and } \Delta m^{(k)}_{n,1} = 0, \Delta m^{(k)}_{n,2} > 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} > m^{(k-1)}_{n,2}. \tag{18c}$$

In this case, the area is divided into three parts. Eqs.(18a)-(18d) respectively correspond to the area (i)-(iii) in Fig.4(a). When the actual travel time is $w'^{\left(k^*_{n-1}\right)}_{n,1} > w'^{\left(k^*_{n-1}\right)}_{n,2}$, the difference equation is described as

$$\Delta m^{(k)}_{n,1} > 0, \Delta m^{(k)}_{n,2} = 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} \le m^{(k-1)}_{n,2} \bigcap m^{(k-1)}_{n,1} < w'^{\left(k^*_{n-1}\right)}_{n,1}, \tag{19a}$$

$$\Delta m^{(k)}_{n,1} \le 0, \Delta m^{(k)}_{n,2} = 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} \le m^{(k-1)}_{n,2} \bigcap m^{(k-1)}_{n,1} \ge w'^{\left(k^*_{n-1}\right)}_{n,1}, \tag{19b}$$

$$\Delta m^{(k)}_{n,1} = 0, \Delta m^{(k)}_{n,2} > 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} > m^{(k-1)}_{n,2} \bigcap m^{(k-1)}_{n,2} < w'^{\left(k^*_{n-1}\right)}_{n,2}, \tag{19c}$$

$$\text{and } \Delta m^{(k)}_{n,1} = 0, \Delta m^{(k)}_{n,2} \le 0 \quad if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} > m^{(k-1)}_{n,2} \bigcap m^{(k-1)}_{n,2} \ge w'^{\left(k^*_{n-1}\right)}_{n,2}. \tag{19d}$$

These equations correspond to (iv)-(vii) in Fig.4(b). If the actual travel time is $w'^{\left(k^*_{n-1}\right)}_{n,1} \le w'^{\left(k^*_{n-1}\right)}_{n,2}$, the n th travellers' route choice always converges to Route 1. In addition, the convergence point can be represented as

$$\left(m^*_{n,1}, m^*_{n,2}\right) = \begin{cases} \left(w^{\left(k^*_{n-1}\right)}_{n,1}, a\right) & if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} \le w'^{\left(k^*_{n-1}\right)}_{n,2} \bigcup m^{\left(k^*_{n-1}\right)}_{n,2} < w'^{\left(k^*_{n-1}\right)}_{n,1} \\ \left(b, w^{\left(k^*_{n-1}\right)}_{n,2}\right) & if \quad w'^{\left(k^*_{n-1}\right)}_{n,1} > w'^{\left(k^*_{n-1}\right)}_{n,2} \bigcap m^{\left(k^*_{n-1}\right)}_{n,2} \ge w'^{\left(k^*_{n-1}\right)}_{n,1} \end{cases} \tag{20}$$

These results described by Eq. (20) imply that, even if information is asymmetrically provided, the deluded equilibrium state can be mitigated, because the probability of choosing the route with the shorter travel time is increased. When the route with travel time information has a shorter actual travel time than the other route, travellers will definitely choose the route with the shorter travel time at the converged state. Even if the actual travel time of the route with travel information is longer, the traveller's route choice will be stable, because repeated switching behaviour does not occur in this system. This feature enables the system to perform quick convergence comparisons to the no-information case.

3. Numerical analysis

In this section, we will illustrate the effects of information accuracy on the day-to-day dynamics of route choice behaviour, under dynamic traffic flow conditions. The discussion in Section 2.6 shows convergence points of individual travellers after route choices of earlier-departing travellers are converged in the day-to-day dynamics. However, the effects of within-day dynamics and its interaction with the day-to-day dynamics were not analytically represented because the deluded equilibrium state was not unique and is path dependant in the proposed system. The numerical analysis in this section is intended to describe how information affects the speed at which travellers converge toward a deluded equilibrium state, and the differences between the deluded equilibrium state and the equilibrium state. The proposed system is determined by $(\alpha, \mu_1, \sigma, N)$. The analysis is conducted using 100 travellers ($N = 100$). The perception updating parameter α was set to 0.2. To compare different capacity allocation situations, the bottleneck capacity of Route 1 was set to $\mu_1 = \{0.1, 0.2, ..., 0.9\}$. The capacity of Route 2 was dependently determined as $\mu_2 = 1 - \mu_1$. The information accuracy was set to $\sigma = (0.00, 0.05, 0.10, 0.15, 0.20)$. The initial states $m_{n,r}^{(0)}$ and $s_{n,r}^{(0)}$ were stochastically distributed as described in Section 2.5. We performed a Monte Carlo simulation with different initial values for the perceived travel time means, because initial values are stochastically distributed and the proposed system depends on the initial values. Two hundred iterations of the simulation were performed for each parameter set.

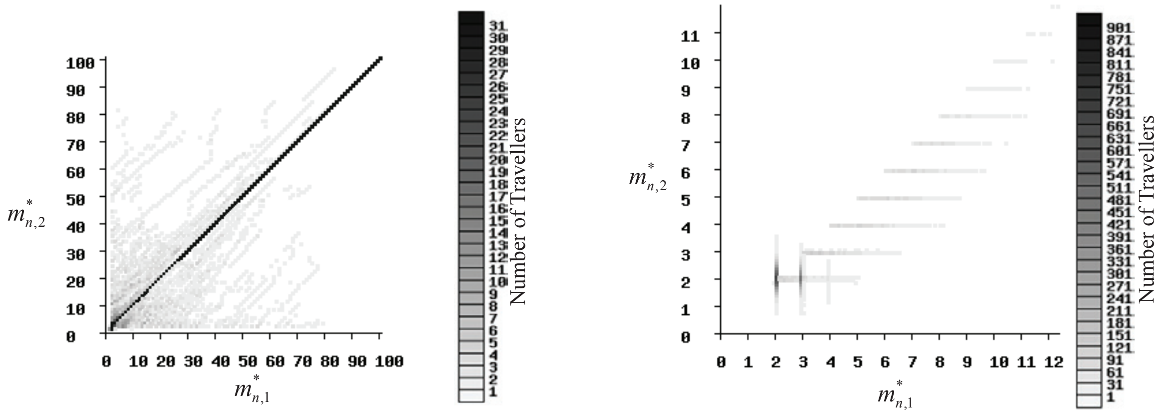
3.1. Information and deluded equilibrium state

This subsection describes the perceived travel time in the deluded equilibrium state. The no-information state was compared with states that provided information having a pre-given accuracy σ. Fig. 5 shows the perceived travel time in the deluded equilibrium state when the capacity of Route 1 was set to $\mu_1 = 0.5$. The figure includes the results of 20,000 travellers, obtained by performing 200 simulations on 100 travellers. The grey scale represents the number of travellers in each pair of perceived travel time values that were derived from each traveller. Case (a) in the figure shows the results of the no-information case, and (b) shows the results for the case where accurate Route 1 travel time information was provided. In Cases (c) and (d), inaccurate travel time information was provided for Route 1. By comparing (a) with the other cases, it is apparent that the distribution of perceived travel time is very scattered when no information is provided. The largest perceived travel time increased by 100 within-day time steps, although it was expected to be only two time steps in the equilibrium state. This deluded equilibrium state implies that the route was not chosen symmetrically, owing to the interaction of within-day and day-to-day dynamics. Synchronous changes in route choices (the so-called hunting phenomenon) result in these long travel times. These changes can be confirmed by frequency of route changes. Travellers changed their routes in 75.6 % days before the convergence on average. The travellers who performed the same route changes as the traveller departed immediately earlier was 74.7 %.This phenomenon occurs because travellers do not know the actual travel time of the route they did not choose. Furthermore, one of the partitions described in Fig. 3 shows repeated route choice changes when the perceived travel times of both routes are less than the actual travel times.

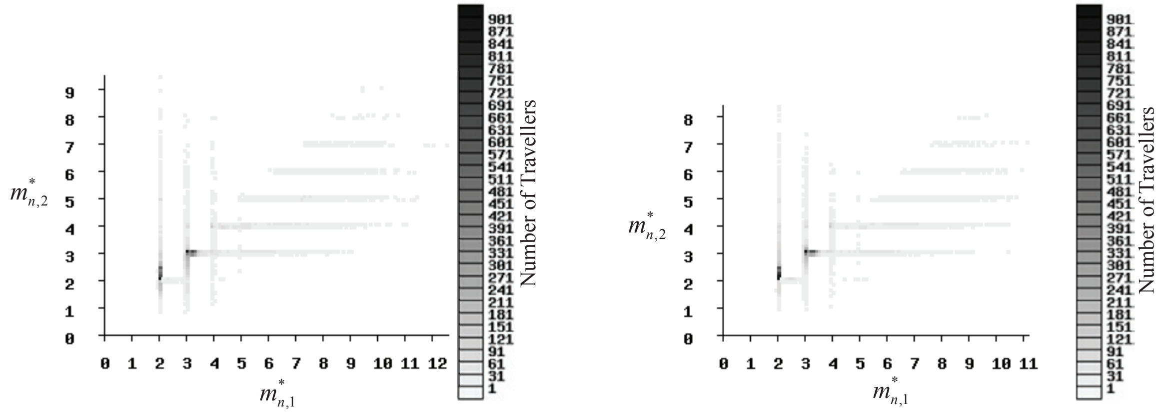
When the information is provided for Route 1, the variability of the perceived travel time is substantially reduced. The convergence points of the perceived travel time coincide with the discussion in Section 2.6. The majority of the trips are converged to the route with a travel time of two time steps. Thus, the perceived travel time moved closer to the user equilibrium state. When the variance of the information became larger, it appeared that the difference between actual and perceived travel time of unchosen route is decreased.

3.2. Effects of information on day-to-day evolution

To describe within-day characteristics, this section describes the choice rate and experienced travel time of each different departure time. The results described in this section aggregate the results of 200 simulations. Fig. 6 shows the choice rate when the bottleneck capacity of Route 1 was 0.2, 0.5, and 0.8. In these cases, when the travel time information was not provided, the choice rate was close to the bottleneck capacity. This result implies that the choice rate was randomly distributed according to the initial values of the perceived travel time and bottleneck capacity. This indicates that within-day time-dependant characteristics were not obtained. In contrast, when the information was provided, the choice rate was cyclically changed. When the capacity was 0.5, a cycle was expected to be two within-day time steps. When the capacity was 0.2 or 0.8, a cycle was expected to be five time steps. These expectations were adopted because the travel time of the routes was balanced in these cycles. For example, travel time of the route whose capacity is 0.2 becomes less than the other route once in five time steps, if the equilibrium state is satisfied. In Fig. 6 (a), two different cycles were observed depending on the initial value of perceived travel time. Both of them had five time step cycles though their phases were different. When σ was more than zero, travellers chose Route 1 in almost 100 % occurrence rate once in the five time steps after 36 time steps. It can be confirmed that larger amplitudes of these cycles were observed as the variations in the information became larger. Route changing was motivated by the variations in the information. It enabled travellers to update the perceived travel time of the route that they had not chosen. When σ was zero, Fig.6(a) showed that occurrence rate of either phase approached to 0.5 as within-day time passes. It might be because several travellers were switched the other phases. In this case, large number of travellers was still enabled travellers to update the perceived travel time. However, the cycle amplitudes became smaller in Case (b) and (c) as time passes. In Fig. 6 (b), two different cycles was found in each information provision cases. Occurrence rate of either phase approached to 0.5 as within-day time passed. In Fig. 6 (c), the cycle was not clearly appeared as (a). In the case when σ was zero, we can find two different cycles which have five step cycles and different phases. Occurrence rate of both cycles approached to 0.5 as time passed. However, when σ was more than zero, several cycles with different phases appeared, and occurrence rate of these phases were complexly varied. It appears that the effect caused by information variations may be changed whether the information is provided or not provided for the lower capacity route.

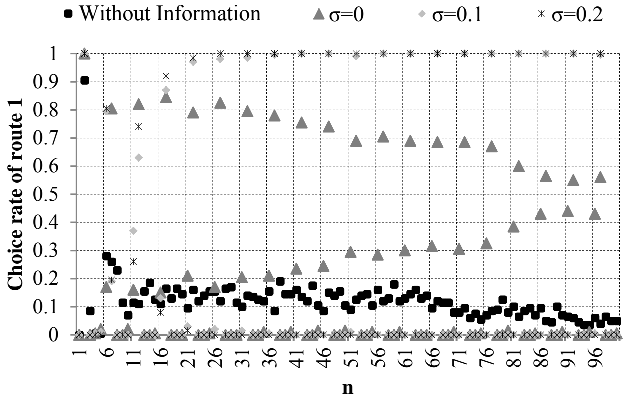


(a) Without information provision (b) $\sigma = 0$

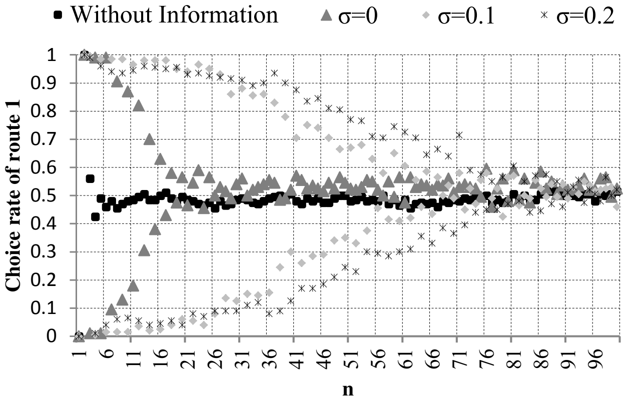


(c) $\sigma = 0.1$ (d) $\sigma = 0.2$

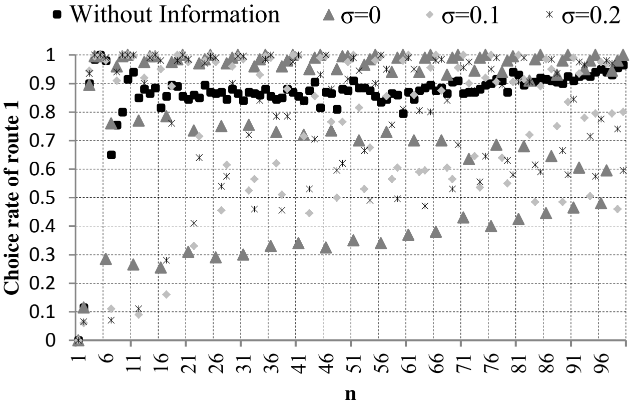
Fig. 5 Perceived travel time of each route in deluded equilibrium state ($\mu_1 = 0.5$)



(a) $\mu_1 = 0.2$

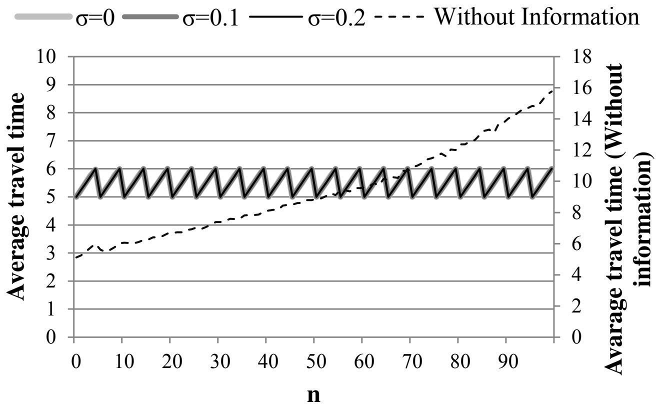


(b) $\mu_1 = 0.5$

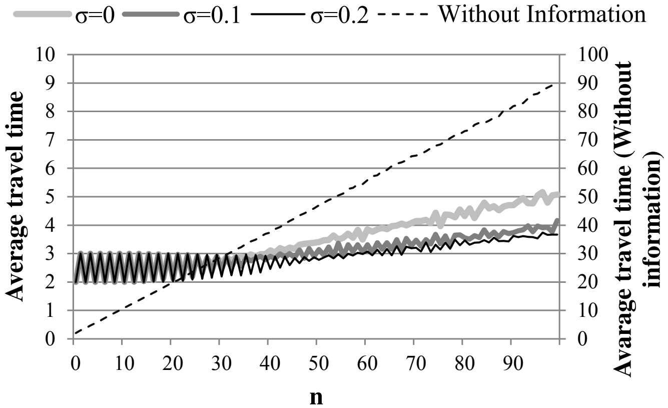


(c) $\mu_1 = 0.8$

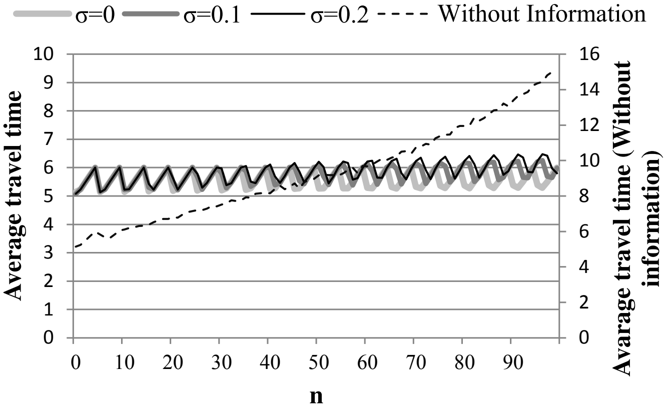
Fig 6 Choice rate of every traveller in the deluded equilibrium state for 200 Monte Carlo simulations



(a) $\mu_1 = 0.2$

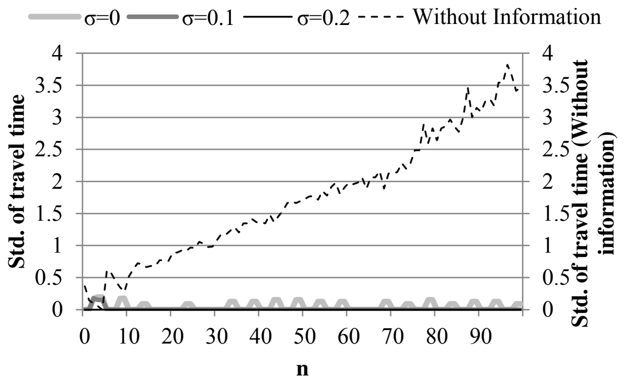


(b) $\mu_1 = 0.5$

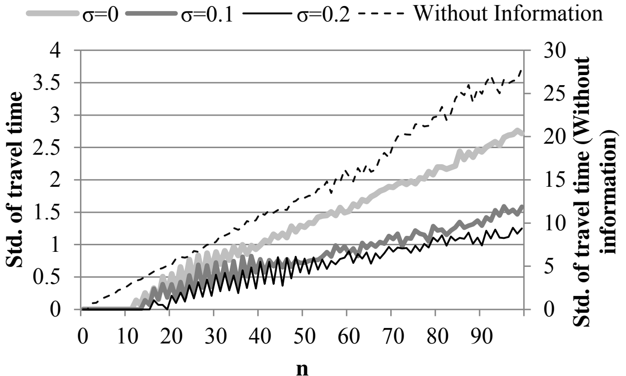


(c) $\mu_1 = 0.8$

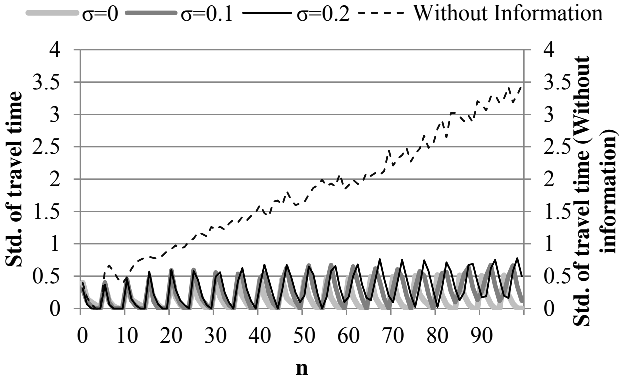
Fig. 7 Mean travel time of every traveller in the deluded equilibrium state for 200 Monte Carlo simulations



(a) $\mu_1 = 0.2$



(b) $\mu_1 = 0.5$



(c) $\mu_1 = 0.8$

Fig. 8 Standard deviation of travel time of every traveller in the deluded equilibrium state in 200 times Monte Carlo simulation

Figures 7 and 8 show the travel time mean and standard deviation (Std) for the route used in the deluded equilibrium state. The horizontal axis represents travellers ordered by their departure time. In all cases, both the mean and Std were dramatically reduced by information provision. Larger decreases were observed for later departing travellers. It appears that later-departing travellers were more likely to suffer from the effects of route changing by earlier-departing travellers, because the deluded equilibrium state appeared in the earlier travellers. When the equilibrium state was satisfied, the average travel time was expected to be around two time steps in (a) and around five time steps in (b) and (c). Within-day characteristics did not change in (a). In (b) and (c), as the amplitude of cyclic choice rate characteristics shown in Fig. 6 decreased, mean travel time was increased. Information variations reduced travel time increases in (a) and (b). However, this relationship was reversed in (c).

In Fig. 8, when information was provided for the lower capacity route, the improvement was larger than cases where information was provided for the higher capacity route. This tendency appears to be affected by travel time's sensitivity to demand. When the sensitivity is large, travellers may experience long travel times due to route changing by the travellers who departed earlier. Accordingly, the traveller's perceived travel time is updated to be longer. Providing travel time information for the higher capacity route will increase the number of cases where the perceived travel time is converged to the route with travel time information. However, when route capacity is lower, those cases may decrease. This situation can be explained as follows. When the relation of the capacities is $\mu_1 > \mu_2$, $m_{n,2}^{(k)}$ possibly takes a larger value than $m_{n,1}^{(k)}$ because the sensitivity of travel time $w_{n,2}'^{(k)}$ to demand becomes larger than $w_{n,1}'^{(k)}$. This situation may lead to $m_{n,2}^{(k)} > w_{n,1}'^{(k)}$. Further, as shown in Fig. 4(b), when $m_{n,2}^{(k)} > w_{n,1}'^{\left(k_{n-1}^{*}\right)}$ is satisfied, the traveller always chooses Route 1.

3.3. Summary of results

Table 1 shows the average number of days that were required to converge to the deluded equilibrium state. When accurate information was provided, the number of days required for convergence was reduced. However, the iterations increased as the Std of the information was increased. This occurred because the information included stochastic error terms; this caused travellers to change routes, and possibly to update the perceived travel time of the other route.

Table 2 shows the average choice rate of Route 1. When the information was not provided, a lower choice rate is observed in the lower capacity route. The table shows that information improves the choice rate. That is, the choice rate moves closer to the allocated capacity. When the capacity was lower than 0.5, the choice rate improved as the variance of the information was increased. However, when the capacity was higher, the choice rate was not improved by the variance of the information. This tendency is more clearly shown in Table 3. Table 3 shows the choice rate of the route where the actual travel time is shorter. If this rate is 1, the state is identical to the equilibrium state. The variation of information possibly improves travellers' choices when the capacities of the two routes are almost identical, or information is provided for the lower capacity route. However, this is not a feasible feature. When the higher capacity was allocated to the route along with an information provision, the travellers' route choices were negatively affected by the large variation of information.

Table 1 Number of days to converge to the deluded equilibrium state.

		Information is provided for Route 1.				
	Without	Std. of error of information σ				
μ_1	Information	0.00	0.05	0.10	0.15	0.20
0.1	129.9	76.4	682.2	691.1	688.8	704.7
0.2	195.6	72.4	1016.5	1018.9	1088.9	1126.2
0.3	114.9	73.6	723.6	984.8	1973.4	6878.0
0.4	136.5	74.7	543.9	802.7	1199.2	2171.9
0.5	202.2	74.7	824.2	1072.5	1380.3	2089.7
0.6	140.9	75.8	450.4	551.3	572.7	665.6
0.7	114.1	78.1	359.3	375.4	433.0	535.6
0.8	181.2	74.7	339.8	383.9	453.6	411.2
0.9	131.6	82.7	210.5	250.4	236.3	244.7

Table 2 Choice rate of Route 1

		Information is provided for Route 1.				
		Std. of error of information σ				
μ_l	Without Information	0.00	0.05	0.10	0.15	0.20
0.1	0.064	0.103	0.100	0.100	0.100	0.100
0.2	0.121	0.205	0.200	0.200	0.200	0.200
0.3	0.218	0.309	0.309	0.307	0.306	0.304
0.4	0.342	0.413	0.411	0.410	0.409	0.409
0.5	0.492	0.530	0.521	0.518	0.517	0.515
0.6	0.658	0.599	0.600	0.601	0.601	0.601
0.7	0.783	0.696	0.700	0.700	0.700	0.700
0.8	0.875	0.795	0.800	0.800	0.800	0.800
0.9	0.936	0.902	0.906	0.908	0.909	0.910

Table 3 Choice rate of the route where the actual travel time is shorter

		Information is provided for Route 1.				
		Std. of error of information σ				
μ_l	Without Information	0.00	0.05	0.10	0.15	0.20
0.1	0.509	0.964	0.985	0.989	0.992	0.992
0.2	0.376	0.857	0.818	0.814	0.813	0.812
0.3	0.406	0.920	0.936	0.956	0.963	0.983
0.4	0.457	0.769	0.788	0.809	0.830	0.841
0.5	0.071	0.598	0.782	0.826	0.845	0.863
0.6	0.453	0.854	0.940	0.911	0.907	0.899
0.7	0.413	0.930	0.936	0.941	0.924	0.905
0.8	0.379	0.962	0.864	0.841	0.837	0.825
0.9	0.504	0.864	0.834	0.808	0.799	0.786

4. Discussion and conclusion

The main focus of these analyses was to understand the effects of information accuracy on the day-to-day dynamics of route choice behaviour, under dynamic traffic flow conditions. In Section 2, we formulated a dynamic traffic assignment model in a two-link network, while considering travellers' learning processes and within-day and day-to-day dynamics. The model assumed that the travellers could obtain information for one of two routes. The model proposed in this study was based on the concept of the deluded equilibrium state which was originally proposed by Nakayama et al. (1999). This state reflects bounded rationality under the day-to-day learning mechanisms of a traveller and static traffic flow system. In this study, we proposed the model to examine the deluded equilibrium state under the dynamic traffic flow system and information provision. The discussion in Section 2.6 showed that information provisions possibly reduced the negative effects of deluded equilibrium, even when the information was only provided for one of the routes. This occurred because route choice behaviour became stable, and it became more likely that converged points would move to the route with the shorter travel time.

Section 3 contained an analysis that used a Monte Carlo simulation. The results of this analysis highlighted the complexity of the proposed dynamical system that used simple and tractable point queue model. In spite of the simple model, insufficient aspects describing characteristics of transport system were found in the behavioural assumptions. For example, although the proposed model includes day-to-day learning process which may reduce so myopic behaviour, synchronous changes in route choices (the so-called hunting phenomenon) were emerged when the information was not provided. The results suggested that simple minimum essential set of behavioural elements of a traveller under the dynamic flow system should be investigated in the future studies. Several learning process such as reinforcement learning (e.g. Chen & Mahmassani, 2009) and heuristics (e.g. Nakayama et al. 1999) will be investigated to find required conditions for behavioural model under the dynamic traffic flow conditions.

As expected from the discussions in Section 2.6, the results of the numerical simulation also suggested that choice behaviour in the deluded equilibrium was improved by information provision. In particular, when the information included variations, information provision for the lower capacity route was more effective than a provision for the higher capacity route. In this study, the maximum standard deviation of information variation was set to 0.20. This value corresponded to 10% of the travel time in the equilibrium state when Route 1 capacity was 0.5. The number of days required to converge to the deluded equilibrium state was increased by the variation of the information. This occurred because travellers changed their route according to outliers in the predictive travel time information. This effect did not worsen the situation when the capacity of the route receiving the information provision was lower than the other route. It is possible that travellers improved their perceived travel time of the other route by changing their route.

References

Avineri, E., Prashker, J.N., 2006. The Impact of Travel Time Information on Travelers' Learning under Uncertainty. Transportation 33, 393–408.

Bie, J., Lo, H.K., 2010. Stability and Attraction Domains of Traffic Equilibria in a Day-to-day Dynamical System Formulation. Transportation Research Part B 44, 90–107.

Chen, R., Mahmassani H. S., 2004. Travel Time Perception and Learning Mechanisms in Traffic Networks. Transportation Research Record 1894, 209–221.

Chen, R., Mahmassani H. S., 2009. Learning and Risk Attitudes in Route Choice Behavior Dynamics, in *"The Expanding Sphere of Travel Behavior Research"*. In: Kitamura, R., Yoshii, T., Yamamoto, T. (Ed.). Emerald Group Publishing Limited, West Yorkshire, pp. 791–818.

Chorus, C. G., Molin, J. E. E., Arentze, T. A., Hoogendoorn, S. P., Timmermans, H. J. P., Van Wee, G. P., 2006. Observing the making of travel choices under uncertainty and information: validation of travel simulator, The 85th Annual Meeting of the Transportation Research Board, Washington, DC, paper 06-0097.

Emmertink, R.H.M., Axhausen, K.W, Nijkamp, P., Rietveld, P., 1995. Effects of Information in Road Transport Networks with Recurrent Congestion. Transportation 22, 21–53.

Ettema, D.F., Timmermans, H.J.P., 2006. Costs of Travel Time Uncertainty and Benefits of Travel Time Information: Conceptual Model and Numerical Examples. Transportation Research Part C 14, 335–350.

Friesz, T.L., Bemstein, D., Mehta, N.J., Tobin, R.L., Ganjalizadeh, S., 1994. Day-to-Day Dynamic Network Disequilibria and Idealized Traveler Information Systems. Operations Research 42, 1120–1136.

Guo, X., Liu, H.X., 2011. Bounded Rationality and Irreversible Network Change. Transportation Research Part B 45, 1606–1618.

Horowitz, J., 1984. The Stability of Stochastic Equilibrium in a Two-link Transportation Network. Transportation Research B 18, 13–28.

Jha, M., Madanat, S., Peeta, S., 1998. Perception Updating and Day-to-Day Travel Choice Dynamics in Traffic Networks with Information Provision. Transportation Research Part C.6, 189–212.

Khattak, A., Polydoropoulou, A., Ben-Akiva, M., 1996. Modeling Revealed and Stated Pretrip Travel Response to Advanced Traveler Information Systems. Transportation Research Record 1537, 46–54.

Mahmassani, H.S., Chang, G. L., 1987. On Bounded Rational User Equilibrium in Transportation Systems. Transportation Science 21, 89–99.

Mahmassani, H.S., Liu, Y. H., 1999. Dynamics of Commuting Decision Behaviour under Advanced, Traveller Information Systems. Transportation Research Part C 7, 91–107.

Nakayama, S., Kitamura, R., Fujii, S., 1999. Driver's Learning and Network Behavior: Dynamic Analysis of Driver-network System as a Complex System. Transportation Research Record 1676, 30–36.

Smith, M.J., 1984: The Stability of a Dynamic Model of Traffic Assignment - An Application of a Method of Lyapunov. Transportation Science 18, 245–252.

Watling, D., 1999. Stability of the Stochastic Equilibrium Assignment Problem: a Dynamical Systems Approach. Transportation Research Part B 33 (4), 281–312.