How to Choose the Shape of the Pan

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Abstract

Nowadays, as the oven is widely used, a variety of pans of different shapes occur to meet different demands of different people. While, when using many kinds of pans, what troubles users is that the food on the corners of the pan is easily overcooked. It is resulted from the uneven distribution of the heat in the corners. Meanwhile, when selecting the pan’s shape, the neglect of considering the shape of the oven may cause the waste of the oven’s space. So we are faced with the problem that how to choose the optimal pan’s shape based on the users’ demands and the shape of the oven. To solve the problem, we develop a model to help decide the shape of pans to choose. In the modelling process, the main factors we consider are the user’s focus and the shape of the oven. When considering the shape of the pans, we do not consider the rare shape for particular use since we develop the model for the use of oven and pans under normal conditions. During the modelling process stated in this paper, we define users’ different demands with continuous data discretization method.

Keywords: Continuous data discretization; optimization; modelling

1. Main text

1.1. Introduction:

Nowadays, thanks to the development of technology and people’s great wisdom, more and more kinds of
household appliances are coming into being. An oven is one of them. When using it, you firstly put the food you want to bake such as brownies into it. Secondly you close the door of the oven and you need to set the coefficients such as the temperature you need to bake your brownies. And at last you press the start button and your brownies will be baked in the oven. An oven is always used together with pans which contain food that is to be baked. And there are many different kinds of pans with different shapes. And it is recognized that different shapes of the pans make a difference on the taste or some other factors of the baked brownies. As more and more people with different demands are using it, it is becoming more and more important to make a choice of the shapes of the pans according to people’s demands. If we want to bake some brownies to sell, our goal is to make the maximal profit. Since the area of the pan is the constant $A$, we may consider two factors: (1) number of pans that the oven can hold (2) valuable ratio of brownies in one pan. So we develop a model to select the best type of pan under some conditions according to one’s demands.

**Nomenclature**

- $r$: radius of the inscribed circle of the pan
- $S_{\text{incircle}}$: area of the inscribed circle of the pan
- $S$: area of a rack
- $A$: area of a pan
- $k$: result of the ratio of width and length of the rack, $W/L$
- $\eta$: area utilization ratio of the rack
- $\Omega$: heat utilization ratio of the pan
- $m$: number of edges of the pan, $m \geq 3$
- $N$: maximal number of pans the oven can hold
- $p$: weight of $\eta$
- $l$: length of the pan’s edge

**2. Model setup**

**2.1. Basic analysis**

1. When putting the maximal number of pans into the oven, we put the maximal number of pans in every rack. So there are at most $N/2$ pans on one rack. And at that time, there is no space wasted. So the area of the rack’s surface is $S = AN/2$.

2. When the heat utilization is the biggest, we use round pan. At this time $\Omega$ is 100%.

3. When selecting the best type of pan, we consider two main factors $\eta$ and $\Omega$. While $p$ and $(1 - p)$ is unknown. So we should divide $p$ into several intervals according to different focus.
2.2. Development

As discussed, we divide $p$ into five intervals, as Fig 1 shows,

![Fig 1](image1)

As $p$ gets bigger, people pay less attention to $\Omega$, and vice versa. So define the five intervals as $I_A$ ($\Omega$ is very important, 80% $< \Omega \leq$ 100%), $I_B$ ($\Omega$ is important, 60% $< \Omega \leq$ 80%), $I_C$ ($\eta$ and $\Omega$ are balanced, 40% $< \Omega \leq$ 60%), $I_D$ ($\Omega$ is not so important, 20% $< \Omega \leq$ 40%), $I_E$ ($\Omega$ is not important at all, 0% $< \Omega \leq$ 20%) as Fig 2,

![Fig 2](image2)

When using round pan, $\Omega$ is 100%, so no matter what the shape of the pan is, the valuable area is the area of the biggest inscribed circle of the pan. So

$$\Omega = \frac{S_{incircle}}{A} \times 100\%$$

According to the definition of the regular polygon’s inscribed circle, the inscribed circle’s radius is the distance from the center to the edge. So

$$r = \sqrt{\frac{2A}{m \sin \frac{\pi}{m}}} \cos \frac{\pi}{m}$$

So

$$S_{incircle} = \pi r^2 = \frac{\pi A \cot \frac{\pi}{m}}{m}$$

So it gets

$$\Omega = \frac{\pi \cot \frac{\pi}{m}}{m} \times 100\% \quad (1)$$

Draw the graph of $\Omega$ with the use of Mathematica, as Fig 3
Based on (1), in different intervals the solutions are,

1. In $I_A$, $80\% < \Omega \leq 100\%$, so $4.13 < m < \infty$
2. In $I_B$, $60\% < \Omega \leq 80\%$, so $2.98 < m \leq 4.13$
3. In $I_C$, $40\% < \Omega \leq 60\%$, so $2.48 < m \leq 2.98$
4. In $I_D$, $20\% < \Omega \leq 40\%$, so $2.19 < m \leq 2.48$
5. In $I_E$, $0 < \Omega \leq 20\%$, so $2 < m \leq 2.19$

Since $m \geq 3$, only $I_A$ and $I_B$ correspond to the reality. Then select $m$ in $I_A$ and $I_B$ considering on $\eta$, the analysis process is as follows,

1. In $I_B$, $m = 3or4$. So if $\Omega$ is important, that is people pay more attention to the heat utilization ratio, it is more likely to choose equilateral triangle pan or square pan. So select pan’s shape according to the different values of $k$. Under the premise that maximize the number of pans used:
   - When using equilateral triangle pans, the smallest value of $k$ is in the scene shown in Fig 4.

In this figure, based on the relation of the rack’s area and the pan’s area as discussed before, and the equilateral triangle’s area is $A$, there is,

$$\frac{1}{2} \times l^2 \times \frac{\sqrt{3}}{2} = A \quad \text{and} \quad \frac{1}{k} \times 3 \times l^2 = \frac{NA}{2} \quad \Rightarrow \quad k = \frac{2\sqrt{3}}{N}$$
Solution 1: If \( k < \frac{2\sqrt{3}}{N} \), no equilateral triangle’s pans can be put into the rack.

When using square pans, the smallest value of \( k \) is in the scene shown in Fig 5, \( k = \frac{2}{N} \), solution 2: if \( k = \frac{2}{N} \), no square pan can be put into the rack.

Fig 5

**Solution:** combining solution 1 and solution 2, if \( \frac{2}{N} \leq k < \frac{2\sqrt{3}}{N} \), no square pan can be put in the rack, so select equilateral triangle’s pans. If \( k = \frac{2}{N} \), select square pan.

- If \( \frac{2\sqrt{3}}{N} < k \leq 1 \),
- If \( k = \frac{2\sqrt{3}}{N} \times \frac{2^{\epsilon}}{2^{\epsilon}} \) (\( \epsilon \) is a positive integer), the changing process is based on Fig 4 and the process is show Fig 6,

Fig 6

**Solution:** select equilateral triangle’s pans.

- If \( k = \frac{2}{N} \times \frac{2^{\epsilon}}{2^{\epsilon}} \), (\( \epsilon \) is a positive integer), the changing process is based on Fig 5, and the process is show as Fig 7
Solution: select square pans.

- Other values of $k$ have no such law, and the difference between equilateral triangle’s pans and square pans is not obvious. So people can select according to their own standards.

(2) In $I_A$, $m \in [5, +\infty)$ (m is an integer). So if $\Omega$ is very important, that is people pay much more attention to the heat utilization ratio, regular polygon with more edges is likely to be selected. As $m$ increases, the outer edges of $\frac{(m-2)}{m} \times \pi$, so if one kind of pans can stitch seamlessly. $2\pi$ is a multiple of the polygon’s internal angle. Assume that a regular polygon with $m$ edges can stitch seamlessly, So $k = \frac{2m}{m-2} = 2 + \frac{4}{m-2}$ different shapes do not have obvious difference. So under this condition, the main factor that should be considered is the interspaces between the pans. A regular polygon’s internal angle is If $k$ is an integer, 4 must be a multiple of $(n-2)$, so $m = 3, 4, 6$ As figures show only equilateral triangle, square and regular hexagon can stitch seamlessly. While in $I_A$, $m \geq 5$. So the best choice is regular hexagon pan. Fig 8 shows four different pans in $I_A$.
(a) Regular pentagon’s combination  
(b) Regular hexagon’s combination  
(c) Regular octagon’s combination

**Solution:** In $I_A$, the best type is regular hexagon pan

3. Conclusion

In this paper, in order to find the optimal shape of the pan, we consider the problem under different users’ different focuses and the shape of the oven. We can find that if the shape of the oven is not as normal, then the shape of the pan we choose is mainly based on the oven’s shape. While under other conditions, we value the focus of the users more. And by means of modeling, we can get the result. In the future work, we should do market research analysis to confirm the correctness and value of our study.

References

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