Dynamic Semantics of Plurals $DPL_Q^{\circledast}$

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Abstract

This paper proposes a dynamic semantics of plurals, $DPL_Q^{\circledast}$, that is an extension of DPL [8] by adding binary generalized quantifiers, plural terms with join-operators as in Link [17]'s semilattice semantics of plurals, dynamic selectors, dynamic distributors and division functions. $DPL_Q^{\circledast}$ provides a formalism for handling dependent plurals, bound plurals, generic plurals, and ambiguity of collective/distributive/cumulative interpretation of plurals.

Key words: dynamic semantics, plurals, binary generalized quantifiers

1 Introduction

Dynamic semantics of natural and logical languages such as *Discourse Representation Theory* (*DRT*) [11], *Dynamic interpretation* [2,19], and *Dynamic Predicate Logic (DPL)* [8] have been developed since the early 80's. In particular, DPL is regarded as the most logically sophisticated system of dynamic semantics since it is equivalent to a $*$-free first-order dynamic logic with a poor test [8,9]. Dynamic semantics of plurals, plural quantifiers, and plural anaphoras have also been previously proposed [11,5,21,14]. However, they are not extensions [2] of DPL in the sense that if expression φ is true in DPL then φ is also true in the system, and are not compatible with the standard static semantics of plurals, *Logic of Plurals and Mass Terms* (LPM) [17].[3] In this

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[2] Although DRT [11] and TAI (the Theory of Anaphoric Information) [5] exploit Link's semilattice semantics of plurals, they have no direct connection with DPL. In particular, DRT introduces the abstraction operator and copying procedures such as DA, Abstraction, and Summation. DRT uses neutral variables which are denoted by small Greek letters, where the plurality and singularity is expressed by unary predicate *atom*.

[3] Link [17] proposes a formal semantics of plurals based on *join semilattice*, i.e., an algebraic structure $\langle L, At, \sqcup \rangle$, where $\sqcup$ is a commutative, idempotent, and associative, binary operator

paper, I will propose *Dynamic Predicate Logic with binary generalized quantifiers, plurals, the dynamic distributor, and the dynamic selector* ($DPL_Q^{\circledast}$), that is an extension of DPL with binary generalized quantifiers [12] (DPL_Q), compatible with LPM, and which handles the following problems relating to dynamics of plurals:

(i) dependent plurals and bound plural anaphoras [11],

(ii) descriptional plural anaphoras

(iii) generic plurals

(iv) plurally quantified antecedents of plural anaphoras [6,14],

(v) basic problems: (a) summation and (b) abstraction and uniqueness.

Section 2 will introduce problems on dynamic interpretations of plurals. Section 3 will introduce DPL_Q (DPL with binary generalized quantifiers), define $DPL_Q^{\circledast}$, and apply it to basic properties of dynamic interpretations of plurals [11]. Section 4 will propose solutions to the problems using dynamic distributors, dynamic selectors, and division functions.

2 Problems Relating to Dynamics of Plurals

2.1 Dependent Plurals and Bound Plural Anaphoras

Kamp & Reyle [11] discusses problems with interpretation of bare plurals and plural anaphora 'they' or 'them' in discourse, including the problems of *dependent plurals* and *bound plural anaphoras.* Kamp & Reyle [11] characterizes dependent plurals as in (1).

(1) a. A dependent plural x depending plural y is interpreted as X such that for each $z \in y$, for some $u \in X$, $R(z,u)$, and for each $u \in X$, for some $z \in y$, $R(z,u)$, where R is a relation.

b. A bare plural NP can be interpreted as a dependent plural only if it can be interpreted as dependent on some other plural NP which occurs in the same clause.

For example, let us consider (2).

(2) The women bought *cars.*

The logical form of (2a) is neither (3a) nor (3b), but (3c) due to condition (1a).

(3) a. $\exists! X.\exists Y.woman^{\circledast}(X) \wedge bought^{\circledast}(X,Y) \wedge car^{\circledast}(Y)$

b. $\exists! X.\exists Y.woman^{\circledast}(X) \wedge car^{\circledast}(Y) \wedge \forall x \in X.\exists y \in Y.bought(x,y)$

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over L and At the set of atoms in L. For example, 'John and Mary' is translated into logical form $\mathbf{j} \oplus \mathbf{m}$ that is interpreted as $j \sqcup m$ ($j, m \in At$), and 'the students' is translated into $\sigma x.student^{\circledast}(x)$ and interpreted as $\bigsqcup\{a \in At | student(a)\}$.

c. $\exists ! X.\exists Y.woman^{\circledast}(X) \wedge car^{\circledast}(Y) \wedge \forall x \in X.\exists y \in Y.bought(x,y) \wedge \forall y \in Y.\exists x \in X.bought(x,y)$

where $P^{\circledast}$ means the pluralized predicate of P and $X, Y, Z, \ldots$ are plural variables. This type of dependency can continue by adding relative clauses to dependent plurals as in (4).

(4) The women bought *cars* which had *automatic transmissions.*

Dependent plurals can be plural pronouns as in (5).

(5) a. Every director gave a present to a child. They found *teachers* who opened *them.*

b. Every director gave a present to a child. They found *a teacher* who opened *them.*

In (5a), 'teachers' is a dependent plural and, furthermore, 'them' depends on it, whereas in (5b) there is no such dependency. Therefore, the logical form of each second sentence in (5) is represented as in (6) and (7), respectively.

(6) a. They found teachers who opened *them.*

b. $\exists Y.found^{\circledast}(X,Y) \wedge teacher^{\circledast}(Y) \wedge \forall y \in Y.\exists z \in Z.opened(y,z) \wedge \forall z \in Z.\exists y \in Y.opened(y,z)$

(7) a. They found a teacher who opened *them.*

b. $\exists y.found^{\circledast}(X,y) \wedge teacher(y) \wedge opened(y,Z)$

However, according to Kamp & Reyle [11], the logical forms of (6) and (7) should be (8a) and (8b), respectively,

(8) a. $\forall uvz(u \in X \wedge director(v) \wedge present(z) \wedge child(u) \wedge gave\ to(v,z,u) \rightarrow \exists \mu.found(u,\mu) \wedge teacher^{\circledast}(\mu) \wedge opened(\mu,z))$

b. $\forall uvz(u \in X \wedge director(v) \wedge present(z) \wedge child(u) \wedge gave\ to(v,z,u) \rightarrow \exists y.found(u,y) \wedge teacher(y) \wedge opened(y,z))$

where μ is a neutral variable (i.e., μ can be singular or plural), via constructing the *Discourse Representation Structures (DRS)* (9a) and (9b), which are semantically equivalent to (8a) and (8b).

(9) a. $\forall u([u^{pl}, v^{pl(u)}, z^{pl(u)}|u \in X, \underline{director(v), present(z), child(u), gave\ to(v,z,u)}], [\mu^{pl,pl(u)}, \xi|teacher^{\circledast}(\mu), opened(\mu,\xi), \xi = z, found(u,\mu)])$

b. $\forall u([u^{pl}, v^{pl(u)}, z^{pl(u)}|u \in X, \underline{director(v), present(z), child(u), gave\ to(v,z,u)}], [y, \xi|teacher(y), opened(y,w), \xi = z, found(u,y)])$

where the superscription *pl* means the 'licensor' of dependent plurals, i.e., the variable that other dependent plurals depend on, and $pl(\alpha)$ marks the discourse referents that depend on discourse referent α, respectively. These annotations have no semantics but they only instruct constructions of the intended

DRSs[4]. These DRSs need the copy operations of the underlined conditions from the their previous contexts. ‘Them’ in these examples are represented by neutral variable ξ and they can be linked with a singular variable marked with *pl* or variable marked with $pl(u)$. Kamp & Reyle’s logical forms in (8) above denote a kind of dependent plurals, dependent plural anaphoras, but they are independent of logical forms of dependent bare plurals as in (3), although both share the property “dependent plurality”. In particular, the second half of definition (1a) is not considered in Kamp & Reyle’s logical forms. Furthermore, their logical forms in (8) need extra-non-semantical annotations such as marking of *pl* and $pl(\alpha)$ and copying of conditions from the previous contexts. In particular, the latter operation prevents the logical form from representing its “character” in the sense of David Kaplan, i.e., the context-independent meaning of the sentences, or in other words, the sentence meaning in the sense of Paul Grice. These two points constitute one of my objections to Kamp & Reyle’s treatment of dependent plurals and dependent plural anaphoras.

Dependent plurals are deeply related to *bound plural anaphora* as in (10a), of which the approximate intended meaning is not (10c) but (10d), although the most appropriate expression of (10d) is (10b).

(10) a. Few lawyers$_i$ hired secretaries $*$he$_i$/they$_i$ liked.
b. Few lawyers$_i$ hired a secretary $*$he$_i$/they$_i$ liked.
c. $Few\ x(lawyer(x), \exists Y.hired(x,y) \wedge secretary^{\circledast}(Y) \wedge liked^{\circledast}(x,Y))$
d. $Few\ x(lawyer(x), \exists y.hired(x,y) \wedge secretary(y) \wedge liked(x,y))$

Kamp & Reyle [11] attributes the inappropriateness of ‘he’ instead of ‘they’ in (10a-b) not to a semantical but to a grammatical reason, i.e., the license by a *pl*-marked variable, as in (11).

(11) a. $Few\ x([x^{pl}|lawyer(x)], [\xi^{pl,pl(x)}, \zeta|hired(x,\xi), secretary(\xi), liked(\zeta,\xi), \zeta = x])$
b. $Few\ x([x^{pl}|lawyer(x)], [y, \zeta|, hired(x,y), secretary(y), liked(\zeta,y), \zeta = x])$

But, as we have seen, from a semantic point of view, ‘they’ depends on ‘secretaries’, but Kamp & Reyle’s (11a) cannot handle this, and thus the more appropriate logical form of (10a) is (12), although even (12) is insufficient for representing dependency between ‘they’ and ‘secretaries’.

(12) $Few\ x(lawyer(x), \exists Y.secretary^{\circledast}(Y) \wedge (\forall y \in Y.hired(x,y) \wedge liked(x,y))$

[4] Although usually DRSs are denotated by Kamp’s “box-notation” (see [11]), the DRSs in (9) are represented in a “formula-notation” similar to [8]. To explain briefly, in this “formula-notations”, in DRS $[x_1,\ldots,x_n|\varphi_1,\ldots,\varphi_m]$, variables $x_1,\ldots,x_n$ (in DRT-terms, discourse referents) bind formulas $\varphi_1,\ldots,\varphi_m$ (in DRT-terms, conditions) and the DRSs subordinated by this DRS in a way similar to existential quantifiers. Generalized quantifiers including $\forall x$ are all binary predicates that take DRSs as their arguments.

To handle dependent plurals in the sense of (1), I consider singular binary generalized quantifiers to be inappropriate for describing dependent plurals and bound plural anaphoras, and I will propose dynamic plural generalized quantifiers in section 3.

2.2 Descriptional Plural Anaphoras

I assume at least four readings for plural anaphora 'they':

(i) demonstrative (this is outside of the scope of this paper)
(ii) bound anaphora (see section 2.1)
(iii) dependent anaphora (see section 2.1)
(iv) descriptional anaphora (this section and see section 2.3)

A descriptional anaphora is an anaphora which is not bound by the antecedent explicitly, but rather it must be read as a definite plural description[5]. For example, Krifka [14] reminds us of the following examples.

(13) a. No student wrote an article. *They* (all) spent their days on the beach.
b. Few students wrote an article. *They* rather spent their days on the beach.

In (13a), *they* means *(all) the students*. On the other hand, in (13b), *they* means *the other students*. His solution to such a various reference of *they* is based on a kind of multiple coindexing as in (14).

(14) a. $\mathrm{No}_{1,2}$ student wrote an article. $They_1$ (all) spent their days on the beach.
b. $\mathrm{Few}_{1,3}$ students wrote an article. $They_3$ rather spent their days on the beach.

where $1 = \{x|student(x)\}$, $2 = \{x|student(x)\} \cap \{x|wrote(x,y), article(y)\}$, and $3 = \{x|student(x)\} - \{x|wrote(x,y), article(y)\}$. However, this solution seems to be ad-hoc and not compatible with DPL. Rather, I adopt the idea that *they* denotes plural definite descriptions such as 'the students' and 'the other students'. That is, in the logical form, *they* can be replaced with a plural definite description. This idea is compatible with $DPL_Q^{\circledast}$ and needs no extra complex indexing.

Furthermore, in the next section, we will see similar cases that are relevant to generic expressions.

[5] Descriptional anaphoras are deeply related to Neale [18]'s *D-type pronouns*. D-type pronouns are pronouns interpreted as definite descriptions of which descriptions are recovered from antecedents and every element *c-commanding* at LF (and non-c-commanding elements). However, description contents of descriptional anaphoras are not always recovered from overt expressions. See (13b) and (16).

2.3 Generic Plurals

English generic noun phrases are classified into at least the following classes:

(i) bare generic plurals: e.g., 'Dogs bark.'

(ii) definite generic singular descriptions: e.g., 'The dog barks.'

(iii) definite generic plural descriptions

(iv) indefinite generic singular descriptions: e.g., 'A wolf takes a mate for life.' [6]

(v) implicit antecedents of generic plural anaphoras

(vi) dependent generic bare plurals

In other words, noun phrases without indefinite plurals can be interpreted as generics.

(15) is an example of (iii).

(15) a. *The children in this city* thrive.

b. *The dogs in this city* do not bark.

According to Kamp & Reyle [11], (15a) is characterized as follows.

> This sentence can be accepted as true even when there are a few children in city who do not thrive. There may be special children, children born with some debilitating malformation, say, who don't thrive. But that does not really threaten the global generalization that [(15a)] means to express. ... [(15a)] could only be true, one feels, if a substantial proportion (presumably a majority) of the city's children thrive. – [11], p. 411

That is, (15a), as well as (15b), does not denote individual properties but generic properties.

The sentences of (16) are examples of (v).

(16) a. Few women from this village came to the feminist rally. No wonder. *They* don't like political rallies very much.

b. If at least one chicken which Ottilie owns had laid an egg, she had a nice breakfast. *They* are very good to eat.

c. John killed a spider because *they* are ugly.

In (16c), *they* denotes generic spiders. Also in (16a-b), *they* denotes generic women and eggs or generic 'women from this village' and 'eggs which Ottilie's chickens lay'.

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[6] In Kamp & Reyle [11], this sentence's DRS is:
$[|[x|wolf(x)] \approx\!> [y|mate(y), takes\ for\ life(x,y)]]$ or
$[|Gen\ x([x|wolf(x)],[y|mate(y), takes\ for\ life(x,y)])]$, where $\approx\!>$ and Gen are "generic implications" or "generic operators". For more detail, see [11], pp.294 – 297 and p.411. Although the semantics of Gen or $\approx\!>$ is not given in [11], we can exploit Asher & Morreau [1]'s semantics based on default conditionals or Cohen [4]'s probabilistic conditionals for them.

(17a) is an example of (vi), cited from [16].

(17) a. Indians make *baskets*.

b. Dogs chase *cats*.

As [16] points out, *baskets* in (17a) means general 'baskets which are made by Indians', while *cats* in (17b) means general 'cats'.

Although here I will not delve deeply to the problems of 'what is genericity', I will consider the problems from the point of view of the interpretation of plurals and plural anaphoras. Since of the above classes, (i), (iii), (v) and (vi) are able to be the antecedents of plural anaphoras, I will treat only them.

From the point of view of 'dynamic semantics of plurals', generic plurals are problematic in the sense of how to represent them in logical forms. Carlson [3] represents 'Dogs bark' as follows.

(18) $(G'(bark))(dog)$

He handles 'dogs' as an expression denoting an object of which sort is called 'kind', and by G' he denotes a sort of type coercive function from sort 'individual' to sort 'kind'. However, since, as in (19a), 'dogs' can be an antecedent of plural anaphoras, the logical form of 'Dogs bark' must be (19b), or the plural anaphora must be singular as in (19c). The logical form should not be (19d), since (19d) means that there is a group or set of dogs which are good pets, that is expressed by (19e).

(19) a. Dogs bark. But *they* are good pets.

b. $\exists X.X = dog \wedge (G'(bark))^{\circledast}(X) \wedge (G'(good\ pet))^{\circledast}(X)$

c. $\exists x.x = dog \wedge (G'(bark))(x) \wedge (G'(good\ pet))(x)$

d. $\exists X.dog^{\circledast}(X) \wedge bark^{\circledast}(X) \wedge good\ pet^{\circledast}(X)$

e. Dogs are barking. But *they* are good pets.

Kamp & Reyle [11] proposes the rules for (iii) and (v) and DRSs for (15) and (16a-b) as in (20).

(20) a. $[X|the\ child\ in\ this\ city^{\circledast}(X), Gen\ x([x|x \in X],[thrive(x)])]$

b. $[X,y,z|woman^{\circledast}(X), this\ village(y), the\ feminist\ rally(z),$
$Few\ x([|woman(x), from(x,y)],[|came\ to(x,z)]), \neg like\_political\_rallies(X)]$

c. $[y,U,V|Ottilie(y), egg^{\circledast}(U), [|\exists x([x|chicken(x), owns(x,y)],$
$[z|egg(z), had\ laid(x,z)])] \rightarrow [u|u = y, had\ a\ nice\ breakfast(u)]$
$V = U, be\ very\ good\ to\ eat^{\circledast}(V)]$

Condition $Gen\ x([x|x \in X],[P(x)])$ handles the genericity of (iii), and conditions $woman^{\circledast}(X)$ in (20b) and $egg^{\circledast}(U)$ in (20c), that have no corresponding expression in (16b-c), play the role of the antecedents of plural anaphoras in (16b-c). However, since the conditions of form of $P^{\circledast}(X)$ only denote that X is a set or group satisfying property P, we need other expressions to denote genericity. Furthermore, it can be problematic that $woman^{\circledast}(X)$ in (20b) and

$egg^{\circledast}(U)$ in (20c) are 'extra'-conditions that have no corresponding expression in (16b-c) if we follow the principle of compositionality of sense.

As for (iv), as far as I know, there has been no proposal of its logical form.

2.4 Plurally Quantified Antecedents

Gillon [6] points out that *they* (originally 'the men' is used instead of 'they') as in (21) is ambiguous as to whether it is *collective*, *distributive*, or other.

(21) They wrote operas or musicals.

If *they* denotes Mozart and Handel, then they each composed their own operas and then it is *distributive*, if Gilbert and Sullivan, then they wrote operas collaboratively and it is *collective*, and if it denotes the group consisting of Mozart, Handel, Gilbert and Sullivan, then it denotes a *partition*, and if Rodgers, Hammerstein, and Hart, then Rodgers and Hammerstein composed musicals collaboratively, and Rodgers and Hart each composed their own musicals, and therefore it denotes a *cover*[7] of the three composers. The problem is not restricted to the interpretation of *they*, but the interpretation of bare plural *operas or musicals*, i.e., a dependent bare plural, is also ambiguous. This ambiguity can be clarified if the bare plural is quantified as in (22).

(22) Three composers visited there. *They* wrote four operas or musicals.

If *they* is distributive, i.e., it means 'each of the three composers', the total number of operas or musicals is twelve. If it is collective, i.e., it means 'the group consisting of the three composers', the total number is four. If *they* is interpreted in relation to the *cumulative reading* of (22), the total number of the composers who visited there and composed operas or musicals is three, and the total number of the operas or musicals composed by the composers who visited there is four, i.e., each of the composers could compose one, two, three, or four musicals or operas, either in collaboration with each others or not, but the total number of the composed pieces must be four. If it denotes a cover, the composed operas or musicals can be overlapped and their total number can vary from four to twelve.[8] Scha [20] defines the cumulative reading of

[7] I define covers as follows: $c(X)$ is a cover of X, i.e., $c(X) \in cov(X)$ if $\bigcup c(X) = X$.

[8] However, this argument depends on a weak consensus of the judgments of interpretability of such multi-quantified sentences. Gillon [6] claims that the *cover* is not appropriate, and instead of it he proposes the *minimal cover* and the *plural cover*. A cover is a minimal cover if any element of the cover is not a subset of the other elements of the cover: e.g., for set $\{a, b\}$, $\{\{a, b\}, \{a\}\}$ is a cover of it but not a minimal cover of it; for set $\{a, b, c\}$, $\{\{a, b\}, \{b, c\}\}$ is a minimal cover of it. But this is criticized by Lasersohn [15]. The minimal cover theory incorrectly verifies sentence *The TAs were paid exactly \$14,000 last year* in the situation such that TA 1 was paid \$7,000, and TA 2, TA 3, too, since $\{\{TA1, TA2\}, \{TA2, TA3\}\}$ is a minimal cover of the set of the TAs. On the other hand, the minimal cover theory incorrectly falsifies sentence *The TAs get paid exactly \$7,000* in the situation such that TA 1 was paid \$7,000 at a class, TA 2 was paid \$7,000 at a class, and TA1 and TA 2 were jointly paid \$7,000 at the other class, since $\{TA1, TA2, \{TA1, TA2\}\}$ is not a minimal cover of $\{TA1, TA2\}$. See also [7]. Similarly, according to Verkuyl [22], for a sentence *Three boys*

sentences of form of 'n Subj Verb m Obj' as follows: let $A = \{x|\exists y.Subj(x) \wedge Verb(x,y) \wedge Obj(y)\}$ and $B = \{y|\exists x.Subj(x) \wedge Verb(x,y) \wedge Obj(y)\}$. Then

- $|A| = n$, $|B| = m$
- $\forall x \in A.\exists y \in B.Verbs(x,y)$
- $\forall x \in B.\exists y \in A.Verbs(x,y)$

Kamp & Reyle [11] generalizes the above definition as follows [9],

- $|A| = n$, $|B| = m$
- parts of A which are $Verb$-related to some part of B exhaust all of A
- parts of B which are $Verb$-related to some part of A exhaust all of B

and denies the following DRS,

(23) $[X,Y|\ |X| = n, |Y| = m, Subj^{\circledast}(X), Obj^{\circledast}(Y), \forall x([x|x \in X], [\forall y([y|y \in Y], [|Verb(x,y)])])]$

but they treat a cumulative reading as an open problem.

Although Krifka [14] proposes a dynamic semantics of cumulative readings, his proposal seems to be insufficient, as we will see in section 3.

2.5 Glued Plurals

Van den Berg [21] points out 'gluing' singular referents as in the following discourse.

(24) Every student borrowed a book$_i$. *They*$_i$ were returned.

As the solution of this problem, whereas in DPL a single assignment is regarded as a state, he regards a set of assignments as a state, and introduces other devices such as the maximization operator and the distribution operator, but the variables are all plural, where the singularity is expressed by unary predicate *sing*.

(25) $(\forall x(student(x) \wedge sing(x) \rightarrow \exists y.book(y) \wedge sing(y) \wedge borrowed(x,y))) \wedge \exists z(z = y \wedge were\ returned(z))$

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bought a boat, Lønning admits the situation such that boy 1 and boy 2 bought boat 1 and boy 3 bought boat 2, but Verkuyl rejects this kind of reading of the sentence.

[9] Their argument is based on some possible readings of 'Three lawyers hired five cleaners'. For example, the first lawyer hired cleaner 1, 2 and 3 on a joint contract and also hired cleaner 4 on a separate contract, and the three lawyers together hired cleaner 5. This case does not satisfy Scha's stipulation. Their stipulation of cumulative readings may include Krifka [13]'s 'event-oriented readings'. For example, *three ships arrived at a harbor* means that there are three arrivals of the same or the other ship at a harbor. This admits only one ship's arrivals three times. However, Krifka's claim is not sufficient to explain the facts. It is possible that there are only two arrivals of one ship, and a total of two ships, and furthermore one of the two ships can be also the arrived ship of another arrival. As a result, only two ships participated in the events.

In my opinion, van den Berg's system may not work well. For example, he gives us the definition: $G\,[\![sing(x)]\!]H$ if $G = H$ and $|G(x)| = 1$, where G, H are a set of assignments such that $G(x) = \{g(x)|g \in G\}$. This means that $sing(x)$ is true in $G = \{g\}$ or G such that $\forall g, h \in G.g(x) = h(x)$. The logical form of this sentence includes $sing(x)$, filtering out all sets of assignments except $G = \{g\}$ or G such that $\forall g, h \in G.g(x) = h(x)$. This prevents the interpreters from 'gluing' referents. Even if the definition is changed as $G\,[\![sing(x)]\!]H$ if $G = H$ and $\forall g \in G.|g(x)| = 1$, I think there is no benefit from adopting assignment sets as states.

If we exploit the Linkian semilattice semantics of plurals, we do not need predicate $sing(x)$, but need the device to explain 'gluing' singular referents. In this paper, I will treat 'glued' plurals by my formalization of *summation* (see section 3.3.1 and 4.5).

3 $DPL_Q^{\circledast}$

3.1 Definitions

I introduce DPL [8] with binary generalized quantifiers [12], DPL_Q, then define $DPL_Q^{\circledast}$, as follows.

Definition 1 ([8]) *Let $Var = \{x_1, \ldots, x_n\}$ be a set of individual variables, $Con = \{a_1, \ldots, a_m\}$ a set of individual constants, $Rel = \{R_i\}_{i \in I}$ a set of relational symbols with arity $\rho : I \to \mathbb{N}$, $Quant = \{Q_j\}_{i \in J}$ a set of binary generalized quantifier symbols, and $\mathcal{M} = \langle D_{\mathcal{M}}, \langle R_{i\mathcal{M}} \rangle_{i \in I}, \langle Q_{j\mathcal{M}} \rangle_{j \in J} \rangle$ be a model of a first order language with binary generalized quantifiers, where $D_{\mathcal{M}}$ is a set of individuals, $R_{i\mathcal{M}} \subseteq D_{\mathcal{M}}^{\rho(i)}$, $Q_{j\mathcal{M}} \subseteq pow(D_{\mathcal{M}}) \times pow(D_{\mathcal{M}})$, and $g, h, i, j, k : Var \to D_{\mathcal{M}}$ variable assignments.*

The syntax of $\varphi \in L_Q$ is defined as follows:

Terms $\tau ::= x|a$

Formulas $\varphi ::= (\tau_1 = \tau_2)|R(\tau_1, \ldots, \tau_n)|\varphi_1 \wedge \varphi_2|\varphi_1 \to \varphi_2|\exists x\varphi|\forall x\varphi|\neg\varphi|Qx(\varphi_1, \varphi_2)$

The semantics of $\varphi \in L_Q$ is defined using relational truth condition '$g\,[\![\varphi]\!]^{\mathcal{M}}h$' that express that formula φ changes g to h in $\mathcal{M}$, as follows:

- $x^{\mathcal{M},g} = g(x)$; $a^{\mathcal{M},g} \in D_{\mathcal{M}}$
- $g\,[\![\tau_1 = \tau_2]\!]^{\mathcal{M}}h \Leftrightarrow g = h$ *and* $\tau_1^{\mathcal{M},g} = \tau^{\mathcal{M},g}$
- $g\,[\![R(\tau_1, \ldots, \tau_n)]\!]^{\mathcal{M}}h \Leftrightarrow g = h \;\&\; \langle \tau_1^{\mathcal{M},g}, \ldots, \tau_n^{\mathcal{M},g} \rangle \in R_{\mathcal{M}}$
- $g\,[\![\varphi_1 \wedge \varphi_2]\!]^{\mathcal{M}}h \Leftrightarrow$ *for some* $i.g\,[\![\varphi_1]\!]^{\mathcal{M}}i$ *and* $i\,[\![\varphi_2]\!]^{\mathcal{M}}h$
- $g\,[\![\varphi_1 \to \varphi_2]\!]^{\mathcal{M}}h \Leftrightarrow g = h$ *and for all* $i : g\,[\![\varphi_1]\!]^{\mathcal{M}}i$. *for some* $j.i\,[\![\varphi_2]\!]^{\mathcal{M}}j$
- $g\,[\![\exists x\varphi]\!]^{\mathcal{M}}h \Leftrightarrow$ *for some* $e \in D_{\mathcal{M}}.g[e/x]\,[\![\varphi]\!]^{\mathcal{M}}h$
- $g\,[\![\forall x\varphi]\!]^{\mathcal{M}}h \Leftrightarrow g = h$ *and for all* $e \in D_{\mathcal{M}}$. *for some* $j.g[e/x]\,[\![\varphi]\!]^{\mathcal{M}}j$.
- $g\,[\![\neg\varphi]\!]^{\mathcal{M}}h \Leftrightarrow g = h$ *and for no* $i.g\,[\![\varphi]\!]^{\mathcal{M}}i$

- $g\,[\![Qx(\varphi_1,\varphi_2)]\!]^{\mathcal{M}}h \;\Leftrightarrow\; g=h$ *and* $\langle\{i(x)|g\,[\![\varphi_1]\!]^{\mathcal{M}}i\},\{i(x)|g\,[\![\varphi_1@\varphi_2]\!]^{\mathcal{M}}i\}\rangle \in Q_{\mathcal{M}}$*, where if* $@=\wedge$*, then the interpretation is called weak, and if* $@=\rightarrow$*, then strong.*

Definition 2 *Adding to definition 1, let* $Var^{\circledast}=\{X_1,\ldots,X_n\}$ *be a set of plural variables and* $\mathcal{M}=\langle D_{\mathcal{M}},D^{\circledast}_{\mathcal{M}},\sqcup,\langle R_{i\mathcal{M}}\rangle_{i\in I},\langle R^{\circledast}_{i\mathcal{M}}\rangle_{i\in I},\langle Q_{j\mathcal{M}}\rangle_{j\in J}\rangle$ *be a model of a first order language with plurals, binary generalized quantifiers and binary plural generalized quantifiers, where* $D^{\circledast}_{\mathcal{M}}$ *is the domain of* $\mathcal{M}$*, a* $\sqcup$*-semilattice generated by* $D_{\mathcal{M}}$*,* $R^{\circledast}_{i\mathcal{M}}\subseteq(D^{\circledast}_{\mathcal{M}})^{\rho(i)}$ *satisfying the following conditions:*

if $\langle\sigma_1,e,\sigma_2\rangle\in R_{\mathcal{M}}$ *and* $\langle\sigma_1,E,\sigma_2\rangle\in R^{\circledast}_{\mathcal{M}}$*, then* $\langle\sigma_1,e\sqcup E,\sigma_2\rangle\in R^{\circledast}_{\mathcal{M}}$*, where* σ_1,σ_2 *are sequences of* $D_{\mathcal{M}}$*,*

if $\langle\Sigma_1,E_1,\Sigma_2\rangle\in R^{\circledast}_{\mathcal{M}}$ *and* $\langle\Sigma_1,E_2,\Sigma_2\rangle\in R^{\circledast}_{\mathcal{M}}$*, then* $\langle\Sigma_1,E_1\sqcup E_2,\Sigma_2\rangle\in R^{\circledast}_{\mathcal{M}}$*, where* Σ_1,Σ_2 *are sequences of* $D^{\circledast}_{\mathcal{M}}$*.*

A variable assignment g *that is a function from* $Var\cup Var^{\circledast}$ *to* $D^{\circledast}_{\mathcal{M}}$*. The syntax and semantics of* $\varphi\in L^{\circledast}_{Q}$ *are defined as follows:*

$$
\begin{array}{llll}
\textit{Term} & \tau & ::= & ...|\tau^{\circledast} \\
\textit{Plural Term} & \tau^{\circledast} & ::= & X|\tau_1\oplus\tau_2|\sigma X.\varphi^{\circledast}|d(\tau^{\circledast}) \\
\textit{Formula} & \varphi & ::= & ...|\varphi^{\circledast}|\varphi^{*} \\
\textit{Plural Formula} & \varphi^{\circledast} & ::= & \tau\epsilon\tau^{\circledast}|\tau\in\tau^{\circledast}|X:=X-\tau|X=\tau^{\circledast}|R^{\circledast}(\tau_1,\ldots,\tau_n) \\
 & & & |Q^{\circledast}X(\varphi^{\circledast}_1,\varphi^{\circledast}_2)|\forall x\epsilon\tau^{\circledast}\varphi|\exists x\epsilon\tau^{\circledast}\varphi|\varphi^{\circledast}_1\wedge\varphi^{\circledast}_2|\varphi^{\circledast}_1\rightarrow\varphi^{\circledast}_2 \\
 & & & |\neg\varphi^{\circledast}|\exists X.\varphi^{\circledast}|\exists! X.\varphi^{\circledast}
\end{array}
$$

where x *of* $\forall x\epsilon...$ *and* $\exists x\epsilon...$ *can be either singular or plural, and* d *is called a division function, that is used in section 4.*

- $X^{\mathcal{M},g}=g(X)\in D^{\circledast}_{\mathcal{M}}\cup List(D^{\circledast}_{\mathcal{M}})$
- $(\tau_1\oplus\tau_2)^{\mathcal{M},g}=\tau_1^{\mathcal{M},g}\sqcup\tau_2^{\mathcal{M},g}$
- $(\sigma X.\varphi^{\circledast})^{\mathcal{M},g}=\bigsqcup\{i(x)|g\,[\![\varphi[x/X]]\!]^{\mathcal{M}}i,$ *for some variable* x *free in* $\varphi\}$*,*
- $d^{\mathcal{M},g}:D^{\circledast}_{\mathcal{M}}\rightarrow List(D^{\circledast}_{\mathcal{M}})$*, satisfying condition* $\bigsqcup\{Y|Y\epsilon d^{\mathcal{M},g}(X)\}=X$ *and if* $X=Y$ *then* $d(X)=d(Y)$*,*
- $g\,[\![\varphi^{*}]\!]^{\mathcal{M}}h\;\Leftrightarrow\;(g,h)\in\bigcup_{n\geq 0}([\![\varphi]\!]^{\mathcal{M}})^{n}$*, where for any relation* R*,* $R^0=Id$ *and* $R^{n+1}=R\circ R^{n}$*),*
- $g[\![\tau\epsilon\tau^{\circledast}]\!]^{\mathcal{M}}h\;\Leftrightarrow\;g=h$ *and* $\tau^{\mathcal{M},g}\epsilon\tau^{\circledast\mathcal{M},g}$*,*
  where $x\epsilon y$ *iff if* y *is a list, then* x *is a member of* y*, otherwise* $x\leq y$*, i.e.,* $x\sqcup y=y$ *and* $x\in D_{\mathcal{M}}$*,*
- $g\,[\![X:=X-\tau]\!]^{\mathcal{M}}h\;\Leftrightarrow\;g=h$ *and* $h(X)=g(X)-\tau^{\mathcal{M},g}$*,*
  where $x-y$ *iff if* $x=\langle y,z\rangle$*, then* $x-y=z$*, else if for some* z*,* $x=z\sqcup y$ *and* $y\not\leq z$*,* $x-y=z$*, otherwise undefined,*

- $g[\![\tau \in \tau^{\circledast}]\!]^{\mathcal{M}} h \Leftrightarrow g = h$ *and* $\tau^{\mathcal{M},g} \leq \tau^{\circledast\mathcal{M},g}$, *and* $\tau^{\mathcal{M},g} \in D_{\mathcal{M}}$,
- $g[\![R^{\circledast}(\tau_1, \ldots, \tau_n)]\!]^{\mathcal{M}} h \Leftrightarrow g = h$ *and* $\langle \tau_1{}^{\mathcal{M},g}, \ldots, \tau_n{}^{\mathcal{M},g} \rangle \in R^{\circledast}_{\mathcal{M}}$
- $g\,[\![\exists X \varphi^{\circledast}]\!]^{\mathcal{M}} h \Leftrightarrow$ *for some* $e.\bot_{D_{\mathcal{M}}} < e \leq (\sigma X \varphi^{\circledast})^{\mathcal{M},g}$, $g[e/X]\,[\![\varphi^{\circledast}]\!]^{\mathcal{M}} h$,
- $g\,[\![\exists X! \varphi^{\circledast}]\!]^{\mathcal{M}} h \Leftrightarrow$ *for some* $e = (\sigma X \varphi^{\circledast})^{\mathcal{M},g}$, $g[e/X]\,[\![\varphi^{\circledast}]\!]^{\mathcal{M}} h$,
- $g[\![\exists x \epsilon \tau^{\circledast}.\varphi]\!]^{\mathcal{M}} h \Leftrightarrow$ *for some* $e \epsilon \tau^{\circledast\mathcal{M},g}$ *and some plural variable* X *free in* φ, $g[e/x]\,[\![X = \tau^{\circledast} \wedge \varphi \wedge X := X - x]\!]^{\mathcal{M}} h$,
- $g[\![\forall x \epsilon \tau^{\circledast}.\varphi]\!]^{\mathcal{M}} h \Leftrightarrow$ *for some plural variable* X *free in* φ, $g[\![X = \tau^{\circledast} \wedge ((X \neq \emptyset) \wedge \exists x \epsilon X.\varphi)^* \wedge X = \emptyset \wedge \neg\varphi]\!]^{\mathcal{M}} h$,
- $g[\![Q^{\circledast} X(\varphi_1^{\circledast}, \varphi_2^{\circledast})]\!]^{\mathcal{M}} h \Leftrightarrow h = g[(\sigma X.\varphi_1^{\circledast} \wedge \varphi_2^{\circledast})^{\mathcal{M},g}/X]$ *and* $g\,[\![Qx(\varphi_1[x/X], \varphi_2[x/X])]\!]^{\mathcal{M}} h$, *where* x *is free in* φ_1 *and* φ_2.

Definition 3 *φ_1 is dynamically equivalent to φ_2, written $\varphi_1 \equiv \varphi_2$ iff for all $g, h\,[\![\varphi_1]\!] h$ implies for some i, $h\,[\![\varphi_2]\!] i$, and for all g, h, $g\,[\![\varphi_2]\!] h$ implies for some i, $h\,[\![\varphi_1]\!] i$.*

3.2 Dynamic Distributor and Dynamic Selector

In $DPL^{\circledast}_{Q}$, new formulae such as $\forall x \epsilon \tau.\varphi$ and $\exists x \epsilon \tau.\varphi$ are introduced in order to handle problems concerning the distributive interpretations of plurals. $\forall x \epsilon \tau$ is called a *dynamic distributor* and $\exists x \epsilon \tau$ a *dynamic selector*. The problems that will be handled by these operators have already been handled in Krifka [14]. Krifka [14] defines a variable assignment change relation $[x/P]$ for some variable x and predicate P over a data structure called *parameterized sum individuals*,[10] but it seems to be non-well-defined. The operator is defined as follows: $g[x/P]h$ iff h is a variable assignment like g, except that every $\langle a, f \rangle \in g(x)$ is replaced by $\langle a, f + i \rangle$ s.t. $\langle g + f, \langle a, f \rangle, g + f + i \rangle \in P$, where P is the denotation of a unary predicate, and $+$ is defined as follows:

$$g + f = \begin{cases} g \cup f & \text{if } RDOM(g) \cap RDOM(h) = \emptyset \\ \bot & \text{otherwise} \end{cases}$$

where $RDOM(g)$ is the set of all variables contained in g. Since $\langle a, f \rangle \in g(x)$, $dom(f) \subseteq RDOM(g)$ and hence $g + f$ is always undefined.

[10] The class of parameterized sum individuals $P(D, I)$ over set of discourse entities D and set of individuals I, and the class of partial assignments $G(D, I)$ from D to $P(D, I)$ are defined by mutual recursion:

$$P^0(D,I) = pow(pow(I) \times G^0(D,I)) - \{\emptyset\} \quad G^0(D,I) = [D \to \{\bot\}]$$
$$P^{\alpha+1}(D,I) = pow(pow(I) \times \textstyle\bigcup_{\beta<\alpha+1} G^{\beta}(D,I)) - \{\emptyset\} \quad G^{\alpha+1}(D,I) = [D \hookrightarrow P^{\alpha}(D,I)]$$
$$P(D,I) = \textstyle\bigcup_{\alpha<\omega} P^{\alpha}(D,I) \quad G(D,I) = \textstyle\bigcup_{\alpha<\omega} G^{\alpha}(D,I)]$$

where $\bot$ is the bottom element denoting the undefinedness, $[X \hookrightarrow Y]$ is the space of partial functions from X into Y.

The operation intended by Krifka can be defined by a dynamic distributor as $\forall y\epsilon x.\varphi$. This formula means the following procedure by definition 2:

$$(\textbf{while}\ x \neq \emptyset\ \textbf{do}\ y :=?; y\epsilon x?; \varphi; x := x - y); \neg\varphi$$

As the result of an execution of this procedure, the following conditions hold about the output assignment h:

- $h(x)$ does not satisfy $P(x)$
- Every $e \leq h(x)$ satisfies $P(y)$

These conditions mean that a dynamic distributor $\forall x\epsilon\tau.\varphi$ do not change the meaning of τ but the meaning of each element a of τ.

Theorem 3.1 *For some g, $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$ implies $h \models \forall x \in y.\exists z \in u.Pxz \wedge \forall z \in u.\exists x \in y.Pxz$.*

Proof. $(\forall x\epsilon y.\exists z\epsilon u.Pxz) \equiv (\textbf{while}\ y \neq \emptyset\ \textbf{do}\ x :=?; x\epsilon y?; z :=?; z\epsilon u; Pxz?; u := u - z; y := y - x); [z :=?; z\epsilon u; Pxz?]\bot$ by definition 2. If $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$ and $h \not\models \forall x \in y.\exists z \in u.Pxz$, i.e., $h \models \exists x \in y.\forall z \in u.\neg Pxz$. Then the part "$x :=?; x\epsilon y?; z :=?; z\epsilon u; Pxz?$" fails and there is no h such that $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$. Therefore, $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$ implies $h \models \forall x \in y.\exists z \in u.Pxz$. If $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$ and $h \not\models \forall z \in u.\exists x \in y.Pxz$, i.e., $h \models \exists z \in u.\forall x \in y.\neg Pxz$. Then the part $[x :=?; z\epsilon u; Pxz?]\bot$ fails and there is no h such that $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$. Therefore, $g\,[\![\forall x\epsilon y.\exists z\epsilon u.Pxz]\!]h$ implies $h \models \forall z \in u.\exists x \in y.Pxz$. $\square$

3.3 Basic Properties of Plural Anaphoras

By the 'basic properties of plural anaphoras', I mean the properties handled in [11], i.e., *summation* and *abstraction* or the *uniqueness effect* [10]. In this subsection, we will see how to handle these basic properties in $DPL_Q^{\circledast}$.

3.3.1 Summation

Summation is an operation to sum all the discourse entities appearing in the previous context, as in the denotation of 'they' in (26a), that is expressed as in (26) in $DPL_Q^{\circledast}$.

(26) a. 'John invited Mary to a party. Bill was also invited to the party by him. *They* were happy.'='John invited Mary and Bill to a party and John, Mary and Bill were happy.'

b. $(\exists X.\exists xyz(x\epsilon X \wedge y\epsilon X \wedge z\epsilon X \wedge \varphi_1(x,y,z)) \wedge \exists v(v\epsilon X \wedge \varphi_2(v,z)) \wedge \varphi_3(X))$
$\equiv (\exists X.\exists xyzv(x\epsilon X \wedge y\epsilon X \wedge z\epsilon X \wedge \varphi_1(x,y,z) \wedge v\epsilon X \wedge \varphi_2(v,z) \wedge \varphi_3(X)))$

3.3.2 Abstraction and Uniqueness

Abstraction is an operation of making the reference to the 'sieve', i.e., the intersection of the quantified noun and the predicate, as in the denotation of 'they' in (27a), that is represented as in (27b) in $DPL_Q^{\circledast}$.

(27) a. 'Most books have been found. *They* are in this room.'='Most books which have been found are in this room.'

b. $Most^{\circledast}X(\varphi_1(X), \varphi_2(X)) \wedge \varphi_3(X)$
$\equiv Most^{\circledast}X(\varphi_1(X), \varphi_2(X)) \wedge \varphi_3(\sigma X.\varphi_1(X) \wedge \varphi_2(X))$

Uniqueness is the property of anaphora, i.e., the entity denoted by an anaphora is unique and if the anaphora is plural, then its denotation is the maximal one, as in (28a), that is expressed as in (28b) in $DPL_Q^{\circledast}$ similarly with the case of abstraction.

(28) a. 'John owns some sheep. Harry vaccinates *them.*' = 'John owns some sheep. Harry vaccinates all the sheep which John owns.'

b. $Some^{\circledast}X(\varphi_1(X), \varphi_2(X)) \wedge \varphi_3(X)$
$\equiv Some^{\circledast}X(\varphi_1(X), \varphi_2(X)) \wedge \varphi_3(\sigma X.\varphi_1(X) \wedge \varphi_2(X))$.

4 Solutions to the Problems

4.1 Dependent Plurals and Plural Anaphoras

Reconsider (10a). This cannot be provided its appropriate logical form in section 2. Now we can give it an appropriate logical form as in (29b).

(29) a. Few lawyers$_i$ hired secretaries $*$he$_i$/they$_i$ liked.

b. $Few^{\circledast}X(lawyer^{\circledast}(X),$
$\exists Y.secretary^{\circledast}(Y) \wedge (\forall y\epsilon Y \exists x\epsilon X.hired(x,y) \wedge liked(x,y)))$

The second argument of $Few^{\circledast}$ must include X and must not include dynamic distributor of X such as $\forall x\epsilon X.\varphi$, since when it is interpreted, by definition 2, X is replaced a free singular variable.

Furthermore, $DPL_Q^{\circledast}$ can distinguish discourse anaphoras which denote the 'sieves' of the plurally quantified antecedents from bound plural anaphoras as in (30).

(30) a. Few lawyers$_i^j$ hired secretaries who they$_i$ liked. They^j were happy.

b. $Few^{\circledast}X(lawyer^{\circledast}(X), \exists Y.secretary^{\circledast}(Y) \wedge (\forall y\epsilon Y.hired^{\circledast}(X,y)$
$\wedge liked(X,y))) \wedge (happy^{\circledast}(X))$

4.2 Descriptional Plural Anaphoras

I assume the interpretational mechanism of plural anaphora, called the *replacement by descriptions*, which is the last resort principle in order to avoid the interpretations' failure, as follows.

(31) Plural anaphora *they* (or *them*, *their*) in sentence ψ is replaced by definite plural description $\sigma X.\varphi$, where X is free in ψ and φ is constructed from inference in the context, if it is free in the logical form.

In this paper, principle (31) is applied to the case of Krifka's examples (13) and 'implicit antecedents of generic plural anaphoras'.

Reconsider Krifka's examples. (32a) are represented as (32b), and (32c) (32d), respectively.

(32) a. No student wrote an article. *They* (all) spent their days on the beach.

b. $No\ x(student(x), \exists y.write(x, y) \wedge article(y)) \wedge$ $(spend\ their\ days\ on\ the\ beach^{\circledast}(\sigma X.student^{\circledast}(X))$

c. Few students wrote an article. *They* rather spent their days on the beach.

d. $Few^{\circledast}X(student^{\circledast}(X), \exists y.write^{\circledast}(X, y) \wedge article(y)) \wedge$ $(spend\ their\ days\ on\ the\ beach^{\circledast}(\sigma Y.student^{\circledast}(Y) \wedge Y \sqcap X = \bot)$

where $\bot$ denotes the bottom of $\mathcal{D}^{\circledast}_{\mathcal{M}}$.

As for 'implicit antecedents of generic plural anaphoras', see the next section.

4.3 Generic Plurals

To handle the generics, I introduce the following devices:

(33) a. function $typical : D_{\mathcal{M}} \times D^{\circledast}_{\mathcal{M}} \rightarrow \{1, 0\}$, satisfying condition $typical(x, X) = 1$ iff x is a typical member [11] of X,

b. binary generalized quantifier $G \in Quant$, where $G_{\mathcal{M}} = \{(X, Y) | \{x \in D_{\mathcal{M}} | x \leq X, typical(x, X) = 1\} \subseteq \{y \in D_{\mathcal{M}} | y \leq Y\}\}$,

c. a sorted subdomain $K_{\mathcal{M}} \subseteq D_{\mathcal{M}}$, of which sort is "kind" (see Carlson [3]),

d. kind name, say dog, denoting $\mathsf{dog}_{\mathcal{M}} \in K_{\mathcal{M}}$, and

e. an instance-of relation ':', denoting $:_{\mathcal{M}} \subseteq D_{\mathcal{M}} \times K_{\mathcal{M}}$.

Generic bare plurals are analyzed as in (34) using (33).

(34) a. Dogs bark. But, they are good pets.

b. $G^{\circledast}X(X :^{\circledast} \mathsf{dog}, bark^{\circledast}(X)) \wedge G^{\circledast}Y(Y \epsilon^{\circledast} X, good\ pet^{\circledast}(Y))$

Generic definite plural descriptions are analyzed as in (35), where (35b) and (35c) are equivalent.

(35) a. The dogs in this city do not bark.

b. $(\exists! X.dog^{\circledast}(X) \wedge in\ this\ city^{\circledast}(X)) \wedge G\ x(x \epsilon X, \neg bark(x))$

c. $G\ x(x \epsilon \sigma X.dog^{\circledast}(X) \wedge in\ this\ city^{\circledast}(X), \neg bark(x))$

Reconsider the examples of implicit antecedents of generic plural anaphoras.

[11] In this paper, I will not define what is a 'typical member'. But it can be defined by a certain measure functions and the standard of 'typicality'.

(36) a. Few women from this village came to the feminist rally, No wander. *They* don't like political rally very much.

b. If at least one chicken which Ottilie owns had laid an egg, she had a nice breakfast. *They* are very good to eat.

c. John killed a spider because *they* are ugly.

Firstly, in 'the most generic' interpretations of (36), (36) are represented using (33) and principle (31) as follows.

(37) a. $Few^{\circledast}\ X(woman^{\circledast}(X) \wedge from^{\circledast}(X,y), come\ to^{\circledast}(X,z)) \wedge$
$G\ u(u\epsilon\sigma Z.woman^{\circledast}(Z), \neg like\_poilical\_rally(u))$

b. $\exists x \exists y.chicken(x) \wedge y = Ottilie \wedge own(x,y) \rightarrow \exists z.egg(z) \wedge lay(x,z)) \rightarrow$
$\exists u.u = y \wedge have\ a\ nice\ breakfast(u) \wedge$
$G\ v(v\epsilon\sigma X.egg^{\circledast}(X), be\ very\ good\ to\ eat(v))$

c. $(\exists x.kill(John,x) \wedge spider(x)) \wedge G\ u(u\epsilon\sigma X.spider^{\circledast}(X), ugly(u))$

where $\sigma Z.woman^{\circledast}(Z)$, $\sigma X.egg^{\circledast}(X)$, and $\sigma X.egg^{\circledast}(X)$ are introduced definite descriptions by (31).

In the 'restricted generic' interpretations of (36a-b), (36a-b) are represented using (33) and principle (31) as follows.

(38) a. $Few^{\circledast}\ X(woman^{\circledast}(X) \wedge from^{\circledast}(X,y), come\ to^{\circledast}(X,z)) \wedge$
$G\ u(u\epsilon\sigma Z.woman^{\circledast}(Z) \wedge from(Z,y), \neg like\_poilical\_rally(u))$

b. $\exists x \exists y.chicken(x) \wedge y = Ottilie \wedge own(y,x) \rightarrow \exists z.egg(z) \wedge lay(x,z)) \rightarrow$
$\exists u.u = y \wedge have\ a\ nice\ breakfast(u) \wedge G\ v(v\epsilon\sigma X.egg^{\circledast}(X) \wedge$
$lay^{\circledast}(\sigma U.chicken^{\circledast} \wedge own^{\circledast}(y,U), X), be\ very\ good\ to\ eat(v))$

where $\sigma Z.woman^{\circledast}(Z) \wedge from(Z,y)$,
$\sigma X.egg^{\circledast}(X) \wedge lay^{\circledast}(\sigma U.chicken^{\circledast} \wedge own^{\circledast}(y,U), X)$ are introduced by (31).

Dependent generic bare plurals are handled by devices in (33) plus dynamic distributor and dynamic selector, as in (39b), while (independent)generic bare plurals are handled only by devices in (33), as in (39d).

(39) a. Indians make baskets.

b. $G^{\circledast}X(X :^{\circledast} \mathsf{Indian}, G^{\circledast}Y(Y :^{\circledast} \mathsf{basket}, \forall y\epsilon Y.\exists x\epsilon X.make(x,y)))$

c. Dogs chase cats.

d. $G^{\circledast}X(X :^{\circledast} \mathsf{dog}, G^{\circledast}Y(Y :^{\circledast} \mathsf{cat}, chase^{\circledast}(X,Y)))$

4.4 Plurally Quantified Antecedents

Reconsider (2).

(40) Three composers visited there. They wrote four operas or musicals.

The distributive reading of (40), i.e., 'three composers visited there and each wrote four operas or musicals', is represented in $DPL_Q^{\circledast}$ as follows:

$$(\exists X.|X| = 3 \wedge \varphi_1(X)) \wedge (\exists Y.|Y| = 4 \wedge \forall x\epsilon X.\varphi_2(x,Y))$$

The collective reading of (40), i.e., 'three composers visited there and jointly wrote four operas or musicals', is represented in $DPL_Q^{\circledast}$ as follows:

$$(\exists X.|X| = 3 \wedge \varphi_1(X)) \wedge (\exists Y.|Y| = 4 \wedge \varphi_2(X, Y))$$

The other readings of (40) include many cases. However, in $DPL_Q^{\circledast}$ they are handled as follows:

$$(\exists X.|X| = 3 \wedge \varphi_1(X)) \wedge (\exists Y.|Y| = 4 \wedge \forall x \epsilon d_i(X).\varphi_2(x, Y))$$

where d_i is a division function, is a map from sets to the list of elements of a cover of the sets. This function is selected depending on its context.[12]

Firstly, let us see about a partition case such as Mozart, Gilbert and Sullivan. In this case, we select function

$$d_1 : Mozart \oplus Gilbert \oplus Sullivan \mapsto \langle Mozart, Gilbert \oplus Sullivan \rangle.$$

Secondly, let us see about a case of a minimal cover such as Rogers, Hammerstein, and Hart. In this case, we select function

$$d_2 : Rogers \oplus Hammerstein \oplus Hart \mapsto \langle Rogers \oplus Hammerstein, Rodgers \oplus Hart \rangle.$$

Thirdly, let us see about a case of a cumulative reading.

(41) Three composers visited there. They wrote four operas or musicals. Two of them were jointly written by two of the composers and one of them by all of them.

In this case, we select division function
$d_3 : Rogers \oplus Hammerstein \oplus Hart \mapsto$
$\langle Rogers \oplus Hammerstein, Rodgers \oplus Hart, Rogers, Rogers \oplus Hammerstein \oplus Hart \rangle$,
and the logical form of the first two sentences is represented as follows:

$$(\exists X.|X| = 3 \wedge \varphi_1(X)) \wedge (\exists Y.|Y| = 4 \wedge \forall y \epsilon Y.\exists x \epsilon d_3(X).\varphi_2(x, y)),$$

that satisfies the subsequent sentences in (41).

(42) is a more complicated case.

(42) Three composers visited there. They wrote four operas or musicals. Only one of them was jointly written by the three composers and the rest were written by only one of them.

[12] We can define division functions for the distributive reading $d_d : X \mapsto \langle x | x \in X \rangle$ and the collective reading $d_c : X \mapsto \langle X \rangle$. This implies that the general logical form of (40) is as follows:

$$(\exists X.|X| = 3 \wedge \varphi_1(X)) \wedge \exists Y.|Y| = 4 \wedge \exists d.\forall x \epsilon d(X).\varphi_2(x, Y))$$

To handle (42), we need two division functions as follows:

$d_4 : Rogers \oplus Hammerstein \oplus Hart \mapsto \langle Rogers, Rogers \oplus Hammerstein \oplus Hart \rangle$,

$d_5 : opus1 \oplus opus2 \oplus opus3 \oplus opus4 \mapsto \langle opus1 \oplus opus2 \oplus opus3, opus4 \rangle$,

and the logical form of the first two sentences is as follows:

$$(\exists X.|X| = 3 \wedge \varphi_1(X)) \wedge (\exists Y.|Y| = 4 \wedge \forall y \epsilon d_5(Y).\exists x \epsilon d_4(X).\varphi_2(x, y))$$

4.5 Glued Plurals

Reconsider the example in section 2.5.

(43) a. Every student borrowed a book$_i$. *They$_i$* were returned.

b. $(\exists XY.\forall x(x \epsilon X \wedge student(x), \exists y.y \epsilon Y \wedge book(y) \wedge borrowed(x, y)))$
$\wedge were\ returned(Y))$

In my framework, as in (43b), the glued plurals are handled by the summation that we have seen in section 3.3.1.

5 Conclusion

As we have seen, in this paper, I have proposed an extension of DPL, $DPL_Q^{\circledast}$, in order to handle plurals. $DPL_Q^{\circledast}$ introduces many new devices, in particular, two new operators: dynamic selector and dynamic distributor and division functions. Since the dynamic distributors are defined using the Kleene star, $DPL^{\circledast}$, i.e., $DPL_Q^{\circledast}$ minus generalized quantifiers is a fragment of non-$*$-free first-order dynamic logic.

I can summarize my treatment of plurals in this paper by exploiting the Montagovian λ-notation, as follows:

Bare plural (*dogs*)	
generic	$\lambda P.G^{\circledast}X(X :^{\circledast} \mathsf{dog}, P)$
dependent generic	$\lambda P.G^{\circledast}X(X :^{\circledast} \mathsf{dog}, \forall x \epsilon X.\exists y \epsilon Y.Pyx)$
indefinite	$\lambda P.\exists X.dog^{\circledast}(X) \wedge P^{\circledast}(X))$
dependent indefinite	$\lambda P.\exists X.dog(^{\circledast}(X) \wedge \forall x \epsilon X.Px)$
Plural anaphora	
variable	X
descriptional	$\sigma X.\varphi^{\circledast}(X)$
(Numeral) quantification (*n dogs*)	
collective	$\lambda P.\exists X.dog^{\circledast}(X) \wedge \lvert X \rvert = n \wedge P^{\circledast}(X)$
distributive	$\lambda P.\exists X.dog^{\circledast}(X) \wedge \lvert X \rvert = n \wedge \forall x \epsilon X.Px$
cover	$\lambda P.\exists X.dog^{\circledast}(X) \wedge \lvert X \rvert = n \wedge \forall x \epsilon d_i(X).Px$
cumulative	$\lambda P.\exists X.dog^{\circledast}(X) \wedge \lvert X \rvert = n \wedge \forall x \epsilon d_j(X).\exists y \epsilon d_i(Y).Pxy$
generalized quantification (*Q dogs*)	$\lambda P.Q^{\circledast}X(dog^{\circledast}(X), P^{\circledast}(X))$

Investigations and applications to other types of problems concerning plurals, and their relation to abstract entities such as events, situations, and cases, are matters to be attended.

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