Far-zone contributions of airborne gravity anomalies' upward/downward continuation

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\textbf{A B S T R A C T}

Airborne gravimetry has become a vital technique in local gravity field approximation, and upward/downward continuation of gravity data is a key process of airborne gravimetry. In these procedures, the integral domain is divided into two parts, namely the near-zone and the far-zone. The far-zone contributions are approximated by the truncation coefficients and a global geo-potential model, and their values are controlled by several issues. This paper investigates the effects of flight height, the size of near-zone cap, and Remove-Compute-Restore (RCR) technique upon far-zone contributions. Results show that at mountainous area the far-zone contributions can be ignored when EIGEN-6C of 360 degree is removed from the gravity data, together with a near-zone cap of $\frac{1}{C_14}$ and a flight height less than 10 km, while at flat area EIGEN-6C of 180 degree is feasible.

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1. Introduction

With the development of airborne gravimetry, upward and downward continuations of gravity data are applied more frequently [1–4]. Poisson's integral is a common solution for upward and downward continuations of gravity data. The integration in Poisson's integral is to be taken over the full solid angle, but for regional geoid determination, in order to use the available data at the limited survey area, a common process is the discretization of Poisson's integral equation, in which the integrating domain is divided into the near-zone and far-zone domain. The near-zone contributions are computed using the detailed gravity data, while the far-zone contributions are estimated from an available global geo-potential model, such as the EGM2008 [5]. A distortion will occur...
in the upward/downward process if the far-zone contributions are treated inappropriately.

A number of issues must be considered for the far-zone contributions, including matters such as the computation of truncation coefficients of Poisson’s integral, the resolution of the airborne gravity data, the type of the geo-potential model, the degree to extend the harmonic expansion, the flight height, and the radius of the near-zone cap. Tenzer et al. [6] computed truncation coefficients of Poisson’s integral using numerical integration method; Tenzer et al. [7] applied truncation coefficients of Stokes’ integral to compute far-zone contributions of Poisson’s integral. For a certain block of airborne gravity data, the spatial resolution (half wavelength) depends on the low-pass filter which is applied to the measured data. Martinec [8] and Huang [9] expanded airborne gravity data, the spatial resolution (half wavelength) depends on the low-pass filter which is applied to the measured data. Martinec [8] and Huang [9] expanded harmonic expansion to 360 degree, but their solutions are for ground observations; Jiang [10] took them as references and expanded harmonic expansion to 360 degree for airborne observations. In this paper there will be a full harmonic expansion. The goals of this paper are to analyze the observations. In this paper there will be a full harmonic expansion. The goals of this paper are to analyze the observations. In this paper there will be a full harmonic expansion.

The far-zone contributions should be considered at all.

2. Far-zone contribution of Poisson’s integral

The domain of Poisson’s integral may be divided to two parts: the near-zone and the far-zone. Thus, the Poisson’s integral can be expressed as

$$\Delta g(r, \Omega) = \frac{R}{4\pi r} \int_{\Omega_1} \Delta g(R, \Omega') K(r, \psi_0, R) d\Omega' + F_{A,0}(r, \Omega)$$  \hspace{1cm} (1)

where the first part on the right of equation (1) are the near-zone contributions; $F_{A,0}(r, \Omega)$ are the far-zone contributions, which can be expressed as

$$F_{A,0}(r, \Omega) = \frac{R}{4\pi r} \int_{\Omega_1} \Delta g(r, \Omega') K(r, \psi_0, r') d\Omega'$$  \hspace{1cm} (2)

With the Poisson kernel

$$K(r, \psi, R) = \frac{r^2 - R^2}{(r^2 + R^2) \cos \psi} = \sum_{l=0}^{\infty} (2l + 1) \left( \frac{R}{r} \right)^{l+1} P_l(\cos \psi)$$  \hspace{1cm} (3)

where $\Delta g$ are gravity anomalies; $(r, \Omega)$ and $(R, \Omega')$ are the computing point and running point respectively; $\Omega$ represents co-latitude and longitude ($\phi, \lambda$); $R$ is the Earth’s mean radius, and $R = 6371$ km is adopted in this paper; $r$ is the geocentric radius of the running point; $\Omega_1$ and $\Omega_1 - \Omega_2$ indicate the near-zone and far-zone; $P_l(\cos \psi)$ is the nth degree Legendre polynomial.

Theoretically, global gravity data is needed for Poisson’s integral. In fact, airborne gravity data can only be obtained in a limited domain. So Remove-Compute-Restore (RCR) technique is always applied, modeling the influence of the gravity data beyond the survey area upon the computing point using a low degree geo-potential model. This paper uses the geo-potential model EIGEN-6C [11] in RCR process; furthermore, the treatment of topographical and atmospheric effects is out of the scope of this study.

As shown in equation (3), $K(r, \psi_0, R)$ is described as an infinite summation, but for an airborne solution, and meanwhile the RCR technique is applied, the Poisson kernel $K(r, \psi_0, R)$ should be modified as

$$K(r, \psi, R) = \sum_{l=0}^{N} (2l + 1) \left( \frac{R}{r} \right)^{l+1} P_l(\cos \psi)$$  \hspace{1cm} (4)

where $N$ is the max degree of airborne gravity; $N_0 - 1$ is the max degree of the low-degree geo-potential model in the RCR process.

The far-zone contributions $F_{A,0}(r, \Omega)$ can be approximated by using the Molodensky-type harmonic expansion technique

$$F_{A,0}(r, \Omega) = \frac{GM}{2\pi r} \sum_{n=1}^{N} \sum_{m=0}^{n} \left[ dC_{nm}(\gamma) \cos(\lambda) + dS_{nm}(\gamma) \sin(\lambda) \right] P_{nm}(\phi)$$  \hspace{1cm} (5)

where $\Delta g_{k}(\Omega)$ is the nth surface harmonics of $\Delta g(\Omega)$; $dC_{nm}$ and $dS_{nm}$ are the fully normalized disturbing potential coefficients; $\gamma$ is the normal gravity; $P_{nm}(\phi)$ are the fully-normalized associated Legendre functions; GM is the geocentric gravitational constant; $Q_{n}(H, \psi_0)$ is the nth truncation coefficient which is a function of the kernel $K(r, \psi_0, r')$ and the angular radius of the far-zone cap $\psi_0$.

$$Q_n(H, \psi_0) = \int_{\psi_0}^{\psi_0} K(r, \psi, R) P_n(\cos \psi) \sin \psi d\psi$$  \hspace{1cm} (6)

As shown in equation (6), $Q_n(H, \psi_0)$ could be computed using numerical integration method. It is worth mentioning that the above expressions assume an Earth-fixed reference frame.

3. Results and discussions

The far-zone contributions to quantities of the Earth’s gravity field are numerically investigated at the area of study bounded by the parallels of 30° N and 33° N latitude and the meridians of 107° E and 110° E longitude, which locates at southwest China. Fig. 1 shows the digital elevation model (DEM) of this area, which is mountainous and suitable for airborne gravimetry. We provide that the resolution of the airborne gravity data is 5', the corresponding spherical harmonic degree is 2160, and EGM2008 of 2160 degree is used to compute the far-zone contributions.

Fig. 2 shows the far-zone contributions of the test area at flight height $H = 3$ km, 5 km, 8 km, 10 km respectively. In the computation, $\psi_0 = 1°$ and $N_0 = 2$ are assumed. Table 1 lists the related statistics of the far-zone contributions. Fig. 2 and Table 1 illustrate that the far-zone contributions increase with the flight height, and when the flight height is 10 km, the far-zone contributions are significant, the max absolute value is as large as 2.5 mGal, which is far beyond the precision of airborne gravimetry, thus the far-zone contributions should be under consideration in the
upward/downward continuation of airborne gravity data in this case.

In order to show the frequency feature of the far-zone contributions, we assume a flight line along the parallel 32°N latitude, the flight height is 10 km, and the longitude varies from 107°E to 110°E. The profile of the far-zone contributions and their power spectral density (PSD) are drawn in Fig. 3. The PSD implies that the power of the far-zone contributions converges at low frequencies, and the high frequencies part is definitely small. Thus theoretically, removing a global geopotential model in the RCR process would be helpful to get rid of the far-zone contributions.

Fig. 4 shows the far-zone contributions of the test area at flight height $H = 3$ km, 5 km, 8 km, 10 km respectively, Table 2 lists the related statistics, considering EIGEN-6C of 120 degree was removed from the gravity data. The far-zone contributions decrease rapidly, and are less than 0.35 mGal when the RCR is applied. These results confirm the necessity of RCR technique in the upward/downward continuation of airborne gravity data.

![Fig. 1 – DEM of the test area.](image)

![Fig. 2 – Far-zone contributions at different flight heights ($N_s = 2, \psi_0 = 1$).](image)

**Table 1 – Statistics of the far-zone contribution at different flight heights ($N_s = 2, \psi_0 = 1$).**

<table>
<thead>
<tr>
<th>Flight height (km)</th>
<th>Min (mGal)</th>
<th>Max (mGal)</th>
<th>Mean (mGal)</th>
<th>Std. (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>−0.8024</td>
<td>−0.3159</td>
<td>−0.5397</td>
<td>0.1145</td>
</tr>
<tr>
<td>5</td>
<td>−1.2604</td>
<td>−0.4697</td>
<td>−0.8326</td>
<td>0.1858</td>
</tr>
<tr>
<td>8</td>
<td>−2.0097</td>
<td>−0.7503</td>
<td>−1.3285</td>
<td>0.2960</td>
</tr>
<tr>
<td>10</td>
<td>−2.5084</td>
<td>−0.9379</td>
<td>−1.6592</td>
<td>0.3692</td>
</tr>
</tbody>
</table>
Fig. 4 — Far-zone contribution at different flight heights ($N_s = 121$, $\nu_0 = 1$).

Table 2 — Statistics of the far-zone contribution at different flight heights ($N_s = 121$, $\nu_0 = 1$).

<table>
<thead>
<tr>
<th>Flight height (km)</th>
<th>Min (mGal)</th>
<th>max (mGal)</th>
<th>Mean (mGal)</th>
<th>Std. (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.0677</td>
<td>0.1037</td>
<td>0.0090</td>
<td>0.0351</td>
</tr>
<tr>
<td>5</td>
<td>-0.1126</td>
<td>0.1725</td>
<td>0.0149</td>
<td>0.0585</td>
</tr>
<tr>
<td>8</td>
<td>-0.1793</td>
<td>0.2749</td>
<td>0.0237</td>
<td>0.0931</td>
</tr>
<tr>
<td>10</td>
<td>-0.2235</td>
<td>0.3428</td>
<td>0.0296</td>
<td>0.1160</td>
</tr>
</tbody>
</table>
Fig. 5 plots the far-zone contributions of the test area for different near-zone caps, Table 3 lists the related statistics. $H = 10$ km and $N_S = 2$ are assumed in the computation. The absolute values of the far-zone contributions decrease with the near-zone caps, especially when the cap changes from $1^\circ$ to $2^\circ$. But for a bigger cap, more detailed gravity data is needed in the computation of near-zone contributions, this condition may not be satisfied in local determination. So it’s unconstructive to reduce the far-zone contributions by extending the near-zone cap.

Fig. 6 shows the far-zone contributions of the test area for different degrees of EIGEN-6C in the RCR process, Table 4 lists the related statistics, $H = 10$ km and $v_o = 1^\circ$ are assumed. 1-cm geoid model is the current aim of Geodesy, the corresponding accuracy of gravity anomalies at low frequencies is 0.1 mGal [12,13]. Hence, the optimal $N_S = 361$ is chosen when the far-zone contributions are below 0.1 mGal. In other words, the far-zone contributions are negligible when 360 degree EIGEN-6C is used in the RCR process and the near-zone cap is $1^\circ$.

There may be another scenario at the flat area. We take the Jianghan plain (29°N—31°N, 111.5°E—113.5°E) for example, Fig. 7 shows the DEM, and Table 5 lists the standard deviations and ranges of the far-zone contributions of this area for different flight heights and different degrees of EIGEN-6C in the RCR process, the near-zone cap $v_o = 1^\circ$ is assumed in the computation. The variation of the far-zone contributions with flight height is the same as in the mountainous area, while the far-zone contributions in Jianghan plain are all less than 0.1 mGal, even the flight height is as high as 10 km. In other words, at flight height below 10 km, EIGEN-6C of 180 degree enables us to ignore the far-zone contributions.

<table>
<thead>
<tr>
<th>$v_o$</th>
<th>Min (mGal)</th>
<th>Max (mGal)</th>
<th>Mean (mGal)</th>
<th>Std. (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^\circ$</td>
<td>-2.5084</td>
<td>-0.9379</td>
<td>-1.6592</td>
<td>0.3692</td>
</tr>
<tr>
<td>$2^\circ$</td>
<td>-0.7756</td>
<td>-0.5689</td>
<td>-0.7037</td>
<td>0.0440</td>
</tr>
<tr>
<td>$3^\circ$</td>
<td>-0.3818</td>
<td>-0.1645</td>
<td>-0.3252</td>
<td>0.0483</td>
</tr>
<tr>
<td>$4^\circ$</td>
<td>-0.1972</td>
<td>-0.1091</td>
<td>-0.1539</td>
<td>0.0223</td>
</tr>
</tbody>
</table>

Fig. 5 — Far-zone contributions for different near-zone radiuses ($NR = 2, H = 10$ km).
4. Conclusion

The effects of flight height, the size of near-zone cap and RCR technique upon far-zone contributions are discussed at the mountainous area and flat area respectively, conclusions are as follow: (1) the far-zone contributions increase with the flight height, while decrease with the near-zone cap, but it’s not wise to lower far-zone contributions by extending the

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Table 4 – Statistics of the far-zone contributions for different $N_S (H = 10\text{ km}, \psi_0 = 1')$.

<table>
<thead>
<tr>
<th>$N_S$</th>
<th>Min (mGal)</th>
<th>Max (mGal)</th>
<th>Mean (mGal)</th>
<th>Std. (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>181</td>
<td>-0.2118</td>
<td>0.2186</td>
<td>0.0105</td>
<td>0.1022</td>
</tr>
<tr>
<td>241</td>
<td>-0.0959</td>
<td>0.1320</td>
<td>0.0028</td>
<td>0.0477</td>
</tr>
<tr>
<td>301</td>
<td>-0.1009</td>
<td>0.1144</td>
<td>0.0019</td>
<td>0.0401</td>
</tr>
<tr>
<td>361</td>
<td>-0.0549</td>
<td>0.0693</td>
<td>0.0012</td>
<td>0.0196</td>
</tr>
</tbody>
</table>

---

Table 5 – Std values and ranges of far-zone contributions in Jianghan plain, units are in $10^{-2}$ mGal ($\psi_0 = 1'$).

<table>
<thead>
<tr>
<th>$N_S$</th>
<th>$H = 5\text{ km}$</th>
<th>$H = 8\text{ km}$</th>
<th>$H = 10\text{ km}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Std.</td>
<td>Range</td>
<td>Std.</td>
</tr>
<tr>
<td>181</td>
<td>1.6</td>
<td>2.77–4.62</td>
<td>2.55</td>
</tr>
<tr>
<td>241</td>
<td>0.71</td>
<td>3.48–2.46</td>
<td>1.13</td>
</tr>
<tr>
<td>301</td>
<td>0.77</td>
<td>2.32–3.06</td>
<td>1.23</td>
</tr>
<tr>
<td>361</td>
<td>0.52</td>
<td>1.45–2.10</td>
<td>0.83</td>
</tr>
</tbody>
</table>

---

Fig. 6 – Far-zone contributions for different $N_S (H = 10\text{ km}, \psi_0 = 1')$.

Fig. 7 – DEM of Jianghan plain.
near-zone cap; (2) the RCR technique can reduce the far-zone contributions effectively; (3) far-zone contributions can be ignored when the near-zone cap is $1^\circ$, and the flight height is less than 10 km, together with EIGEN-6C of 360 degree removed from the gravity data at the mountainous area and EIGEN-6C of 180 $^\circ$ removed from the gravity data at the flat area. The analysis for far-zone contributions of airborne gravity disturbances’ upward/downward continuations would be analogous.

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