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An optimization method for metamorphic mechanisms based on multidisciplinary design optimization

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Abstract The optimization of metamorphic mechanisms is different from that of the conventional mechanisms for its characteristics of multi-configuration. There exist complex coupled design variables and constraints in its multiple different configuration optimization models. To achieve the compatible optimized results of these coupled design variables, an optimization method for metamorphic mechanisms is developed in the paper based on the principle of multidisciplinary design optimization (MDO). Firstly, the optimization characteristics of the metamorphic mechanism are summarized distinctly by proposing the classification of design variables and constraints as well as coupling interactions among its different configuration optimization models. Further, collaborative optimization technique which is used in MDO is adopted for achieving the overall optimization performance. The whole optimization process is then proposed by constructing a two-level hierarchical scheme with global optimizer and configuration optimizer loops. The method is demonstrated by optimizing a planar five-bar metamorphic mechanism which has two configurations, and results show that it can achieve coordinated optimization results for the same parameters in different configuration optimization models.

1. Introduction

The concept of metamorphic mechanism, a new kind of mechanism originated from the configuration and mobility researches of decorative carton folds, was firstly proposed by Dai and Jones[1] in 1999. In contrast to the traditional mechanism, the mechanism has the characteristics of multi-configuration, variable constraint and multi-function. It forms a class of mechanisms that has the ability to change configuration sequentially from one to another as a resultant change of the

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number of effective links and topological structure to achieve different tasks.

The research on metamorphic mechanisms mainly concentrates on configuration analysis, mobility analysis and structural synthesis of the existing mechanisms which have metamorphic characteristics. In 2002, Liu and Dai[2] presented a general representation of carton manipulation with a hereditary connectivity matrix and a configuration matrix. Then, Dai and Jones[3] developed an adjacency matrix form to describe the topological configurations of metamorphic mechanisms and proposed an EU-elementary matrix operation to produce the configuration transformation. Further, they proposed a general categorization of metamorphic mechanisms and introduced their mathematical models of the topological configuration changes.[4] Liu and Yang[5] summarized the basic metamorphic ways and the metamorphic characteristics. Yan et al.[6,7] studied the topological representations of variable joints and proposed a systematic methodology to synthesize all possible design configurations of mechanisms with variable topologies. Zhang et al.[8,9] developed a method for synthesis and configuration design of metamorphic mechanisms based on biological modeling and genetic evolution with biological building blocks. Li et al.[10] proposed a constraint graph representation for topological structure of compliant metamorphic mechanisms. Ding et al.[11,12] proposed three evaluating criteria for metamorphic mechanisms, which are linear ratio, area ratio and volume ratio for providing some fundamental ideas for the design of metamorphic mechanisms by investigating the topology and configuration of a assembly-circles artifact. As for the kinematics and dynamics of metamorphic mechanisms, Jin et al.[13] described different configurations of a metamorphic mechanism through the method of Huston 1ower body arrays and gave the kinematics analyses with generalized topological structures including the velocity, angular velocity acceleration and angular acceleration.

However, less attention has been paid to the optimization design of metamorphic mechanisms. It is quite different from the traditional regular mechanism for its characteristics of multi-configuration. An optimization model for a single sub-mechanism can be established and solved by using traditional optimization methods, which lead to locally optimal solution at best.[14] But in view of the general mechanism, there exist interrelations and differences among these optimization models to make the solution infeasible. For example, optimized results of the same design variables in adjacent configuration optimization models are identifiably inconsistent. So it is necessary to eliminate the inconsistency for achieving the uniform optimized results for all configurations as well as guaranteeing the continuity of configuration transformation.

In this paper, the characteristics of the optimization for metamorphic mechanisms are analyzed with emphasis on their coupling interactions among the different configuration optimization models. Further, MDO is adopted for the optimization of metamorphic mechanisms to achieve the overall performance. Based on this, an optimization process of establishing the corresponding optimization models is proposed.

2. Optimization for metamorphic mechanisms

A metamorphic mechanism is considered as a mechanism set whose total number of all configurations is n can be expressed as

$$T = \left\{T^{(1)}, T^{(2)}, \cdots, T^{(i)}, \cdots, T^{(n)}\right\} \ (i = 1, 2, ..., n) \tag{1}$$

where $T^{(i)}$ represents the sub-mechanism in configuration i. Supposing that the metamorphic mechanism works uninterruptedly during a working cycle, it can go through configuration transformation n times as the order of configuration $1 \rightarrow$ configuration $2 \rightarrow \ldots \rightarrow$ configuration $n \rightarrow$ configuration 1. All the sub-mechanisms in this set can be considered evolving from a special original metamorphic mechanism as well.[9,15]

It is shown that there exist three basic metamorphic ways.[5] They are incorporation or increase of links, changing adjacent relations of links and changing properties of kinematic pairs leading to the variation of topological structures. Setting, limiting and adjusting appropriate constraints on kinematic pairs or components can realize these three variations as well as changing the corresponding topological structures.[16] As stated previously, these variations are reflected on the similarities, differences and interrelations among the design variables, constraints and objectives in the corresponding optimization models.

The general optimization model of the metamorphic mechanism is composed of a series mechanism optimization models in all configurations. For the purpose of researching the optimization method for metamorphic mechanisms, it is necessary to summarize its optimization characteristics with emphasis on the relations between the design variables, constraints and objectives in different optimization models firstly. Therefore, a planar five-bar metamorphic mechanism which has two configurations is investigated here to help summarize the rules.

2.1. An optimization example

The schematic graphs of the planar five-bar metamorphic mechanism which has two configurations are shown in Fig. 1, respectively.

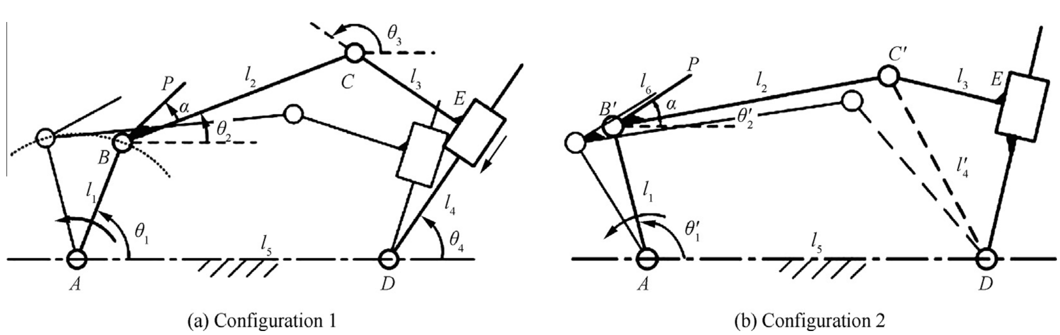


Fig. 1 Schematic graph of a planar five-bar metamorphic mechanism.

The mechanism in Fig. 1(a) which can be considered as the original metamorphic mechanism is a 2-DOF (Degree of Freedom) five-bar mechanism composed by components *AB*, *BC*, *CE*, *DE* and *AD*. The mechanism in Fig. 1(b) originates from it by combining the components *CE* and *DE*. As the original five-bar metamorphic mechanism has its own working task, in the paper it is considered as the sub-mechanism in configuration 1 as well as the four-bar mechanism in Fig. 1(b).

In configuration 1, the components *AB* and *CE* are two driving links whose movements are rotation about the point *A* and translation along the component *DE*, respectively. The component *CE* is always perpendicular to the component *DE*. The specified angular output of the component *DE* meets the precise moving requirements of the hybrid-driven drawing press. After the component *AB* rotates a single complete cycle, the components *CE* and *DE* are fixed together by locking the linear motor fixed on the component *DE* to realize configuration transformation. Thus, the mechanism is transformed to a 1-DOF four-bar mechanism in this configuration, which is easy to control and capable of specific task. Meanwhile the output component is changed to the endpoint *P* fixed on the component *BC* for meeting a given trajectory.

2.1.1. Optimization model of mechanism in configuration 1

In Fig. 1(a), l_1, l_2, l_3, l_4 and l_5 represent the length of each component respectively. θ_1, θ_2, θ_3 and θ_4 represent the angles between the corresponding components and the horizontal axis respectively. $\theta_{1,0}$ represents the initial angle between the component *AB* and the horizontal axis.

Design variables are described as

$$\boldsymbol{x_1} = [x_{1,1}, x_{1,2}, x_{1,3}]^{\mathrm{T}} \tag{2}$$

where $x_{1,1} = \dfrac{l_1}{l_5}$, $x_{1,2} = \dfrac{l_2}{l_5}$, $x_{1,3} = \dfrac{l_3}{l_5}$.

The objective function is expressed as

$$F^{(1)} = l_{4\max} - l_{4\min} \tag{3}$$

where $l_{4\max}$ and $l_{4\min}$ represent the maximum and minimum effective length of the link *DE*, respectively. It will get the minimum displacement of the component *CE* along the link *DE* under given constraints.

Constraints are summarized as follows.

(1) Constraints of crank exist.

$$\begin{cases} l_1 + l_5 - l_2 - \sqrt{l_3^2 + l_{4\min}^2} < 0 \\ l_1 + \sqrt{l_3^2 + l_{4\min}^2} - l_5 - l_2 < 0 \\ l_1 + l_2 - \sqrt{l_3^2 + l_{4\min}^2} - l_5 < 0 \\ l_1 - \min\left(l_2, \sqrt{l_3^2 + l_{4\min}^2}, l_5\right) < 0 \end{cases} \tag{4}$$

(2) Equivalent transmission angle constraints.

$$\begin{cases} \dfrac{{l_2}^2 + l_3^2 + l_4^2 - (l_5 - l_1)^2}{2l_2\sqrt{l_3^2 + l_4^2}} - \cos[\gamma] \leqslant 0 \\ \dfrac{{l_2}^2 + l_3^2 + l_4^2 - (l_5 + l_1)^2}{2l_2\sqrt{l_3^2 + l_4^2}} - \cos[\gamma] \leqslant 0 \end{cases} \tag{5}$$

where $[\gamma] = 40°$ represents the allowable transmission angle.

(3) Trajectory constraints

$$\theta_4 = \begin{cases} H\left[\dfrac{\theta_1}{\pi} - \dfrac{1}{2\pi}\sin(2\theta_1)\right] + 0.5\sin\left(\dfrac{\theta_1}{2}\right) (0 \leqslant \theta_1 \leqslant \pi) \\ H - H\left[\dfrac{\theta_1 - \pi}{\pi} - \dfrac{1}{2\pi}\sin 2(\theta_1 - \pi)\right] \\ +0.5\sin\left(\dfrac{\theta_1}{2}\right) \quad (\pi < \theta_1 < 2\pi) \end{cases} \tag{6}$$

where the value of H is $\dfrac{16}{45}\pi$.

(4) Geometric relation constraints

$$\theta_4 = 2\tan^{-1}\left(\frac{M + \sqrt{L^2 + M^2 - N^2}}{L - N}\right) \tag{7}$$

where $L = l_3 l_5 + l_1 l_4 \sin\theta_1 - l_2 l_3 \cos\theta_1$, $M = l_4 l_5 - l_1 l_3 \sin\theta_1 - l_1 l_3 \sin\theta_1$, $N = \dfrac{{l_1}^2 + l_3^2 + l_4^2 - l_2^2 - 2l_1 l_5 \cos\theta_1}{2}$.

(5) Boundary constraints

$$x_{1,1}, x_{1,2}, x_{1,3} > 0 \tag{8}$$

2.1.2. Optimization model of mechanism in configuration 2

The mechanism in configuration 2 is located at position $AB'C'D$ in Fig. 1(b). $\theta'_{1,0}$ represents the initial angle between the corresponding component *AB* and the horizontal axis. θ'_1 and θ'_2 represent the angles between the components *AB*, *BC* and the horizontal axis. α is the angle between the components *BP* and *BC*. l'_4 and l_6 represent the equivalent length of the components *CD* and *BP*, respectively.

Design variables are described as

$$\boldsymbol{x_2} = [x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4}, x_{2,5}]^{\mathrm{T}} \tag{9}$$

where $x_{2,1} = \dfrac{l_1}{l_5}$, $x_{2,2} = \dfrac{l_2}{l_5}$, $x_{2,3} = \dfrac{l'_4}{l_5}$, $x_{2,4} = \dfrac{l_6}{l_5}$, $x_{2,5} = \alpha$.

The objective function is to get the minimum deviation between the given points $Q(x_q^m, y_q^m)$ listed in Table 1 and the trajectory points $P\ (x_p^m, y_p^m)$ are expressed as

$$\begin{cases} F^{(2)} = \sum\limits_{m=1}^{12}\sqrt{(x_p^m - x_q^m)^2 + (y_p^m - y_q^m)^2} \\ x_p^m = l_1\cos\theta'_1 + l_6\cos(\theta'_2 + \alpha) \\ y_p^m = l_1\sin\theta'_1 + l_6\sin(\theta'_2 + \alpha) \quad (m = 1, 2, ..., 12) \end{cases} \tag{10}$$

Table 1 Coordinate values of the given points.

Input variable θ'_1	Output variable (x_q^m, y_q^m)
π/4	(90.637, 51.7388)
5π/12	(76.8194, 59.6564)
7π/12	(60.1725, 61.1179)
9π/12	(45.4632, 59.246)
11π/12	(36.8254, 42.3444)
13π/12	(35.4143, 28.9602)
15π/12	(41.3717, 18.4987)
17π/12	(50.156, 12.3348)
19π/12	(70.3132, 11.8870)
21π/12	(85.8555, 17.2435)
23π/12	(95.8937, 27.5398)
25π/12	(97.6261, 40.1082)

Constraints are summarized as follows.

(1) Constraints of crank exist.

$$\begin{cases} l_5 + l'_4 - l_1 - l_2 > 0 \\ l_2 + l_5 - l_1 - l'_4 > 0 \\ l_2 + l'_4 - l_1 - l_5 > 0 \\ \min(l_2, l'_4, l_5) - l_1 > 0 \end{cases} \tag{11}$$

(2) Transmission constraints

$$\begin{cases} \dfrac{l_2^{\,2} + (l'_4)^2 - (l_1 - l_5)^2}{2l_2 l'_4} - \cos[\gamma] \leqslant 0 \\ \dfrac{l_2^{\,2} + (l'_4)^2 - (l_1 + l_5)^2}{2l_2 l'_4} - \cos[\gamma] \leqslant 0 \end{cases} \tag{12}$$

where $[\gamma] = 40°$ represents the allowable transmission angle.

(3) Geometric relation constraints

$$\theta'_2 = \arctan\left(\frac{Q + \sqrt{Q^2 + R^2 - S^2}}{R - S}\right) \tag{13}$$

where $Q = 2l_1l_2\sin\theta'_1$, $R = 2l_2(l_1\cos\theta'_1 - l_5)$, $S = l_2^2 + l_1^2 + l_5^2 - (l'_4)^2 + 2l_1l_5\cos\theta'_1$.

(4) Boundary constraints

$$x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4} > 0 \tag{14}$$

2.2. Example analysis

To get a clear look at the coupling interactions distinctly, the optimization models of mechanisms in different configurations should be researched firstly. Owing to the continuity of configuration transformation, the related mechanism parameters in adjacent configurations keep constant at the moment of the configuration transformation. Therefore, there exist the following constraint functions from the geometric constraints after the component AB rotates a single complete cycle, expressed as

$$\begin{cases} l_1\cos\theta_{1,0} + l_2\cos\theta_{2,0} = -l_3\sin\theta_{4,0} + l_4^*\cos\theta_{4,0} + l_5 \\ l_1\sin\theta_{1,0} + l_2\sin\theta_{2,0} = l_3\cos\theta_{4,0} + l_4^*\sin\theta_{4,0} \\ l'_4 = \sqrt{l_3^2 + (l_4^*)^2} \end{cases} \tag{15}$$

where $\theta_{1,0}$, $\theta_{2,0}$ and $\theta_{4,0}$ represent the initial angles between the components AB, BC, DE and the horizontal axis respectively in both of the two configurations, and l_4^* the length of DE at the moment of configuration transformation.

(1) Design variables

There exist several identical design variables in the corresponding optimization models which are expressed as

$$\{x_{1,1} = x_{2,1}, x_{1,2} = x_{2,2}\} \in x^{(g)} \tag{16}$$

where $x^{(g)}$, called global design variables, shows that the design variables exist in both of the two configurations.

In addition, design variables expressed as $x^{(i)}$ (i = 1,2) are defined as local variables or configuration variables only exist in their respective optimization models, such as

$$\{x_{2,4}, x_{2,5}\} \in x^{(2)} \tag{17}$$

Although $x_{1,3}$ and $x_{2,3}$ only exist in their respective optimization models, the relations between the two design variables can be shown in Eq. (15). They can be defined as coupled variables and expressed as $y^{(1,2)}$ and $y^{(2,1)}$, respectively.

(2) Constraints

The constraints in the two optimization models are classified as global constraints, configuration constraints and coupled constraints in accordance with the classification of design variables. Constraints existing in the two configurations are simultaneously defined as global constraints such as Eq. (5). In contrast, the configuration constraints refer to those constraints only exist in the course of respective configuration such as Eq. (4). And the coupled constraints show the relations between the coupled design variables such as Eq. (15).

2.3. Optimization characteristics of metamorphic mechanisms

From the given example above, the optimization characteristics of the metamorphic mechanism are summarized as Fig. 2 shows.

Fig. 2 shows the detailed cross-coupled relations between the optimization models of a metamorphic mechanism which has three configurations. $D^{(i)}$ denotes the mechanism optimization model in configuration i. $x^{(g)}$ shows all global design variables which exist in each optimization model. $x^{(i)}$ denotes the local variables or configuration variables which only exist in the optimization model of configuration i. $y^{(i,j)}$ represents the coupled variables which not only belongs to the variables in configuration i , but the input variables in configuration j. The constraints are composed of global constraints ($C^{(g)}$), configuration constraints ($C^{(i)}$) and coupled constraints ($C^{(i,j)}$). It should be noted that the coupled constraints ($C^{(i,j)}$) represent that the functions about the coupled variables $y^{(i,j)}$ from configuration i to configuration j need to be added in the corresponding constraint set. $F^{(i)}$ represents the objective function set of the mechanism in configuration i.

As shown in Fig. 2, there exist complex coupling interactions between the optimization models in adjacent configurations. And as the number of configuration increases, so does the complexity of the optimization. The solution to each optimization model can lead to the differences of the optimized result of the coupled variables and global variables undoubtedly. Thus, traditional optimization methods are not suitable for solving the complex coupled problems existing in the optimization of metamorphic mechanisms.

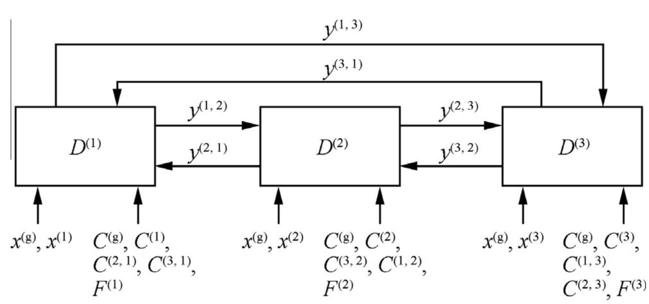


Fig. 2 Relations of optimization models of a metamorphic mechanism (three configurations).

3. Optimization method for metamorphic mechanisms

As summarized above, the optimization characteristics of the metamorphic mechanism are achieved in detail. A method for constructing a configuration-complete optimization was attempted to achieve the unified optimization results of the coupled variables.[17] But the optimization efficiency becomes lower with the increase of designing variables and constraints resulting from the number of configuration. In view of its optimization characteristics, it is obvious that the idea of multidiscipline design optimization is well-suited to solve the coupling interactions of the optimization for metamorphic mechanisms.

MDO is described as a methodology for the design of systems where the interaction between several disciplines must be considered, and where the designer is free to significantly affect the system performance in more than one discipline.[18] Much MDO research has been conducted over the last decade, particularly in the aerospace industry. In recent years, the yield of MDO application has been expanded to the field of the mechanical design. Chen and Yang[19] proposed a multidisciplinary design optimization procedure for integrating optimization of mechanisms and structures. Zhang et al.[20] applied the technique of MDO to the design of the mechanism and the control performance. Zhang[21] established a fuzzy optimization model of multidisciplinary design for planar linkage mechanism based on the fuzzy theory and an idea of multidisciplinary design optimization. Li and Zhang[22] employed MDO in designing distribution cam mechanism for improving the diesel engine system performance. Collaborative optimization, which is a solution method for MDO, has a design architecture intending to improve efficiency while simplifying the structure of multidisciplinary, computation-intensive design problems involving many analysis disciplines and design variables. In this approach, the general optimization is decomposed into some subspace optimization problems that are driven towards interdisciplinary compatibility and a system-level coordination optimization.[23,24] It is a two-level scheme with two optimizer loops, one at each discipline, and one acting globally. The global optimizer drives the design and coupling variables towards an optimal solution that minimizes the objective, while constraining to zero the sum of the squares of the residuals between variables in global optimizer and those in local optimizers. Each local optimizer operates on its own discipline, driving its design variables while minimizing the residual between the actual value of the design variables and the values commanded by the global optimizer.

For applying the principle of collaborative optimization in the optimization of the metamorphic mechanism, each configuration is treated as a discipline. And according to its main framework, a system-level optimization model and several configuration-level optimization models of the metamorphic mechanism need to be established. The system-level optimization model exchanges the coupled variables and the global variables among the configuration-level optimization models continuously until getting the optimum results. The schematic diagram of the collaborative optimization for the metamorphic mechanism can be summarized in Fig. 3.

In Fig. 3, the main framework and components including their interactions based on the principle of collaborative optimization are shown. And the whole optimization process can also be divided into three steps as follows.

(1) Analysis of the mechanism optimization models in all configurations.

The classification of the design variables and constraints, interactions among different configuration optimization models and the corresponding connection diagram are achieved in this step.

(2) Establishment of the system-level optimization model and all configuration-level optimization models.

The system-level optimization model and all configuration-level optimization models are established respectively as Fig. 3 shows according to the results of the above step. In the system-level optimization model, the design variables are given as $Z=\{z_1, z_2, ..., z_{nj}\}$ which are the same with $x^{(g)}$. nj is the number of the global variables. $G^{(i)}(Z,x)=\sum_{j=1}^{nj_i}(z_j^{\in i}-x_j^{\in i})^2$ is defined as the deviation function of global variations in the system-level optimizer for reflecting their unconformity with that in configuration i. ε is a small positive number for limiting the unconformity of variables.

A configuration-level optimization model is different from the original traditional optimization model for adding more design variables and constraints. The deviation function $G^{(i)}$

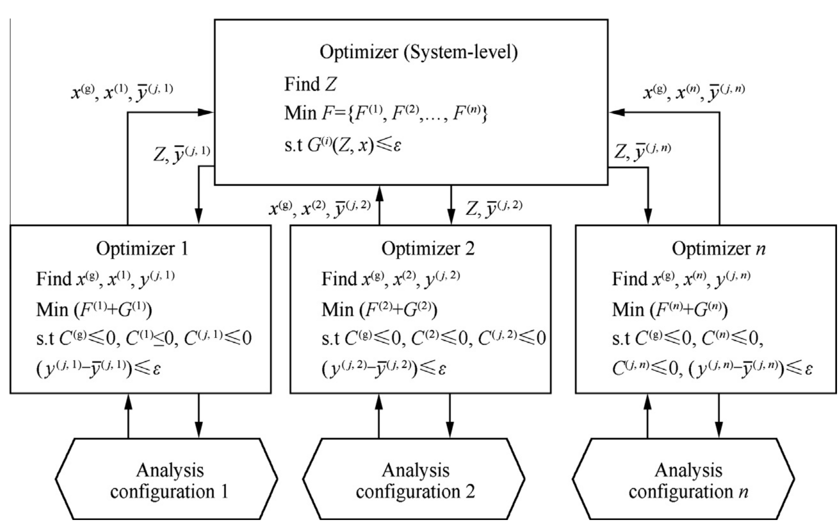


Fig. 3 Schematic diagram of collaborative optimization for metamorphic mechanism.

Table 2 Optimized results.

Items	Optimization model		
	Configuration 1	Configuration 2	MDO
l_1	11.685	31.11699	31.225
l_2	294.9362	324.8551	308.9803
l_3	16.4368	69.2112	22.4833
l'_4	18.3356	93.8949	45.0382
l_5	300	300	300
l_6	74.0295	75.0772	74.9552
α	0.4437	0.2104	0.3719
$F^{(1)}$	26.554	166.9076	36.6296
$F^{(2)}$	219.7981	7.5738	10.3925
F	123.1761	87.2407	23.5111

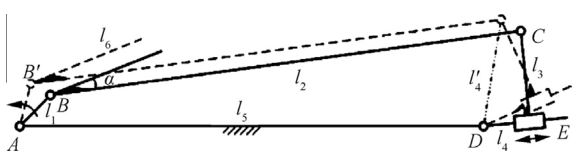


Fig. 4 Schematic graphs of optimized mechanism.

of the global variations is added to the objective functions in configuration-level optimization models for minimizing the unconformity. $\bar{y}^{(j,i)}$ represents the auxiliary variables of input coupled variables ($y^{(j,i)}$) in the optimization models of configuration i. If $F^{(i)}$ are multiple objectives, it should be converted into a single objective by weighting method. $C^{(j,i)}$ represents the coupled constraints from the analysis on the interactions of adjacent configurations.

(3) Solution to the established optimization models

The optimization solution process begins with the configuration-level. After selecting the optimum solution of the configuration-level, the optimum design variables are achieved and passed to the system-level as constant parameters and the system-level optimization proceeds to get the optimum results. Then the results will be put back to the configuration-level as constant parameters. The optimization process goes back and forth between the system-level and the configuration-level until the convergence is reached.

4. Case study

According to the proposed optimization method for the metamorphic mechanism, the example in Section 3 is solved. Analysis on the mechanism in all configurations has been completed. And the next step is the establishment of the optimization models.

The system-level optimization model is established as

$$\begin{cases} \text{Find } Z = \{z_1, z_2\} \\ \text{Min } F = (F^{(1)} + F^{(2)})/2 \\ \text{s.t. } G^{(1)}(Z,x) = (x_{1,1} - z_1)^2 + (x_{2,1} - z_2)^2 \leqslant 0.001 \\ \quad G^{(2)}(Z,x) = (x_{2,1} - z_1)^2 + (x_{2,2} - z_2)^2 \leqslant 0.001 \end{cases} \tag{18}$$

where represents the global design variables. Two configuration-level optimization models are established as

$$\begin{cases} \text{Find } \{x_{1,1}, x_{1,2}, x_{1,3}\} \\ \text{Min } (F^{(1)} + G^{(1)}) \\ = (l_{4\max} - l_{4\min}) + (x_{1,1} - z_1)^2 + (x_{1,2} - z_2)^2 \\ \text{s.t. } (x_{1,3} - \bar{x}_{1,3}) \leqslant 0.001, \\ \text{Eqs. } (4), (5), (6), (7), (8), (15) \end{cases} \tag{19}$$

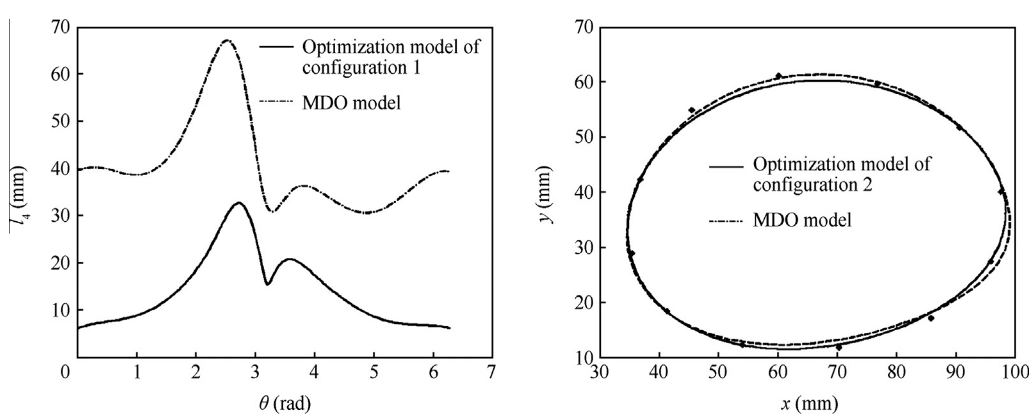
$$\begin{cases} \text{Find } \{x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4}, x_{2,5}\} \\ \text{Min } (F^{(2)} + G^{(2)}) = F^{(2)} + (x_{2,1} - z_1)^2 + (x_{2,2} - z_2)^2 \\ \text{s.t. } (x_{2,3} - \bar{x}_{2,3}) \leqslant 0.001 \\ \text{Eqs. } (11), (12), (13), (14), (15) \end{cases} \tag{20}$$

where $\bar{x}_{1,3}$ and $\bar{x}_{2,3}$represent the auxiliary variables of $x_{1,3}$ and $x_{2,3}$, respectively.

The system-level model and configuration-level optimization models are solved using many methods in iSIGHT, such as adaptive simulated annealing and pointer automatic optimizer. And the results are illustrated in Table 2.

The schematic graphs of the global optimized mechanisms in configuration 1 and configuration 2 are shown in Fig. 4, respectively. $ABCED$ represents the mechanism in configuration 1 and $AB'C'D$ represents the mechanism in configuration 2.

The variation of l_4 is shown in Fig. 5(a) by solving the optimization model of configuration 1 and the MDO model respectively. Solid line gives the variation shape of l_4 from



(a) Configuration 1 (b) Configuration 2

Fig. 5 Comparison of optimization results.

the solution to the optimization model of configuration 1, and the deviation between the maximum and minimum value is $F_1^{(1)} = 32.7168–6.1628 = 26.554$. Dotted line shows its variation shape achieved by solving the MDO model, and its deviation value is $F_{\text{MDO}}^{(1)} = 67.2265–30.5969 = 36.6296$.

Fig. 5(b) shows the optimized trajectories of point P by solving the optimization model of configuration 2 and the MDO model respectively. The star-points show the given points listed in Table 1. And the optimized trajectories from the optimization model in configuration 2 and the MDO model are shown in dotted line and solid line respectively. The value of objective function in single configuration 2 is $F_2^{(2)} = 7.5738$ which is prior to MDO model $F_{\text{MDO}}^{(2)} = 10.3925$.

As shown in Table 1 and Fig. 5, the optimum results obtained from each individual optimization model are prior to the results from the solution to the MDO model. However, the coupled variables and global variables in different configurations, such as $x_{1,1}$, $x_{1,2}$, are inconsistent, leading to the difficulties of optimization. If the optimized results of the global variables by solving an individual optimization model are used in other adjacent configurations, the object value is also inferior to the result by using MDO method. As shown in Table 1, both $F_1 = 123.1761$ and $F_2 = 87.2407$ are inferior to $F_{\text{MDO}} = 23.5111$. Therefore, the uniform optimized results are achieved by taking all configurations into consideration simultaneously by applying the proposed method.

5. Conclusions

(1) The classifications of design variables and constraints in multiple different configuration models are proposed for illustrating coupling interactions between the mechanisms in different configurations distinctly.
(2) An optimization method and its whole optimization process for metamorphic mechanisms based on collaborative optimization are presented by constructing a two-level hierarchical scheme with global optimizer and configuration optimizer loops.
(3) The method is verified by optimizing a planar five-bar metamorphic mechanism which has two configurations, and the optimization results show that the method could get the general optimum results of coupled variables for a complete operation cycle.

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