Gravity gradient distribution in mainland China from GOCE satellite gravity gradiometry data

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Abstract

At present, gravity field and steady-state ocean circulation explorer (GOCE) gravity data are always used to compute regional gravity anomaly and geoid height. In this study, the latest GOCE gravity field model data (from Oct. 2009 to Jul. 2010) are used to compute the gravity gradient of mainland China according to a rigorous recursion formula (in all the six directions). The results show that the numerical values of the gravity gradients are larger in the $T_{rr}$ direction than those in the other directions. They reflect the terrain characteristics in detail and correlate with the regional tectonics; however, in the $T_{ql}$ and $T_{rl}$ directions, the numerical values are relatively smaller and the gravity gradients in the $T_{rl}$ direction do not reflect the terrain characteristics in detail.

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1. Introduction

The gravity field and steady-state ocean circulation explorer (GOCE) gravity satellite was successfully launched by the European Space Agency in March 2009 [1,2] with a main task to measure Earth’s gravity field and static current detection. GOCE is expected to provide a gravity model with an accuracy of 1 mGal for the gravity anomaly and 1 cm for the geoid. This satellite’s mission will hopefully produce spherical harmonic coefficients of the Earth’s gravity field to a degree and order of 300. It can measure gravity gradients through the perpendicular gravity gradiometer directly. We know that the gravity gradient is the second derivative of the gravitational potential. As a result, the gravity gradient is more sensitive to the distance. The GOCE gravity data set has been released since 2010; it includes original gravity gradient data in the Earth Fixed Reference Frame (EFRF) and the Local North Oriented Frame, precise science orbit data, and final gravity field model data. The data are always used to compute the regional gravity anomaly and

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geoid height; the use of these data is in a fledging period in geosciences research. Many scholars at home and abroad are conducting fruitful studies using simulative GOCE data. For example, Gruber [3] analyzed the validation of GOCE gravity field models; Knudsen [4] achieved a global mean dynamic topography and ocean circulation estimation using a GOCE gravity model; Bouman [5] researched the GOCE gravitational gradients along the orbit; Luo [6] and Li [7] studied the method to recover Earth's gravity field; Wu [8,9] discussed the least squares collocation of harmonic analysis using the radial GOCE satellite gravity gradiometry (SGG) data; Wu [10] studied the method for preprocessing GOCE gravity gradient data in detail; Xu [11] discussed the accuracy evaluation method of GOCE SGG data based on satellite crossovers; Zhong [12,13] and Liu [14,15] researched the gridding method of the GOCE SGG data; Yang [16] acquired the gravity gradient tensor from gravity anomaly data. Nevertheless, studies on seismic activity using actual GOCE gravity field data are few. Zhang [17] studied the strong M9.0 earthquake in Japan and computed the gravity anomaly of the strong earthquake region with GOCE gravity field model data. Xu [18] obtained the gravity anomaly of mainland China using GOCE gravity field model data. Their result shows that the gravity anomaly computed from GOCE data is consistent with the tectonics of the earthquake region. Strong earthquakes usually occur in steep gravity gradient zones. As is known, the gravity gradient is the second derivative of the gravitational potential. As a result, it is more sensitive than the gravity anomaly. It is interesting to know whether the gravity gradients correlate with the tectonics distribution. Thus far, there have been no studies using actual GOCE measured gravity data to compute the entire gravity gradients for a specific region, to the best of our knowledge. In this study, the gravity gradient distribution in EFRF was obtained using actual measured GOCE gravity data.

2. Theoretical model and computing formula

The gravity anomaly has two definitions. One is the gravity value subtracted from the normal gravity value at the same point. In this case, it is also called the pure gravity anomaly. Another is the result after the gravity value on the geoid is subtracted from the normal gravity value on the corresponding reference ellipsoid; it is also called the mixed gravity anomaly. In this study, the mixed gravity anomaly was computed using GOCE data. The gravitational potential spherical harmonic series is given by Heiskanen/Moritz [19]:

\[
V(r, \theta, \lambda) = W(r, \theta, \lambda) - Q(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=0}^{N_{\text{max}}} \sum_{m=-n}^{n} \left( C_{nm} \cos \lambda + S_{nm} \sin \lambda \right) P_{nm}(\cos \theta)\]

(1)

where \(V\) is the gravitational potential at the computation point, \(W\) is the gravity potential, \(Q\) is the centrifugal potential, \(GM\) is the gravitational constant times total mass of the Earth (solid Earth, atmosphere, ocean). Furthermore, \(a\) is equatorial radius of the Earth ellipsoid used for the determination of the harmonic coefficients, \(n\) is degree of spherical harmonic series, \(N_{\text{max}}\) is maximum degree of spherical harmonic series, \(r\) is radius distance of the computation point from the geocenter, \(\theta\) is geocentric co-latitude of computation point, \(\lambda\) is geocentric longitude of computation point, \(P_{nm}\) represents normalized associated Legendre functions of degree \(n\) and order \(m\), \(C_{nm}, S_{nm}\) are coefficients of the spherical harmonic series. After rescaling the spherical harmonic coefficients to the set of constants of the reference potential, the disturbing potential at a point \(P\) can be computed by

\[
T_p = W_p - U_p
\]

where \(W_p\) is gravity potential at point \(P\) (including centrifugal potential). \(U_p\) is normal potential of the reference ellipsoid at point \(P\) (including centrifugal potential), and \(T_p\) is the disturbing potential at point \(P\).

If the gravity potential and normal gravity potential have the same centrifugal potential, the disturbing potential does not contain the effect of the centrifugal potential.

\[
T_p(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=2}^{N_{\text{max}}} \sum_{m=-n}^{n} \left( (n+1)(n+2) \frac{R^2}{\tau} \right) \frac{r^{n+1}}{\tau} t_{nm} Q_m(\lambda) P_{nm}(\cos \theta)
\]

(2)

The second partial derivatives are as follows:

\[
\begin{align*}
T_r(P) &= \frac{GM}{R^2 \sin \theta} \sum_{n=2}^{N_{\text{max}}} \sum_{m=-n}^{n} \left( (n+1)(n+2) \frac{R^2}{\tau} \right) \frac{r^{n+1}}{\tau} t_{nm} Q_m(\lambda) P_{nm}(\cos \theta) \\
T_{rr}(P) &= \frac{GM}{R^2 \sin \theta} \sum_{n=2}^{N_{\text{max}}} \sum_{m=-n}^{n} \left( (n+1)(n+2) \frac{R^2}{\tau} \right) \frac{r^{n+1}}{\tau} t_{nm} Q_m(\lambda) P_{nm}(\cos \theta) \\
T_{

\]

Formula (5) refers to the first and second derivative of the Legendre function. The terms of \(T_{r\theta}, T_{\theta\theta}, T_{rr}\) contain 1/sin \(\theta\). In order to avoid the singular point during the computing process, i.e., in the south and north poles, sin(\(\theta\)) = 0, a rigorous recursion formula was used in this study [20]. The specific calculation process is as follows:
In formula (6) and henceforth, $\delta$ is the Kronecker sign function:

$$
\frac{\partial^2 P_{n[m]}}{\partial \theta^2} = b_i^{n[m]} P_{n[m]-2} + b_i^{n[m]} P_{n[m]+2} + b_i^{n[m]} P_{n[m]+2}
$$

(7)

$$
b_i^{n[m]} = \frac{1}{4} \left[ n + |m| \sqrt{n + |m|} |m| + 2 \sqrt{2 - \delta_{m|0}} \sqrt{2 - \delta_{m|2}} \right]
$$

(8)

$$
m \frac{P_{n|m}}{\sin \theta} = c_i^{n|m} P_{n|m-1} + c_i^{n|m} P_{n|m+1} + c_i^{n|m} P_{n|m+2}
$$

(9)

$$
\frac{\partial}{\partial \theta} \frac{m}{\sin \theta} = d_i^{n|m} P_{n|m-1} + d_i^{n|m} P_{n|m+1} + d_i^{n|m} P_{n|m+2}
$$

(10)

$$
d_i^{n|m} = \frac{m^2}{2|m|} \left[ 2 - \delta_{m|0} \right] \left[ 2 - \delta_{m|2} \right]
$$

(11)

$$
m \frac{\partial P_{n|m}}{\partial \theta} = e_i^{n|m} P_{n|m-2} + e_i^{n|m} P_{n|m-1} + e_i^{n|m} P_{n|m+2}
$$

(12)

$$
e_i^{n|m} = \frac{m}{2|n|} \left[ (2 - \delta_{n|0}) (2n + 1) \right]
$$

(13)

$$
e_i^{n|m} = \frac{m}{2|n|} \left[ (2 - \delta_{n|0}) (2n + 1) \right]
$$

(14)

We can obtain the gravity gradient distribution in a specific region through the above formulas.

### 3. The gravity gradient distribution in mainland China in EFRF

The study region is located at 70°–130° E and 15°–55° N. GOCE gravity field model data for the period Sep. 2009 to Jul. 2010 was used in this study. The order of the spherical harmonic coefficient was 250. The sample interval was 0.5° × 0.5° and the average orbit altitude was 250 km. The terrain distribution in mainland China is shown in Fig. 1.

Fig. 1 shows the terrain of mainland China. The most famous mountain is the Qinghai-Tibet Plateau. There are many fault zones, such as the Altun fault, Kunlun fault, and Longmenshan fault.

Using the formulas (5–14), the gravity gradients of mainland China were computed in the $T_{n0}$, $T_{n1}$, $T_{n2}$, $T_{o0}$, $T_{o1}$, $T_{o2}$ directions respectively. The computing results are shown in Fig. 2(a–f).

The gravity gradient in mainland China was computed using GOCE gravity field model data for the first time. In Fig. 2, The results show that in the $T_{n0}$ direction, the gravity gradients are the largest as compared with the other directions and the numerical value range is between $-3.5 \times 10^{-8}$ s$^2$ and $+5.0 \times 10^{-8}$ s$^2$. The data reflect the terrain characteristics in detail and correlate well with the tectonics. The gradient in the $T_{o0}$ and $T_{o1}$ directions reflect the terrain characteristics on the whole. However, the sign of the numerical value is opposite to that in the $T_{n}$ direction. The gradients of $T_{o0}$ and $T_{o1}$ are relatively small compared with the other directions and the numerical value range is between $-2.0 \times 10^{-8}$ s$^2$ and $+2.0 \times 10^{-8}$ s$^2$. In the $T_{o2}$ direction, the characteristics of the topography are revealed and they correlate well with the regional tectonics. In the $T_{o2}$ direction, the gravity gradients do not reflect the terrain characteristics in detail.
In summary, the gravity gradients computed using actual GOCE gravity field data correlate well with the geology at large scales, especially in the $T_{rr}$ direction. In the near future, higher resolution images using higher orders of GOCE gravity data will be obtained.

4. Conclusion

The results of this study show that the gravity gradients in the $T_{rr}$ direction are the largest among all the six directions. Additionally, the gravity gradients in the $T_{rr}$ direction correlate well with the terrain distribution in mainland China. The most probable reason for this may be that the gravity gradients in the $T_{rr}$ direction (also called the radial direction) are the nearest to the ground. As a result, the gravity gradients are more sensitive than those in the other directions.

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Xu Haijun gained his doctor degree from Chang’an University in 2012 and his major is geodesy and survey engineering. His research area is satellite gravity survey, particle swarm algorithm inversion and the theory and application of three-dimensional laser scanner. He has published papers in domestic journals such as Journal of Geodesy and Geodynamics, Progress in Geophysics, Geophysical and Geochemical Exploration, Northwestern Seismological Journal, and so on. He works as a lecturer in North China University of Water Resources and Electric Power from July 2012.

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