Noël Gastinel and Computational Mathematics

Jean Della Dora
François Robert
Université de Grenoble
Saint-Martin-d'Hères Cedex
France

Submitted by Hans Schneider

Noël Gastinel passed away in Grenoble on September 11, 1984, after a very serious illness: a tongue cancer which had been discovered in February of 1983.

He was born in Le Muy, a small town in southern France, on December 25, 1925 (indeed, Noël means Chistmas in French!). A brilliant student of mathematics, he was preparing in Toulon the "Concours d'entrée dans les grandes écoles d'ingénieurs françaises" during the second world war. Hurt and much pained by the bombing of this town (his house "fell down over himself" as he often said), he was obliged to discontinue his studies. He spent the last months of the war clandestinely in the small village of Aups.

After the war, he resumed his studies at the Faculté des Sciences de Marseille (where he was enthused by astronomy). He passed the "Agregation de Mathématiques" and became a teacher in Toulon for a number of years. In 1957, Professor Jean Kuntzmann, who founded the first French engineering school in Applied Mathematics and Computer Science (called ENSIMAG today), invited him to come to the University of Grenoble.

Noël Gastinel's career continued at Grenoble until his death. He was for many years the Head of the "Centre Interuniversitaire de Calcul de Grenoble" (C.I.C.G.). During the last difficult months of his life, he showed a great clarity of thought and an exemplary courage. Even during his last days, he was still writing programs for the Apple II-e computer which his colleagues and students had presented him as token of their affection.

HIS WORK

The scientific life of N. Gastinel was entirely devoted to Computational Mathematics. His own conception of computation evolved all through his life. We shall now examine different stages of his work.

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1. His Thesis and its Developments (1960–1970)

Noël Gastinel's "thèse d'Etat" was presented at the University of Grenoble in 1960, with Professor J. Kuntzmann as the advisor. It is entitled "Matrices du Second Degré et Normes Générales en Analyse Numérique Linéaire" (Matrices of the Second Degree and General Norms in Linear Numerical Analysis). It covers the following main themes:

Second degree matrices: that is, matrices with minimal polynomial of degree 2, especially matrices of rank 1, of the form

$$K = xy', \text{ as } K^2 - y'xK = 0.$$

N. Gastinel developed this general and useful tool for a good presentation of the basic algorithms of matrix algebra, linear systems (especially Gaussian elimination), and eigenvalue methods.

General norms on finite dimensional spaces: N. Gastinel was very fond of this subject, and of its geometrical aspects in particular. He devoted to it many hours of work, lectures, and discussions, and directed the research of a number of his students on this theme (F. Robert, J. F. Maitre, Pham Dinh Tao, to name a few). He was fascinated by the notion of the condition number which *he* defined as

$$\gamma_\varphi(A) = \frac{1}{S_\varphi(A)S_\varphi(A^{-1})}$$

(and not the inverse

$$K_\varphi(A) = S_\varphi(A)S_\varphi(A^{-1})$$

as is generally employed in the literature!).

At this epoch-making stage of numerical linear algebra, N. Gastinel was very interested in and impressed by the pioneering work of A. Householder, R. S. Varga, J. Wilkinson, F. L. Bauer, A. Ostrowski, O. Taussky, and others. He felt that this kind of concrete mathematics had a better future with the advent of computers than the abstract mathematics often studied in France during that period.

He discovered an original method for the iterative solution of linear systems, sometimes referred to as "Gastinel's Method." He had named it as a "projection method" associated with a decomposition of a norm. He presented it at the IFIP meeting in München in 1962.

Gastinel's pioneering work in linear algebra was recognized by constant invitations to the Gatlinburg meetings, organized initially by Professor A. Householder. He was very fond of these travels to the United States. He visited quite often, especially when he was the Head of the Computer Center. N. Gastinel was much impressed by the American dynamism in scientific and technical work, and spoke often about it with enthusiasm.

Later, N. Gastinel generalized his method to over-decomposition of general norms (*Numerische Mathematik*, 1963). All his life, he was especially fond of the geometrical properties of the unit balls of norms in R^n, and of the unit balls of matricial norms. He directed several students on this subject, and in 1965 published a paper in *Numerische Mathematik*, where he geometrically characterizes the unit balls of the matricial norms he denoted S_φ. In 1971, he published a paper in *Numerische Mathematik*, where he tried to extend to the whole algebra $\mathcal{M}_n$ a multiplicative norm defined only on a subalgebra of $\mathcal{M}_n$. This paper leaves open an interesting field of research. Noël Gastinel was on the editorial boards of various journals, including *R.A.I.R.O.*, *Numerische Mathematik*, *Calcolo*, as well as *Linear Algebra and Its Applications.*

N. Gastinel had also a genuine interest in functional analysis; he wrote a number of pages (yet unpublished) on angle constants for normed vector spaces, and studied a posteriori errors for linear problems in Hilbert Spaces. In a rather theoretical joint-work with J. P. Bertrandias, he considered the general problem of approximate solutions of linear problems in general topological vector spaces. He generalized the LR method to linear transformations on vector spaces of sequences. He was a great admirer of the Rutishauser algorithm, and directed the theses of F. Chatelin, J. Wolf, and J. Della Dora on eigenproblems.

Most of his contributions remain unpublished. Many of them can be found either in the many theses he directed, or in the numerous papers he wrote for the "Numerical Analysis Seminar" he directed at Grenoble for more than 15 years. For some years he gave a very original course on linear problems on spaces of sequences. He wrote a paper entitled "Inverse Problems in Numerical Analysis," in which he tried to characterize the sequences generated by a given computational iterative algorithm.

Finally, his thesis paid much attention to error analysis in the numerical solution of linear systems. He gave a formal inverse of a (generalized) Hilbert matrix:

$$h_{ij} = \frac{1}{b_i - \beta_j} \qquad (i, j = 1,2 \ldots n),$$

which can be used as a test-matrix. He published a short note on the bad

condition number of such a matrix and he directed Mrs. Pichat's thesis on the control of rounding errors in numerical analysis.

In fact, linear numerical algebra was a subject close to Noël Gastinel's heart throughout his scientific life. His book *Analyse Numérique Linéaire* was published in 1966 by Hermann. It is a textbook, full of rich information on the subject at this time, but somewhat difficult to read. It has been translated into both English and Spanish.

2. *The Centre Universitaire de Calcul de Grenoble (CICG) and the development of Applied Mathematics*

N. Gastinel was very fond of machines and technology, especially computers. He explored many aspects of them, rapidly becoming the Director of CICG. To himself, this was indeed the primary focus of his mission.

Through his influence, this Center became a torchbearer among the French University Computer Centers and received very modern equipment. N. Gastinel was able to foresee the evolution of machines. He was a visionary in the field of computing techniques.

During this period there arose a "Grenoble Vision" of Computational Mathematics: the evolution of the machines, the adaptation of the algorithms, and the creation of new mathematics to master this progress.

N. Gastinel gave his tireless help to many users of computers and numerical methods. He thus encountered many interesting problems:

Signal processing. As early as 1963, he suggested to A. Eberhard work on FFT along the lines of the Cooley-Tuckey algorithm. The programs have been of great use at the Faculty of Medicine of Grenoble. In 1969, he wrote a paper on form retrieval with D. Chenais.

Image processing. He was much interested in the numerical treatment of discrete, biological images. He spent much time and energy in helping J. M. Chassery to conceive of a biological colored image processor, which is now commercially manufactured.

Physics. He investigated several connected problems with Prof. R. Maynard and B. Lacolle: Ising model, spin glass theory (which gave rise to complex problems of counting), and percolation problems. He directed an interdisciplinary seminar on these subjects.

Singularities problems and various generalizations. Analytical prolongation and conformal mapping, localization of singularities for a complex function of the complex variable; Padé, Padé-Hermite and Hardy-type approximants: these were the subjects of J. Della Dora's "These d'Etat" on rational approximation.

O.D.E. In the context of J. Kuntzmann's work in the early years, around 1960, N. Gastinel directed several students on Runge-Kutta methods.

Partial Differential Equations. N. Gastinel was mainly interested in the numerical aspects of this field (finite difference schemes, the Peaceman-Rachford method, etc.). However, he considered, along with J. C. Miellou, more fundamental aspects, such as the existence of a solution of elliptic operator equations in a variational context.

A. Poncet, one of his students, wrote the first French university general package for the finite elements method. N. Gastinel directed several theses on the applied aspects of the P.D.E.

From a technical point of view, N. Gastinel was very fascinated, inside the Computer Center, by virtual memories and by computer networks.

3. The Second Half of His Career (1970–1984)

During the second part of his scientific life, N. Gastinel pursued passionately nonlinear algebraic problems. Strassen's important discovery on the computation of the product of matrices (1969) fascinated him. As early as 1971, N. Gastinel published a paper in *Numerische Mathematik* in which he showed that Strassen's product is based on the representation of an Hadamard product of (2×2) matrices in a seven-dimensional space. He gave several lectures on the subject, and realized the importance of the tensor rank of a set of matrices for evaluating bilinear forms. His contributions about complexity of algorithms remain unpublished—except for the Proceedings of his seminar. N. Gastinel directed J. C. Lafon's "These d'Etat" on this subject and he came back to it very often.

N. Gastinel was then led to consider the problem of optimizing algebraic computing programs. He gave, as early as 1974, very simple and interesting examples. In 1975 he enlivened a seminar on matricial problems of automatics (identification of linear systems).

During these years, he encouraged some of his students to take interest in Algebraic Computation (in 1970, Y. Siret had presented, under his direction, a "Thèse d'Etat," about the use of LISP as a basic tool for Algebraic Computation). He saw in this domain a wide range of possibilities for Computational Mathematics. N. Gastinel was himself a user of algebraic codes for handling problems of automatics, and for the computation of tensorial ranks. During the final years of his life, he conducted a workshop on the use of Algebraic Computation on linear differential equations with polynomial coefficients (to obtain asymptotic solutions).

As early as 1975, N. Gastinel was working with several research workers (M. Cosnard, A. Eberhard) on problems about iteration of functions. He gave lectures on this very rich subject, which engaged his attention until his last days. Death, alas, kept him from presiding at C. Masse's thesis on Newton iterations.

It is necessary to quote many other subjects to get an idea of his wide range of competence:

Approximation theory: L_1, L_∞ approximation, spline functions, Padé approximants.

Computer Graphics: graphical representations, *B*-splines, and Béziers curves.

Automata networks, cellular automata: he brought back this subject from the United States as early as 1972. He was fascinated by the idea of a global behavior resulting from local interactions between finite cells, as defined by J. Von Neumann and S. Ulam. After him, F. Robert and M. Tchuente were both motivated by these subjects, especially on discrete iterations and on parallel computation on a network.

Interpolation in several variables: in 1973 he wrote a short note on a method for studying the interpolation error for functions of several variables by using Sard's theorem. He worked on this subject with P. Chenin, who presented his thesis under his direction. With C. F. Ducateau, he extended Witney's process for general interpolation.

CONCLUSION

N. Gastinel's scientific life is probably exceptional: though he published very few papers, his personal contribution to the developments of numerical analysis has been really quite large. Every day he would give his students suggestions, short notes, hours of common work at the blackboard, which were for them a mine of original ideas, and this continued till the last months of his life (C. Masse working on Newton's method, and J. M. Muller working on the computation of elementary functions benefited from his help as late as Spring of 1984). The incredible number of theses he personally directed in Grenoble is the best testimony to his exceptional scientific work; indeed, one counts up to:

18 "Thèses d'Etat" (1966–1984)
34 "Thèses de 3° cycle et Docteur Ingénieur" (1962–1980).

Moreover, he gave direction to the work of many researchers outside of Grenoble, even though he was not officially their advisor.

Noël Gastinel gave much to Mathematics and Computation. A profound, original spirit, with a strong character, he contributed to creating a school which tried to bring to Computation the necessary Mathematics. He liked to

quote this sentence from Hilbert:

> *It is nonsense to search for methods without having a well defined problem to solve.*

His modesty (and his great shyness) made him withdraw, to some extent, from scientific and academic life. His implacable judgements did not always make him friends. He requested from everybody the same intellectual and moral uprightness that he brought to his vocation. This was not always easy.

SOME PAPERS BY NOËL GASTINEL

His Thesis

Matrices du second degré et normes générales en Analyse Numérique Linéaire, Faculté des Sciences de Grenoble, December 17, 1960.

In Chiffres

Inversion d'une matrice généralisant la matrice de Hilbert. 3:149–152 (1960).

Sur le choix des paramètres de sur-relaxation. (Procédé de Peaceman-Rachford). 2:109–128 (1962).

Correction automatique de tests à l'Université de Grenoble. 4:237–245 (1960).

In Numerische Mathematik

Sur-décomposition de normes générales et procédés itératifs. 5:142–151 (1963).

Propriétés de certains ensembles normés de matrices. 7:255–260 (1965).

Sur l'extension de normes sur des algèbres de matrices. 17:71–83 (1971).

Sur le calcul des produits de matrices. 17:222–229 (1971).

In Linear Algebra and Its Applications

Condition numbers and projection method (joint paper with J. L. Joly). 3:185–224 (1970).

In IFIP Congresses

Sur certains procédés itératifs non linéaires de résolution de systèmes d'équations du 1 er degré, Munich, 1962, pp. 97–107.

Pattern recognition in a signal (joint paper with D. Chenais), Amsterdam, 1969, pp. 1447–1450.

In C.R.A.S. (Comptes-rendus à l'Académie des Sciences de Paris)

Procédé itératif pour la résolution numérique d'un système d'équations linéaires, May 5, 1958.

Sur le choix des pivots dans l'élimination de Gauss pour la résolution de systèmes linéaires, January 11, 1960.

Utilisation de matrices vérifiant une équation du 2ème degré pour la transmutation de matrices, March 7, 1960.

Résolution d'équations fonctionnelles par la méthode de décomposition de norme. Application à l'approximation de la solution de problèmes aux limites elliptiques variationnels du second degré (joint paper with J. C. Miellou), January 7, 1963.

Conditionnement d'un système d'équations linéaires, May 11, 1969.

In Congrès de l'Afcalti-Afiro-Afcet

Quelques problèmes récents relatifs aux normes de vecteurs et de matrices, Versailles, 1964, pp. 193–199.

Quelques problèmes inverses de convergence en analyse numérique, Lille, 1966, pp. 463–465.

Tableaux des valeurs d'une fonction sur des subdivisions de leur intervalle de définition, Nancy, 1967, pp. 8–13.

Quelques procédés itératifs pour la résolution de systèmes linéaires associés, par la méthode des différences, à des équations aux dérivées partielles, p. 39–46.

COMPLETE LIST OF ALL THÊSES DIRECTED BY NOËL GASTINEL IN GRENOBLE (BETWEEN 1962 AND 1984)

Thèses d'Etat

M. Atteia, Etude de certains noyaux et théorie des fonctions splines en analyse numérique, June 20, 1966.

F. Robert, Etude et utilisation de normes vectorielles en analyse numérique linéaire, November 12, 1968.

Y. Siret, Contribution au calcul formel sur ordinateur, January 1, 1970.

J. C. Miellou, Sur une notion de monotonie conduisant à une extension de l'application de la méthode variationnelle dans l'étude des systèmes d'équations et d'inéquations aux dérivées partielles; opérateurs paramonotones, October 19, 1970.

C. F. Ducateau, Etude de quelques problèmes d'interpolation, March 12, 1971.

F. Chatelin, Méthodes numériques de calcul des valeurs propres et vecteurs propres d'un opérateur linéaire, March 12, 1971.

C. Brezinski, Méthodes d'accélération de la convergence en analyse numérique, April 29, 1971.
M. Duc Jacquet, Approximation des fonctionnelles linéaires sur les espaces hilbertiens autoreproduisants, March 23, 1973.
J. Wolf, Analyse numérique de quelques problèmes liès au traitement des signaux, October 18, 1974.
J. F. Maitre, Sur certaines normes et fonctionnelles dans les espaces de matrices et d'opérateurs, November 30, 1974.
M. Pichat, Contribution à l'étude des erreurs d'arrondi en arithmétique à virgule flottante, May 14, 1976.
J. C. Lafon, Complexité de l'évaluation de plusieurs formes bilinéaires et des principaux calculs matriciels, November 29, 1976.
C. Belissant, Contribution à l'analyse et à la reconnaissance automatique de la parole, September 23, 1978.
A. Poncet, Autour de l'écriture d'un code d'éléments finis, March 8, 1979.
J. Della Dora, Contribution à l'approximation de fonctions de la variable complexe au sens de Hermite-Padé et de Hardy, June 20, 1980.
Pham Dinh Tao, Contribution à la théorie des normes et ses applications à l'analyse numérique, February 19, 1981.
M. Tchuente, Contribution à l'étude des méthodes de calcul pour des systèmés de type coopératif, December 3, 1982.
B. Lacolle, Sur certaines méthodes de calcul de la Physique statistique, June 29, 1984.

Thèses de Docteur Ingénieur

J. Cunge, Etude d'un schéma de différences finies appliqué à l'intégration numérique d'un certain type d'équation hyperbolique d'écoulement, May 1966.
P. Bourret, Méthodes de calcul de certains écoulements, July 15, 1968.
A. Eberhard, Algorithmes de l'analyse harmonique numérique, June 22, 1970.
C. De Polignac, Méthodes optimales de calcul de produits de matrices, June 22, 1970.
A. Aussens, Une mise en oeuvre frontale des méthodes d'éléments finis, December 14, 1972.

Théses de 3ème Cycle

G. Schauer, Utilisation de quelques types de normes de vecteurs dans des méthodes itératives de résolution de systèmes linéaires, September 29, 1962.
M. Liot, Etude de la propagation des erreurs de calcul dans deux méthodes classique de résolution de l'équation de la chaleur, October 23, 1964.

C. Di Crescenzo, Résolution explicite du système linéaire représentant une approximation de Dirichlet sur un rectangle, March 10, 1965.
J. Ville, Méthodes numériques de recherche de la meilleure approximation, March 19, 1965.
C. Joubran, Etude numérique du procédé de Peaceman Rachford pour la resolution de problèmes elliptiques, April 6, 1965.
J. C. Sabonnadière, Applications des calculatrices numériques en Electrotechnique, June 25, 1965.
J. Wolf, Méthodes de calcul des valeurs propres d'une matrice quelconque par utilisation de transformations unitaires, November 6, 1965.
J. Chion, Deux méthodes de résolution d'équations algébriques; le procédé des réduites et l'algorithme de Routh, November 26, 1965.
M. Duc Jacquet, Résolution de quelques problèmes d'analyse numérique linéaire par des matrices antiscalaires, January 24, 1968.
J. Ducrocq, Méthodes d'estimation a posteriori d'erreurs, February 5, 1968.
J. L. Guyot, Etude de quelques méthodes de calcul du polynome caractéristique et des valeurs propres, March 15, 1969.
J. Cosnier, Sur la stabilité de la solution numérique pour un problème particulier d'équations aux dérivées partielles de type hyperbolique, June 26, 1969.
M. Petit, Quelques programmes graphiques d'analyse numérique, September 25, 1971.
E. Tournier, Un exemple d'utilisation du calcul sur ordinateur: méthode générale de localisation des racines d'une équation algébrique à coefficients complexes, September 25, 1971.
Pham Dinh Tao, Etude d'une classe de normes dans les espaces vectoriels à dimension finie générées par les normes des espaces fonctionnels de Banach. Applications, February 14, 1972.
F. Veillon, Quelques nouvelles méthodes pour le calcul numérique de la transformée de Laplace, March 11, 1972.
J. Della Dora, Sur quelques algorithmes de recherche de valeurs propres, July 4, 1973.
J. E. Mazille, Etude et résolution numérique de quelques problèmes d'aérothermochimie, December 5, 1973.
M. Bouhier, Quelques questions d'analyse numérique traitées du point de vue de la calculabilité, February 28, 1974.
P. Chenin, Quelques problèmes d'interpolation à plusieurs variables, June 26, 1974.
S. Rachidi, Méthode des sécantes pour la résolution de systèmes d'équations algébriques non linéaires, June 30, 1976.
B. Lacolle, Quelques pròblemes de restructuration dans un environnement paginé, September 14, 1976.
J. M. Chassery, Quelques problèmes liés au traitement d'images numériques

obtenues par un système automatique de microphotométrie à balayage, December 15, 1976.

C. Rubat, Etude de quelques problèmes liés à l'analyse d'images. Recherche de directions privilégiées et projection d'images, May 13, 1977.

C. Espinoza, Contribution à la résolution numérique de certains systèmes d'équations, July 25, 1977.

A. Galli, Quelques propriétés et algorithmes de calcul formel de polynomes symétriques et antisymétriques, May 11, 1979.

A. Chalabi, Analyse numérique de quelques problèmes relatifs aux ondes longues en eau peu profonde, June 11, 1980.

J. R. Roche, Application des approximants de Padé au calcul de l'exponentielle d'une matrice, June 20, 1980.

M. Chenin Mordojovich, Résolution numérique des équations des ondes longues dans un réseau de caractéristiques, June 26, 1980.