Physics Letters B 716 (2012) 236–239

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Maximally helicity violating disk amplitudes, twistors and transcendental integrals

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ARTICLE INFO

Article history:
Received 9 August 2012
Accepted 10 August 2012
Available online 14 August 2012
Editor: L. Alvarez-Gaumé

ABSTRACT

We obtain simple expressions for tree-level maximally helicity violating amplitudes of N gauge bosons from disk world-sheets of open superstrings. The amplitudes are written in terms of $(N-3)!$ hypergeometric integrals depending on kinematic parameters, weighted by certain kinematic factors. The integrals are transcendental in a strict sense defined in this work. The respective kinematic factors can be succinctly written in terms of "dual" momentum-twistors. The amplitudes are computed by using the prescription proposed by Berkovits and Maldacena.

We observe physical phenomena by detecting photons, gluons, neutrinos and other particles scattered or emitted at various distance scales. The scattering amplitudes must contain full information not only about the physical processes, but also about the nature of spacetime. Some of the simplest amplitudes describe the scattering of gauge bosons (gluons) in (supersymmetric) Yang–Mills theories [1]. In 2003, Edward Witten proposed a twistor string theory description of such amplitudes [2]. Ever since then, there has been a growing evidence that twistors play an important role in the S-matrix description of spacetime, with the ambient spacetime related to twistor space.

Strings propagate in ambient spacetime. Open superstrings offer a self-consistent generalization of Yang–Mills theory and introduce the Regge slope parameter α' that determines the mass scale $M = 1/\sqrt{\alpha'}$ of higher spin excitations of the gauge supermultiplet. The scattering amplitudes receive contributions from all two-dimensional world-sheets with and without boundaries. Our goal is to obtain compact expressions for some full-fledged but possibly simplest superstring amplitudes. We also want to see if twistors play any special role in a theory in which (spacetime) superconformal invariance is explicitly broken by the presence of mass scale M. We will consider the semi-classical contributions of disk world-sheets to multi-particle amplitudes involving gluons in maximally helicity violating (MHV) configurations which, by experience with Yang–Mills theory, are expected to have the simplest form.

Disk amplitudes have been studied for almost fifty years. Some time ago, we performed a detailed analysis of multi-gluon amplitudes, deriving explicit expressions for up to $N = 7$ gluons [3–7]. More recently, more compact expression have been obtained by using the pure spinor formalism [8,9]. The general structure is, however, always the same. For a specific ordering of non-abelian gauge group (Chan–Paton) factors, the N-gluon (partial) amplitudes are usually written as sums of $(N-3)!$ terms: $\sum_{i=1}^{i=(N-3)!} I_i K_i$, where I_i are some integrals over $N-3$ positions $z_4, \dots, z_N$ of vertex operators at the disk boundary (z_1, z_2, z_3 are fixed by $PSL(2,\mathbb{R})$ invariance), while K_i are certain kinematic factors that depend on the polarization vectors and momenta of external gluons. In this work, we discuss the integrals and kinematic factors describing MHV amplitudes. Instead of performing traditional computations, we follow the prescription of Berkovits and Maldacena [10] which offers a shortcut for MHV configurations.

We will be considering the N-gluon partial MHV disk amplitude associated to the Chan–Paton (color) factor $\mathrm{Tr}(T^{a_1} \cdots T^{a_N})$, involving two helicity minus gluons (labeled by 1 and 2) and $N-2$ helicity plus gluons. Berkovits and Maldacena [10] argued that this amplitude is given by the following correlation function:

$$A_N(\lambda_k, \tilde\lambda_k) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\Big\langle V_1(z_1)V_2(z_2)V_3(z_3)\int_{z_3}^{\infty} dz_4 U_4(z_4) \cdots \int_{z_{N-1}}^{\infty} dz_N\, U_N(z_N)\Big\rangle \tag{1}$$

where $V_k(z_k) = e^{ip_k X(z_k)} = e^{i\lambda_k\tilde\lambda_k X(z_k)}$ and

$$U_k(z_k) = \big(\epsilon_k^a \tilde\lambda_k^{\dot a} \partial X_{a\dot a} + \psi_{\dot a}\bar\psi_{\dot b}\tilde\lambda_k^{\dot a}\tilde\lambda_k^{\dot b}\big) e^{ip_k X(z_k)}. \tag{2}$$

Here, X are the standard bosonic coordinates while $\psi_{\dot a}$, $\bar\psi_{\dot b}$ are fermions of conformal weight $(\frac{1}{2}, 0)$. ϵ_k^a are arbitrary "reference spinors" normalized by $\epsilon_k^a \lambda_{ak} = 1$. In this work, we follow

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http://dx.doi.org/10.1016/j.physletb.2012.08.018

the same conventions as in Ref. [5], always choosing $(z_1, z_2, z_3) = (-\infty, 0, 1)$, and integrate in Eq. (1) over $N-3$ real variables ordered as $z_4 < \cdots < z_N$ in the interval $(1, \infty)$.

It should be made clear that Eq. (1) is a conjecture supported by several tests. Berkovits and Maldacena have already checked that Eq. (1) does indeed yield correct Yang–Mills amplitudes in the $\alpha' \to 0$ limit. Furthermore, they verified that for $N = 4$ and $N = 5$, it reproduces the known string amplitudes. As a warm-up, we will first examine the $N = 6$ case, using this as an opportunity to establish some notation.

For $N = 6$, the correlator of Eq. (1) becomes

$$\left\langle V_1(z_1)V_2(z_2)V_3(z_3) \prod_{k=4,5,6} \int dz_k \big(\varepsilon_k \partial X(z_k) + \psi(z_k)\tilde{\lambda}_k \bar{\psi}(z_k)\tilde{\lambda}_k\big) V_k(z_k) \right\rangle \tag{3}$$

with the polarization vectors $\varepsilon_k^{a\dot{a}} = \epsilon_k^a \tilde{\lambda}_k^{\dot{a}}$. A choice of "parallel" reference spinors, say $\epsilon_k^a = \lambda_3^a / \langle 3k \rangle$ as in Ref. [10], greatly simplifies the computations because it eliminates all bosonic contractions involving the products $\varepsilon_k \varepsilon_l$ and potentially producing double poles $(z_i - z_j)^{-2}$. In order to discuss this correlator, it is convenient to introduce the following notation for the boundary integrals, useful not only for $N = 6$ but also for arbitrary N:

$$\big[\!\big[(i_1 i_2)(i_3 i_4) \cdots (i_{n-1} i_n)\big]\!\big]_N = \int_1^\infty dz_4 \cdots \int_{z_{N-1}}^\infty dz_N \, (z_{i_1 i_2} z_{i_3 i_4} \cdots z_{i_{n-1} i_n})^{-1} \prod_{2 \leqslant k < l \leqslant N} |z_{kl}|^{2\alpha' p_k p_l} \tag{4}$$

where $z_{ij} \equiv z_i - z_j$, $\{i_1, i_2, \ldots, i_n\}$ take arbitrary values in the set $\{2, 3, \ldots, N\}$, $z_2 = 0$ and $z_3 = 1$. At the face value, for the gauge choice of $\epsilon_k^a = \lambda_3^a / \langle 3k \rangle$, there are 16 integrals originating from the free-field correlators appearing in Eq. (3). By using partial fractioning, however, it is possible to express all of them in terms of a six-element basis. In some bases, the amplitude becomes particularly simple, for instance[1]

$$\begin{aligned} A_6 = \frac{\langle 12 \rangle^3}{\langle 34 \rangle \langle 35 \rangle \langle 36 \rangle} \big\{ &[42][5|2+4|3\rangle[61] \big[\!\big[(24)(45)(56)\big]\!\big]_6 \\ &+ [42][6|2+4|3\rangle[51] \big[\!\big[(24)(46)(65)\big]\!\big]_6 \\ &+ [52][4|2+5|3\rangle[61] \big[\!\big[(25)(54)(46)\big]\!\big]_6 \\ &+ [52][6|2+5|3\rangle[41] \big[\!\big[(25)(56)(64)\big]\!\big]_6 \\ &+ [62][4|2+6|3\rangle[51] \big[\!\big[(26)(64)(45)\big]\!\big]_6 \\ &+ [62][5|2+6|3\rangle[41] \big[\!\big[(26)(65)(54)\big]\!\big]_6 \big\}. \end{aligned} \tag{5}$$

Note the presence of spurious singularities at $\langle 35 \rangle = 0$ and at $\langle 36 \rangle = 0$ which appear as a consequence of the gauge choice.

The six-gluon MHV amplitude has been given before in Refs. [4,5], where it appears in a rather complicated form. For quite a long time, however, we knew [12] that it can be simplified to

$$\begin{aligned} A_6 = \frac{\langle 12 \rangle^3}{\langle 34 \rangle \langle 56 \rangle} \big\{ &[12][35][46] \big[\!\big[(25)(26)(35)(46)\big]\!\big]_6 \\ &+ [15][24][36] \big[\!\big[(25)(26)(24)(36)\big]\!\big]_6 \\ &+ [16][23][45] \big[\!\big[(25)(26)(23)(45)\big]\!\big]_6 \\ &- [12][36][45] \big[\!\big[(25)(26)(36)(45)\big]\!\big]_6 \\ &- [15][23][46] \big[\!\big[(25)(26)(23)(46)\big]\!\big]_6 \\ &- [16][24][35] \big[\!\big[(25)(26)(24)(35)\big]\!\big]_6 \big\}. \end{aligned} \tag{6}$$

The above expression is free of spurious poles, albeit not as "symmetric" as Eq. (5). By a repeated use of partial fractioning, $(z_{ik} z_{ij})^{-1} = (z_{kj} z_{ik})^{-1} - (z_{kj} z_{ij})^{-1}$, and partial integrations of the integrals, combined with some spinor algebra, we managed to show that the right hand sides of Eqs. (5) and (6) are indeed equal.

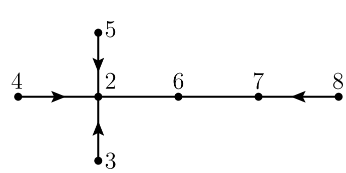
Now we proceed to the general N-gluon case. As in the previous case, a good gauge choice leads to tremendous simplifications. Here, we make a slightly different choice, $\epsilon_k^a = \lambda_2^a / \langle 2k \rangle$, which ensures $\varepsilon_k \varepsilon_l = 0$ for all polarization vectors, in addition to $\varepsilon_k p_2 = \varepsilon_k p_k = 0$. Then the bosonic correlation functions contribute to the integrands as

$$\begin{aligned} &\left\langle \varepsilon_{k_1} \partial X(z_{k_1}) \varepsilon_{k_2} \partial X(z_{k_2}) \cdots \prod_{j=1}^N V_j(z_j) \right\rangle \\ &\quad = \sum_{i_1 \neq 1,2,k_1} \sum_{i_2 \neq 1,2,k_2} \cdots \frac{\varepsilon_{k_1} p_{i_1}}{z_{k_1 i_1}} \frac{\varepsilon_{k_2} p_{i_2}}{z_{k_2 i_2}} \cdots \prod_{2 \leqslant k < l \leqslant N} |z_{kl}|^{2\alpha' p_k p_l}. \end{aligned} \tag{7}$$

Hence effectively, each i takes $N-3$ possible values and a correlator involving a product of m ∂X's yields $(N-3)^m$ terms. The above correlator includes terms with double poles, $(z_{ij})^{-2}$ which, in bosonic string theory, would give rise to tachyonic singularities. Such terms, however, cancel after including the correlation functions arising from the fermionic parts of the vertices, *cf.* Eq. (2). Actually, fermion correlators cancel not only double poles $(z_{ij} z_{ji})^{-1}$ but also all terms involving longer cycles, $(z_{ij} z_{jk} \cdots z_{mi})^{-1}$. In this way, in N-gluon amplitudes, one ends up with the integrals of the form (4) but without any closed cycles, *i.e.* all integrals involving closed loops of indices, like $(i_1 i_2)(i_2 i_3) \cdots (i_k i_1)$, are eliminated. There are $(N-2)^{(N-4)}$ integrals remaining.

It seems that the calculations following the proposal of Berkovits and Maldacena yield integrals with one factor of z_{ij}^{-1} less than in the standard RNS formalism, *cf.* Eqs. (5) and (6). From now on, we insert the "missing" factor of $1 = z_{32} = -z_{23}$, so that in both formalisms, all N-gluon integrals (4) contain $n/2 = N-2$ brackets.

The integrals (4) can be represented by diagrams. To each z_i, $i = 2, \ldots, N$, we associate a point and to each bracket, *i.e.* to each $(z_{ij})^{-1}$ factor, we associate a link. Since there are no loops of indices, all integrals under consideration can be represented by tree diagrams with $N-2$ links. Note that any point repeating more than two times will produce a branching. As an example, we draw below a typical diagram contributing to $N = 8$ amplitudes:



$$= \big[\!\big[(42)(52)(32)(87)(76)(62)\big]\!\big]_8. \tag{8}$$

We introduced arrows to indicate the ordering of indices inside brackets. Integrals associated to tree diagrams will be called *transcendental*, for the reasons explained below.

Some properties of the integrals (4) become more transparent after changing the integration variables according to $z_4 = 1/x_1, z_5 = 1/(x_1 x_2), \ldots, z_N = 1/(x_1 x_2 \cdots x_{N-3})$ [5]. We obtain

[1] Our notation follows Ref. [11].

$$\begin{aligned}&[\![(i_1 i_2)(i_3 i_4)\cdots(i_{n-1} i_n)]\!]_N\\&\quad=\left(\prod_{i=1}^{N-3}\int_0^1 dx_i\right)\prod_{a=1}^{N-3} x_a^{\alpha' s_{23\cdots a+2}+n_a}\\&\qquad\times\prod_{b=a}^{N-3}\left(1-\prod_{j=a}^{b}x_j\right)^{\alpha' s_{a+2,b+3}+n_{a,b}},\end{aligned}\tag{9}$$

depending on the kinematic invariants $s_{i,j}\equiv s_{ij}=(p_i+p_j)^2$ and $s_{i_1\cdots i_l}=(p_{i_1}+\cdots+p_{i_l})^2$. The integers n_a and n_{ab} are determined by

$$\begin{aligned}&n_a=1+a-N-\sum_{j=3+a}^{N}\sum_{i=2}^{j-1}\tilde{n}_{ij},\\&n_{a,b}=\tilde{n}_{a+2,b+3},\quad a=1,\ldots,N-3,\ b=a,\ldots,N-3,\end{aligned}\tag{10}$$

where $\tilde{n}_{ij}=-1$ for each link (ij), otherwise $\tilde{n}_{ij}=0$. The integrals (9) represent generalized Euler integrals, which integrate to multiple Gaussian hypergeometric functions [3]. They depend parametrically on the kinematic invariants, which are constrained by the momentum conservation law and the mass-shell condition $p_i^2=0$.

The representation (9) is particularly suitable for studying the low-energy $\alpha'\to 0$ limit of the integrals. For N gluons, the leading terms $\mathcal{O}[(\alpha')^{3-N}]$ yield the Yang–Mills limit of the amplitude, with the kinematic singularities associated to massless gauge bosons propagating in intermediate channels. We call the integrals associated to tree diagrams "transcendental" because their low-energy expansions are rather special: the powers of α' are always accompanied by one zeta function or products thereof with a fixed "degree of transcendentality" DT. For the objects of interest, the latter (sometimes called transcendentality level) is defined as [13]

$$\begin{aligned}&DT(\pi)=1,\qquad DT\big(\zeta(n)\big)=n,\\&DT\big(\zeta(n_1,\ldots,n_r)\big)=\sum_{l=1}^{r}n_l,\end{aligned}\tag{11}$$

and $DT(x\cdot y)=DT(x)+DT(y)$ for products. The expansions of (9) have the form

$$\begin{aligned}&[\![(i_1 i_2)(i_3 i_4)\cdots(i_{n-1} i_n)]\!]_N\\&\quad=\big(\alpha'\big)^{3-N}p_{3-N}+\big(\alpha'\big)^{5-N}p_{5-N}\zeta(2)\\&\qquad+\big(\alpha'\big)^{6-N}p_{6-N}\zeta(3)+\cdots,\end{aligned}\tag{12}$$

where p_l are degree l homogenous rational functions of the kinematic invariants $s_{i\ldots j}$. In (12) at each order $5-N+m$ in α' only products of zeta functions of total degree of transcendentality $DT=m+2$ show up. For $N\geqslant 5$ multiple zeta values $\zeta(n_1,\ldots,n_r)$ of depth greater than one ($r>1$) occur at weight eight and higher [14].

In Refs. [5,6,9] evidence has been given, that any integral (9) can be expressed in terms of $(N-3)!$ basis integrals (all referring to the *same* color ordering) with coefficients being rational functions of the kinematic invariants. This is to be contrasted with recent results on string amplitude relations [15,16], which express a given subamplitude in terms of a basis of $(N-3)!$ other subamplitudes (of *different* color ordering). However, in [9] arguments are given, that these two $(N-3)!$-dimensional bases are actually complementary. In fact, here we confirm the conclusions of [5,6,9].

In order to cast the N-gluon amplitude in a simplest possible form, the $(N-2)^{(N-4)}$ transcendental integrals, remaining after combining bosonic and fermionic contractions, should be expressed in a suitable basis. To that end, we introduce the "chain" basis $\{\mathscr{C}_\sigma^N\}$ labeled by $(N-3)!$ permutations σ of the set $\{4,5,\ldots,N\}$:

$$\begin{aligned}\mathscr{C}_\sigma^N&=\overset{2}{\bullet}\!\longrightarrow\!\overset{3}{\bullet}\!-\!\!-\!\!-\overset{4_\sigma}{\bullet}\cdots\cdots\overset{(N-1)_\sigma}{\bullet}\!-\!\!-\!\!-\overset{N_\sigma}{\bullet}\\&=[\![(23)(34_\sigma)\cdots\big((N-1)_\sigma N_\sigma\big)]\!]_N\end{aligned}\tag{13}$$

where $i_\sigma\equiv\sigma(i)$. Expressing $(N-2)^{(N-4)}$ functions in this particular chain basis is a purely algebraic operation involving partial fractioning only; no partial integrations are necessary to accomplish it. After this step, followed by some spinor algebra, we obtain

$$\begin{aligned}A_N=&\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\left(\prod_{k=4}^{N}\langle 2k\rangle\right)^{-1}\sum_\sigma\mathscr{C}_\sigma^N\langle 2|3|4_\sigma]\langle 2|3+4_\sigma|5_\sigma]\\&\cdots\langle 2|3+4_\sigma+\cdots+(N-1)_\sigma|N_\sigma].\end{aligned}\tag{14}$$

The amplitude (14) contains poles at $\langle 2k\rangle=0$, originating from the polarization vectors associated to the gauge choice $\varepsilon_k p_2=0$: $\varepsilon_k^\mu=\sigma^\mu_{a\dot{a}}\lambda_2^a\tilde{\lambda}_k^{\dot{a}}/\langle 2k\rangle$. The gauge invariance of the amplitude, demonstrated in Ref. [10], guarantees that these singularities are spurious. Actually, depending on the gauge choice and the choice of basis, the amplitude can be rewritten in various ways, with different spurious singularities. One can ask if there exists a basis in which the amplitude is manifestly free of such singularities. Eq. (6) shows that it does exist for $N=6$. We addressed this question in the case of $N=7$. Indeed, spurious singularities can be eliminated by manipulating Eq. (14), leaving physical poles only, say at $\langle 34\rangle=0$, $\langle 56\rangle=0$, $\langle 67\rangle=0$. Such manipulations, however, in particular the partial integrations, destroy the symmetry of chain basis and the resultant expression is not as simple as Eq. (14).

An important new insight into the structure of N-gluon tree-level Yang–Mills amplitudes has been recently gained by rewriting them in terms of the "position" coordinates dual to momentum variables. These coordinates are defined implicitly by $p_k=x_k-x_{k-1}$, with the momentum conservation expressed by $x_0=x_N$. Then the amplitudes exhibit a "dual" superconformal symmetry [17]. In 2009, Andrew Hodges [18] introduced "momentum" twistors associated to the dual superconformal group. Tree-level Yang–Mills amplitudes become manifestly covariant when written in terms of momentum twistor variables. No such symmetry is expected to hold for full-fledged superstring amplitudes because dilatational symmetry is manifestly violated by the dependence on α'. Nevertheless we can try to express Eq. (14) in terms of momentum-twistors, in order to compare Yang–Mills with the full-fledged superstring amplitudes.

The momentum-twistors are defined as $Z_k=(\lambda_k,\tilde{\mu}_k{=}\lambda_k x_k)$ and the dual momentum-twistors as $W_k=(\mu_k{=}x_k\tilde{\lambda}_k,\tilde{\lambda}_k)$. With the choice of $x_1=0$ as the origin of the dual coordinate space, $\mu_1=\tilde{\mu}_1=\mu_2=\tilde{\mu}_2=0$ and $x_n=\sum_{k=2}^{n}p_k$. Now the N-gluon amplitude (14) can be rewritten as

$$\begin{aligned}A_N=&\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\left(\prod_{k=4}^{N}\langle 2k\rangle\right)^{-1}\sum_\sigma\mathscr{C}_\sigma^N\big(Z_2W_4^\sigma\big)\big(Z_2W_5^\sigma\big)\\&\cdots\big(Z_2W_N^\sigma\big).\end{aligned}\tag{15}$$

To summarize, we derived a simple formula, given in Eqs. (14) and (15), for the superstring MHV amplitude at the disk level. It is possible that the twistor-dependence of kinematic factors is purely coincidental. On the other hand, it may point to a deeper role of momentum-twistors in superstring theory.

Acknowledgements

St.St. is grateful to Carlos Mafra and Oliver Schlotterer for valuable discussions. T.R.T. thanks Jacob Bourjaily and James Drummond for useful discussions and correspondence. Both authors are grateful to the Theory Division of CERN for hospitality and financial support during various stages of this work. This material is based in part upon work supported by the National Science Foundation under Grant No. PHY-0757959. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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