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Nuclear Physics B 883 (2014) 598–614

NUCLEAR PHYSICS B

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Electric charges and magnetic monopoles in Gravity's Rainbow

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Received 24 February 2014; accepted 8 April 2014

Available online 13 April 2014

Editor: Stephan Stieberger

Abstract

In this work, we explore the possibility that quantum fluctuations induce an electric or magnetic charge or both, in the context of Gravity's Rainbow. A semi-classical approach is adopted, where the graviton one-loop contribution to a classical energy in a background spacetime is computed through a variational approach with Gaussian trial wave functionals. The energy density of the graviton one-loop contribution, in this context, acts as a source for the electric/magnetic charge. The ultraviolet (UV) divergences, which arise analyzing this procedure, are kept under control with the help of an appropriate choice of the Rainbow's functions. In this way we avoid the introduction of any regularization/renormalization scheme. A comparison with the observed data leads us to determine the size of the electron and of the magnetic monopole which appear to be of Planckian size. Both results seem to be of the same order for a Schwarzschild and a de Sitter background, respectively. Estimates on the magnetic monopole size have been done with the help of the Dirac quantization procedure. We find that the monopole radius is larger than the electron radius. Even in this case the ratio between the electric and magnetic monopole radius appears to be of the same order for both geometries.

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http://dx.doi.org/10.1016/j.nuclphysb.2014.04.005

1. Introduction

It was Andrei Sakharov in 1967 [1] who first conjectured the idea of *Induced gravity* (or emergent gravity), namely spacetime background emerges as a mean field approximation of underlying microscopic degrees of freedom, similarly to the fluid mechanics approximation of Bose–Einstein condensates. This means that some basic ingredients of General Relativity like the gravitational Newton's constant can be computed by means of quantum fluctuations of some matter fields. This idea is opposed to the concept of "*charge without charge*" and "*mass without mass*" arising from the *spacetime foam* picture of John A. Wheeler [2], where the matter properties emerge as a geometrical feature of spacetime. In a *foamy spacetime* topological fluctuation appear at the Planck scale, meaning that spacetime itself undergoes a deep and rapid transformation in its structure. Wheeler also considered wormhole-type solutions as objects of the quantum spacetime foam connecting different regions of spacetime at the Planck scale. Although the Sakharov approach has the appealing property of being renormalizable "*ab initio*" because it involves only quantum fluctuations of matter fields described by bosons and fermions, the Wheeler picture involves quantum fluctuations of the gravitational field alone and since one of the purposes of Quantum Gravity should be a realization of a theory combining Quantum Field Theory with General Relativity, it appears that spacetime foam is the right candidate for such a description. Unfortunately, every proposal of Quantum Gravity except string theory has to face with Ultra Violet (UV) divergences. Recently a proposal which uses a distortion of the gravitational field at the Planck scale, named as *Gravity's Rainbow* [3,4] has been considered to compute Zero Point Energy (ZPE) to one loop [5,6]. The interesting point is that such a distortion enters into the background metric and becomes active at Planck's scale keeping under control UV divergences. Briefly, the situation is the following: one introduces two arbitrary functions $g_1(E/E_P)$ and $g_2(E/E_P)$, denoted as *Rainbow's functions*, with the only assumption that

$$\lim_{E/E_P\to 0} g_1(E/E_P)=1 \quad \text{and} \quad \lim_{E/E_P\to 0} g_2(E/E_P)=1. \tag{1}$$

On a general spherical symmetric metric such functions come into play in the following manner

$$ds^2=-N^2(r)\frac{dt^2}{g_1^2(E/E_P)}+\frac{dr^2}{g_2^2(E/E_P)(1-\frac{b(r)}{r})}+\frac{r^2}{g_2^2(E/E_P)}\left(d\theta^2+\sin^2\theta\, d\phi^2\right), \tag{2}$$

where $N(r)$ is the lapse function, $b(r)$ is denoted as the shape function and E_P is the Planck energy. The purpose of this paper is to approach one of the aspects of Wheeler's ideas, namely "*charge without charge*". In particular, we will investigate if quantum fluctuations of the gravitational field can be considered as a source for the electric/magnetic charge. Note that a similar approach to realize "*charge without charge*" has been described in Ref. [7]. However due to UV divergences a regularization/renormalization was used to obtain finite results and if a renormalization group like equation has been used, the final result would depend on the renormalization point scale μ_0. Here the renormalization point is fixed at the Planck scale due the Rainbow's functions. It is clear that, if an electric/magnetic charge can be generated, this information is encoded in the Einstein's field equations. These equations are simply summarized by

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\kappa T_{\mu\nu}, \tag{3}$$

where

$$T_{\mu\nu}=\frac{1}{4\pi}\left[F_{\mu\gamma}F_{\nu}^{\gamma}-\frac{1}{4}g_{\mu\nu}F_{\gamma\delta}F^{\gamma\delta}\right] \tag{4}$$

is the energy–momentum tensor of the electromagnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\kappa = 8\pi G$, with G the Newton's constant and here we have neglected the contribution of the cosmological constant Λ_c. The electromagnetic field strength tensor $F_{\mu\nu}$ can be computed with the help of the electromagnetic potential A_μ which, in the case of a pure electric field assumes the form $A_\mu = (Q_e/r, 0, 0, 0)$ while in the case of pure magnetic field, the form is $A_\mu = (0, 0, 0, -Q_m \cos\theta)$. Q_e and Q_m are the electric and magnetic charge respectively. It is interesting to note that Q_e and Q_m contribute in the same way to the electromagnetic Hamiltonian density. Indeed, for the electric charge, the on-shell contribution of $T_{\alpha\beta}u^\alpha u^\beta$ is

$$T_{\mu\nu}u^\mu u^\nu = \frac{1}{8\pi}(F_{01})^2 = \frac{1}{8\pi}\frac{Q_e^2}{r^4} = \rho_e, \tag{5}$$

and when we consider the magnetic charge, we get

$$T_{\mu\nu}u^\mu u^\nu = \frac{1}{8\pi}(F_{23})^2 = \frac{1}{8\pi}\frac{Q_m^2}{r^4} = \rho_m. \tag{6}$$

u^μ is a time-like unit vector such that $u \cdot u = -1$. However, while the electric charge exists, for the magnetic charge or magnetic monopole, there is no experimental evidence for its existence.[1] The magnetic monopole search has a long history in theoretical physics: predicted by Paul Dirac in 1931, he showed that QED allows the existence of point-like magnetic monopole with charge

$$Q_m = \frac{2\pi}{Q_e} \tag{7}$$

or an integer multiple of it [10]. Subsequently this prediction was also confirmed by Gerard 't Hooft and Alexander Polyakov who showed that magnetic monopoles are predicted by all Grand Unified Theories (GUTs) [11]. Although monopoles of grand unified theories would have masses typically of the order of the unification scale ($m \sim 10^{16}$ GeV) but generally there are no tight theoretical constraints on the mass of a monopole. For this reason, the reference value of our calculation will be that of the electric charge. It is important to remark that in a system of units in which $\hbar = c = k = 1$, that will be used throughout the paper[2]

$$e^2 = \frac{1}{137}. \tag{9}$$

The rest of the paper is organized in the following manner. In Section 2 we introduce the charge operator, in Section 3 we introduce the charge operator in presence of Gravity's Rainbow specified to the Schwarzschild and to the de Sitter metric, in Section 4 we will apply the charge operator to the magnetic monopole case and in Section 5 we will summarize and conclude.

2. The charge operator

To build the charge operator, we have to recognize the gravitational field as a fundamental field and see what implications we have on Q_e and Q_m. For example, in Ref. [12], the rôle of Q_e

[1] Recently, it has been discovered that spin ices, frustrated magnetic systems, have effective quasiparticle excitations with magnetic charges very close to magnetic monopoles [9].

[2] For example, in SI units

$$\frac{e^2}{4\pi\hbar c\epsilon_0} = \frac{1}{137}. \tag{8}$$

and Q_m has been played by a cosmological constant interpreted as an eigenvalue of an associated Sturm–Liouville problem. To do this, we have introduced the Wheeler–DeWitt equation (WDW) [13] by rearranging the Einstein's field equations, to get:

$$\mathcal{H}_\Lambda = (2\kappa) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa}\left({}^3R - 2\Lambda_c\right) = 0, \tag{10}$$

for the sourceless case and in presence of a cosmological term.

$$\mathcal{H}_Q = (2\kappa) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa}\left({}^3R - \mathcal{H}_M\right) = 0, \tag{11}$$

with a matter term and in absence of a cosmological constant. Note the formal similarity between Eqs. (10) and (11). G_{ijkl} is the *supermetric* defined as

$$G_{ijkl} = \frac{1}{2\sqrt{g}}(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl}) \tag{12}$$

and ${}^3R$ is the scalar curvature in three dimensions. π^{ij} is called the supermomentum. This is the time–time component of the Einstein's field equations. It represents the invariance under *time* reparametrization and it works as a constraint at the classical level. Fixing our attention on the constraint (11), the explicit form of $\mathcal{H}_M$ is easily obtained with the help of Eqs. (5) and (6), where one finds

$$\mathcal{H}_M = 2\kappa T_{\alpha\beta}u^\alpha u^\beta = \frac{\kappa}{4\pi}\frac{Q_e^2 + Q_m^2}{r^4}. \tag{13}$$

Thus, the classical constraint $\mathcal{H}_Q$ becomes

$$\mathcal{H}_Q = (2\kappa) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa}\left({}^3R - \frac{\kappa}{4\pi}\frac{Q_e^2 + Q_m^2}{r^4}\right) = 0. \tag{14}$$

For a spherically symmetric metric described by (2) with $g_1(E/E_P) = g_2(E/E_P) = 1$, it is easy to recognize that the classical constraint reduces to

$${}^3R = 2G\frac{Q_e^2 + Q_m^2}{r^4} \quad \Longrightarrow \quad b'(r) = G\frac{Q_e^2 + Q_m^2}{r^2}, \tag{15}$$

whose solution represents the Reissner–Nordström (RN) metric if

$$N^2(r) = \left[1 - \frac{b(r)}{r}\right]^{-1} \tag{16}$$

and

$$b(r) = 2MG - \frac{G(Q_e^2 + Q_m^2)}{r}. \tag{17}$$

On the other hand, changing the point of view, one could fix the background to see if there are other combinations solving the classical constraint (15). For example, if one fixes the background metric to be the Schwarzschild metric, one finds that the only solution compatible with the classical constraint is the trivial solution $Q_e = Q_m = 0$. The same situation happens for the de Sitter (dS) and Anti-de Sitter (AdS) metric. Things can change if we consider quantum fluctuations of the gravitational field. Indeed, these can be a source of nontrivial solutions as shown in Ref. [7]. To this purpose, we promote $\mathcal{H}_Q$ to be an operator and the WDW equation in the presence of an electromagnetic field becomes

$$\mathcal{H}_Q\Psi=\left[(2\kappa)G_{ijkl}\pi^{ij}\pi^{kl}-\frac{\sqrt{g}}{2\kappa}\left({}^3R-\frac{\kappa}{4\pi}\frac{Q_e^2+Q_m^2}{r^4}\right)\right]\Psi=0. \tag{18}$$

The WDW equation can be cast into the form

$$\hat{Q}_\Sigma\Psi[g_{ij}]=-\frac{\sqrt{g}}{8\pi r^4}\left(Q_e^2+Q_m^2\right)\Psi[g_{ij}], \tag{19}$$

where

$$\hat{Q}_\Sigma=(2\kappa)G_{ijkl}\pi^{ij}\pi^{kl}-\frac{\sqrt{g}}{2\kappa}{}^3R \tag{20}$$

is the charge operator. Now we see that this equation formally looks like an eigenvalue equation. To further proceed, we multiply Eq. (19) by $\Psi^*[g_{ij}]$ and we functionally integrate over the three spatial metric g_{ij}, to obtain

$$\int\mathcal{D}[g_{ij}]\Psi^*[g_{ij}]\hat{Q}_\Sigma\Psi[g_{ij}]=-\frac{1}{8\pi}\int\mathcal{D}[g_{ij}]\Psi^*[g_{ij}]\left(\sqrt{g}\frac{Q_e^2+Q_m^2}{r^4}\right)\Psi[g_{ij}]. \tag{21}$$

Finally one can formally re-write the WDW equation as

$$\frac{\langle\Psi|\int_\Sigma d^3x\,\hat{Q}_\Sigma|\Psi\rangle}{\langle\Psi|\Psi\rangle}=-\frac{Q_e^2+Q_m^2}{8\pi}\frac{\langle\Psi|\int_\Sigma d^3x(\sqrt{g}/r^4)|\Psi\rangle}{\langle\Psi|\Psi\rangle}, \tag{22}$$

where we have integrated over the hypersurface Σ. The l.h.s. of Eq. (22) can be interpreted as an expectation value and the r.h.s. can be regarded as the associated eigenvalue with a weight. In principle, one should expand in perturbations even the determinant to one loop. This means that

$$\frac{\langle\Psi|\int_\Sigma d^3x\,\hat{Q}_\Sigma|\Psi\rangle}{\langle\Psi|\Psi\rangle}=-\frac{Q_e^2+Q_m^2}{8\pi}\frac{\langle\Psi|\int_\Sigma d^3x\frac{\sqrt{g}^{(0)}+\sqrt{g}^{(1)}+\sqrt{g}^{(2)}+\ldots+\sqrt{g}^{(n)}}{r^4}|\Psi\rangle}{\langle\Psi|\Psi\rangle}, \tag{23}$$

where $\sqrt{g}^{(n)}$ is the order of the approximation. However one may also adopt an alternative approach, where one fixes the background on the l.h.s. of Eq. (22) and consequently let the quantum fluctuations evolve, and then one verifies what kind of solutions one can extract from the r.h.s. in a recursive way. Therefore the first step begins with the r.h.s. of Eq. (22) which further can be reduced to

$$\frac{\langle\Psi|\int_\Sigma d^3x\,\hat{Q}_\Sigma|\Psi\rangle}{\langle\Psi|\Psi\rangle}=-\frac{Q_e^2+Q_m^2}{8\pi}\int_\Sigma d^3x\frac{\sqrt{g}^{(0)}}{r^4}. \tag{24}$$

If Eq. (24) gives the desired nontrivial eigenvalues at zero order, it means that an electric or magnetic charge (or both) has been created. This means that after the charge creation, the correct background will be represented by a Reissner–Nordström metric. This also means that Eq. (24) cannot be used anymore. In the next section we will discuss some subtleties arising in dealing with quantum fluctuations of the determinant of the r.h.s. of Eq. (22). If we consider $Q_e^2(Q_m^2)$ as eigenvalues of the Sturm–Liouville problem for some fixed background, we unavoidably find that the one loop calculation is plagued by UV divergences. Therefore a regularization/renormalization scheme is needed to remove the divergences [7]. Nevertheless, the purpose of this paper is to propose a procedure to avoid such a scheme: the computation of $Q_e^2(Q_m^2)$ in presence of Gravity's Rainbow which introduces only one scale, the Planck scale E_P.

3. The charge operator in presence of Gravity's Rainbow

To compute the electric/magnetic charge in Gravity's Rainbow, we begin with the line element (2). The form of the background is such that the *shift function*

$$N^i = -Nu^i = g_0^{4i} = 0 \tag{25}$$

vanishes, while N is the previously defined *lapse function*. Thus the definition of K_{ij} implies

$$K_{ij} = -\frac{\dot{g}_{ij}}{2N} = \frac{g_1(E)}{g_2^2(E)}\tilde{K}_{ij}, \tag{26}$$

where the dot denotes differentiation with respect to the time t and the tilde indicates the quantity computed in absence of Rainbow's functions $g_1(E)$ and $g_2(E)$. For simplicity, we have set $E_P = 1$ in $g_1(E/E_P)$ and $g_2(E/E_P)$ but later we will bring it back for relative comparison. The trace of the extrinsic curvature, therefore becomes

$$K = g^{ij}K_{ij} = g_1(E)\tilde{K} \tag{27}$$

and the momentum π^{ij} conjugate to the three-metric g_{ij} of Σ is

$$\pi^{ij} = \frac{\sqrt{g}}{2\kappa}\left(Kg^{ij} - K^{ij}\right) = \frac{g_1(E)}{g_2(E)}\tilde{\pi}^{ij}. \tag{28}$$

Recalling that $u_\mu = (-N, 0, 0, 0)$, in presence of Gravity's Rainbow we have the following modification

$$u_\mu = \frac{\tilde{u}_\mu}{g_1(E)} \quad \Longrightarrow \quad u^\mu = g_1(E)\tilde{u}^\mu \tag{29}$$

which is useful to compute the distorted electromagnetic energy–momentum tensor. Indeed, from Eqs. (5) and (6), we find

$$T_{\mu\nu}u^\mu u^\nu = \frac{g_2^2(E)\tilde{g}^{11}}{8\pi}(\tilde{F}_{01})^2\tilde{u}^\mu\tilde{u}^\nu g_1^2(E) = \frac{1}{8\pi}\frac{Q_e^2}{r^4}g_1^2(E)g_2^2(E), \tag{30}$$

for the electric charge, while when we consider the magnetic charge, we get

$$T_{\mu\nu}u^\mu u^\nu = \frac{\tilde{g}_{00}}{8\pi g_1^2(E)}(\tilde{F}_{23})^2\tilde{u}^\mu\tilde{u}^\nu\tilde{g}^{22}\tilde{g}^{33}g_1^2(E)g_2^4(E) = \frac{1}{8\pi}\frac{Q_m^2}{r^4}g_2^4(E). \tag{31}$$

Since the scalar curvature R has the following property

$$R = g^{ij}R_{ij} = g_2^2(E)\tilde{R}, \tag{32}$$

we find that the WDW equation becomes

$$\mathcal{H}\Psi = \left[(2\kappa)\frac{g_1^2(E)}{g_2^3(E)}\tilde{G}_{ijkl}\tilde{\pi}^{ij}\tilde{\pi}^{kl} - \frac{\sqrt{\tilde{g}}}{2\kappa g_2(E)}\left(\tilde{R} - \frac{\kappa}{4\pi r^4}Q^2_{eg_1;mg_2}\right)\right]\Psi = 0, \tag{33}$$

where we have defined

$$Q^2_{eg_1;mg_2} = Q_e^2 g_1^2(E) + Q_m^2 g_2^2(E) \tag{34}$$

and where

$$G_{ijkl} = \frac{1}{2\sqrt{g}}(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl}) = \frac{\tilde{G}_{ijkl}}{g_2(E)}. \tag{35}$$

By repeating the same steps that have led to Eq. (22), we find

$$\frac{\langle\Psi| \int_\Sigma d^3x\, \hat{Q}_\Sigma |\Psi\rangle}{\langle\Psi|\Psi\rangle} = -\frac{1}{8\pi}\frac{g_1^2(E)}{g_2(E)}\frac{\langle\Psi| \int_\Sigma d^3x (Q^2_{eg_1;mg_2}\sqrt{\tilde{g}}/r^4)|\Psi\rangle}{\langle\Psi|\Psi\rangle}, \tag{36}$$

where we have defined the distorted charge operator

$$\hat{Q}_\Sigma = 2\kappa \frac{g_1^2(E)}{g_2^3(E)} \tilde{G}_{ijkl}\, \tilde{\pi}^{ij}\, \tilde{\pi}^{kl} - \frac{\sqrt{\tilde{g}}}{2\kappa g_2(E)}\, \tilde{R}. \tag{37}$$

Since Eq. (36) as well as Eq. (22) cannot be solved exactly, we adopt a variational procedure with trial wave functionals of the Gaussian type. To further proceed, we fix a background metric $\bar{g}_{ij}$ and we consider quantum fluctuation around the background of the form $g_{ij} = \bar{g}_{ij} + h_{ij}$. Following the procedure in Ref. [5], we canonically separate the degrees of freedom and since only the transverse traceless (TT) tensor contribution becomes relevant, we find

$$\hat{Q}_\Sigma = \frac{1}{4}\int_\Sigma d^3x \sqrt{\tilde{\bar{g}}}\, \tilde{G}^{ijkl}\left[2\kappa \frac{g_1^2(E)}{g_2^3(E)} \tilde{K}^{-1\perp}(x,x)_{ijkl} + \frac{1}{2\kappa g_2(E)}\left\{\tilde{\Delta}_L^m \tilde{K}^\perp(x,x)\right\}_{ijkl}\right], \tag{38}$$

where we have functionally integrated over Gaussian trial wave functionals. $\tilde{\Delta}_L^m$ represents the modified Lichnerowicz operator whose expression is

$$(\hat{\Delta}_L^m h^\perp)_{ij} = (\Delta_L h^\perp)_{ij} - 4R_i^k h_{kj}^\perp + {}^3R h_{ij}^\perp. \tag{39}$$

Now when we consider the eigenvalue equation

$$(\hat{\Delta}_L^m h^\perp)_{ij} = E^2 h_{ij}^\perp \tag{40}$$

we find

$$(\tilde{\Delta}_L^m \tilde{h}^\perp)_{ij} = \frac{E^2}{g_2^2(E)}\tilde{h}_{ij}^\perp \tag{41}$$

and the propagator $K^\perp(x,y)_{iakl}$ can be represented as

$$K^\perp(\vec{x},\vec{y})_{iakl} = \tilde{K}^\perp(\vec{x},\vec{y})_{iakl} = \sum_\tau \frac{\tilde{h}_{ia}^{(\tau)\perp}(\vec{x})\tilde{h}_{kl}^{(\tau)\perp}(\vec{y})}{2\lambda(\tau)g_2^4(E)} \tag{42}$$

where $\tilde{h}_{ia}^{(\tau)\perp}(\vec{x})$ are the eigenfunctions of $\tilde{\Delta}_L^m$. τ denotes a complete set of indices and $\lambda(\tau)$ are a set of variational parameters to be determined by the minimization of Eq. (38). The expectation value of $\hat{Q}_\Sigma^\perp$ is obtained by plugging the propagator in Eq. (38) and minimizing with respect to the variational function $\lambda(\tau)$. Therefore the one-loop charge in Gravity's Rainbow for the TT tensors is

$$Q_\Sigma = -\frac{1}{2}\sum_\tau g_1(E)g_2(E)\left[\sqrt{E_1^2(\tau)} + \sqrt{E_2^2(\tau)}\right], \tag{43}$$

where

$$Q_\Sigma = \frac{1}{8\pi}\frac{g_1^2(E)}{g_2(E)}\int_\Sigma d^3x\,\sqrt{{}^3\tilde{g}}\,\frac{Q^2_{eg_1;mg_2}}{r^4}. \tag{44}$$

It is important to remark that if we had considered quantum fluctuations of the r.h.s. of Eq. (22), then the r.h.s. of Eq. (41) would have been modified with the introduction of the charge term which is possible only when Eq. (24) is solved. The expression in Eq. (43) makes sense when $E_i^2(\tau) > 0$, where E_i^2 are the eigenvalues of $\tilde{\Delta}_L^m$. Using the WKB approximation as used by 't Hooft in the brick wall problem we can evaluate Eq. (43) explicitly. Extracting the energy density, we can write

$$\frac{1}{2}\frac{g_1^2(E)}{g_2(E)}\frac{Q^2_{eg_1;mg_2}}{r^4} = -\frac{1}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{\infty} E_i g_1(E)g_2(E)\frac{d}{dE_i}\left[\frac{E_i^2}{g_2^2(E)} - m_i^2(r)\right]^{\frac{3}{2}} dE_i, \tag{45}$$

where E^* is the value which annihilates the argument of the root and where we have defined two r-dependent effective masses $m_1^2(r)$ and $m_2^2(r)$

$$\begin{cases} m_1^2(r) = \dfrac{6}{r^2}\left(1 - \dfrac{b(r)}{r}\right) + \dfrac{3}{2r^2}b'(r) - \dfrac{3}{2r^3}b(r) \\ m_2^2(r) = \dfrac{6}{r^2}\left(1 - \dfrac{b(r)}{r}\right) + \dfrac{1}{2r^2}b'(r) + \dfrac{3}{2r^3}b(r) \end{cases} \quad \big(r \equiv r(x)\big). \tag{46}$$

We have hitherto used a generic form of the background. We now fix the attention on some backgrounds which have the following property

$$m_0^2(r) = m_2^2(r) = -m_1^2(r), \quad \forall r \in (r_t, r_1). \tag{47}$$

For example, the Schwarzschild background represented by the choice $b(r) = r_t = 2MG$ satisfies the property (47) in the range $r \in [r_t, 5r_t/4]$. Similar backgrounds are the Schwarzschild–de Sitter and Schwarzschild–Anti-de Sitter. On the other hand, other backgrounds, like dS, AdS and Minkowski have the property

$$m_0^2(r) = m_2^2(r) = m_1^2(r), \quad \forall r \in (r_t, \infty). \tag{48}$$

3.1. *The Schwarzschild case*

Before going on, we examine the classical constraint for the Schwarzschild metric. From Eq. (33), the condition $\mathcal{H} = 0$ reduces to

$$\tilde{R} - \kappa\frac{Q^2_{eg_1;mg_2}}{4\pi r^4} = 0 \quad \Longrightarrow \quad \frac{Q_e^2}{r^4}g_1^2(E) + \frac{Q_m^2}{r^4}g_2^2(E) = 0, \tag{49}$$

leading to the only trivial classical solution $Q_e = Q_m = 0$. Note that even in Minkowski space, we have a trivial solution with vanishing charges. This situation persists even at the quantum level, because there is no parameter which fixes the scale like the Schwarzschild mass can do. To further proceed, we observe that the Schwarzschild background satisfies condition (47) and Eq. (45) becomes

$$\frac{1}{2}\frac{g_1^2(E/E_P)}{g_2(E/E_P)}\frac{Q^2_{eg_1;mg_2}}{r^4} = -\frac{1}{3\pi^2}(I_+ + I_-), \tag{50}$$

where

$$I_{+}=3\int_{0}^{\infty}E^{2}g_{1}(E/E_{P})\sqrt{\frac{E^{2}}{g_{2}^{2}(E/E_{P})}+m_{0}^{2}(r)}\frac{d}{dE}\left(\frac{E}{g_{2}(E/E_{P})}\right)dE \tag{51}$$

and

$$I_{-}=3\int_{E^{*}}^{\infty}E^{2}g_{1}(E/E_{P})\sqrt{\frac{E^{2}}{g_{2}^{2}(E/E_{P})}-m_{0}^{2}(r)}\frac{d}{dE}\left(\frac{E}{g_{2}(E/E_{P})}\right)dE. \tag{52}$$

For convenience we have reintroduced the Planck energy scale E_p in Eqs. (51) and (52). It is clear that the final result is strongly dependent on the choices we can do about $g_1(E/E_P)$ and $g_2(E/E_P)$. Nevertheless, some classes of functions cannot be considered because they do not lead to a finite result. For example, fixing

$$g_1(E/E_P)=1-\eta(E/E_P)^n \quad \text{and} \quad g_2(E/E_P)=1, \tag{53}$$

with η a dimensionless parameter and n an integer [8], Eq. (50) does not lead to a finite result and therefore will be discarded. Other examples that we have to discard without involving a specific form of $g_1(E/E_P)$ and $g_2(E/E_P)$ are:

$$g_2(E/E_P)=g_1^4(E/E_P), \tag{54}$$

$$g_2(E/E_P)=g_1^{-2}(E/E_P) \tag{55}$$

and

$$g_2^{-2}(E/E_P)=g_1(E/E_P). \tag{56}$$

When we adopt the choice (54), the electric charge becomes independent on the Rainbow's functions and Eq. (50) becomes

$$\frac{1}{2r^4}\left(Q_e^2+Q_m^2g_1^6(E/E_P)\right)=-\frac{1}{3\pi^2}(I_{+}+I_{-}). \tag{57}$$

In order to have real results, the argument of the square root in I_- must be positive for $E\gg E_P$ and this happens when $g_1(E/E_P)$ is of the form

$$g_1(E/E_P)=\sqrt[4]{1+E/E_P} \tag{58}$$

leading to a divergent result. The same situation happens for choice (55) where the magnetic charge becomes independent on the Rainbow's functions

$$\frac{1}{2r^4}\left(Q_e^2g_1^6(E/E_P)+Q_m^2\right)=-\frac{1}{3\pi^2}(I_{+}+I_{-}). \tag{59}$$

To have real results for $E\gg E_P$ we have to impose

$$g_1(E/E_P)=(1+E/E_P)^{-\frac{1}{2}} \tag{60}$$

but also in this case I_+ and I_- diverge. Finally for the choice (56) we find that the integrals I_+ and I_- in Eq. (50) become finite but with a negative sign in front of the r.h.s. of Eq. (50) which means that Q_e^2 (Q_m^2) should be everywhere negative, a result which is not compatible with observation. Since the choices (54)–(56) do not give the desired result, we need to fix independently the form of $g_1(E/E_P)$ and $g_2(E/E_P)$. It is immediate to observe that $g_1(E/E_P)$ must have a shape such that I_+ and I_- be convergent. Following Ref. [5], we consider

$$g_1(E/E_P) = \left(1 + \beta \frac{E}{E_P}\right) \exp\left(-\alpha \frac{E^2}{E_P^2}\right) \qquad g_2(E/E_P) = 1 \quad \alpha, \beta \in \mathbb{R}. \tag{61}$$

In this configuration, we know that the integrals I_+ and I_- are finite and the introduction of the parameter β allows to change the sign in the one loop term. Note that the choice of the Gaussian was dictated by a comparison with a cosmological constant computation in the framework of Noncommutative theory [14]. By defining

$$x = \sqrt{\frac{m_0^2(r)}{E_P^2}} \tag{62}$$

and following the same steps of Ref. [5], one finds

$$\frac{Q_e^2 g_1^4(E/E_P) + Q_m^2 g_1^2(E/E_P)}{2E_P^4 r^4} = -\frac{1}{2\pi^2} f(\alpha, \beta; x), \tag{63}$$

where

$$\begin{aligned} f(\alpha, \beta; x) = \bigg[& \frac{x^2}{\alpha} \cosh\left(\frac{\alpha x^2}{2}\right) K_1\left(\frac{\alpha x^2}{2}\right) \\ & + \beta \left(\frac{3x}{2\alpha^2} - \frac{x^2 \sqrt{\pi}}{\alpha^{\frac{3}{2}}} \sinh(\alpha x^2) + \frac{3\sqrt{\pi}}{2\alpha^{\frac{5}{2}}} \cosh(\alpha x^2) \\ & + \frac{\sqrt{\pi}}{2\alpha^{\frac{3}{2}}} \left(x^2 - \frac{3}{2\alpha}\right) e^{\alpha x^2} \operatorname{erf}(\sqrt{\alpha} x) \right) \bigg] \end{aligned} \tag{64}$$

and where $K_1(x)$ is the Bessel function of the first kind and $\operatorname{erf}(x)$ is the error function. For the Schwarzschild background, Eq. (62) becomes

$$x = \sqrt{\frac{m_0^2(r)}{E_P^2}} = \begin{cases} \sqrt{\frac{3MG}{r^3 E_P^2}} & r > 2MG \\ \sqrt{\frac{3}{8(MG)^2 E_P^2}} & r = 2MG \end{cases} \tag{65}$$

and its behavior is

$$x \to \begin{cases} \infty & \text{when } M \to 0 \text{ for } r = 2MG, \\ 0 & \text{when } M \to 0 \text{ for } r > 2MG, \end{cases} \tag{66}$$

while

$$x \to \begin{cases} 0 & \text{when } M \to \infty \text{ for } r = 2MG, \\ \infty & \text{when } M \to \infty \text{ for } r > 2MG. \end{cases} \tag{67}$$

The behavior in Eq. (67) will be discarded, because it does not represent a physical realization. Therefore, we fix our attention on Eq. (66). For large x, the r.h.s. of Eq. (63) becomes:

$$\begin{aligned} \frac{g_1^2(E/E_P) Q_{eg_1;mg_2}^2}{2E_P^4 r^4} \simeq & -\frac{(2\beta\alpha^{3/2} + \sqrt{\pi}\alpha^2)x}{4\pi^2\alpha^{7/2}} - \frac{8\beta\alpha^{5/2} + 3\sqrt{\pi}\alpha^3}{16\pi^2\alpha^{11/2}x} \\ & + \frac{3}{128\pi^2} \frac{16\beta\alpha^{7/2} + 5\sqrt{\pi}\alpha^4}{\alpha^{15/2}x^3} + O\left(x^{-4}\right), \end{aligned} \tag{68}$$

while for small x we obtain

$$\frac{g_1^2(E/E_P)Q^2_{eg1;mg2}}{2E_P^4r^4} \simeq -\frac{4\alpha^{5/2}+3\sqrt{\pi}\beta\alpha^2}{4\pi^2\alpha^{9/2}} - \frac{(2\sqrt{\alpha}\gamma+2\sqrt{\alpha}\ln(\frac{x^2}{4}\alpha\sqrt{e})-2\sqrt{\pi}\beta)x^4}{16\pi^2\sqrt{\alpha}} - \frac{2\beta}{15\pi^2}x^5+O\left(x^7\right), \tag{69}$$

where γ is the Euler's constant. To keep the procedure as general as possible, in Eq. (63) we have kept the combination between the electric and magnetic charge coming from the energy–momentum tensor expressed in Eq. (34). It is also interesting to note that every choice we can do on the function $g_1(E/E_P)$ satisfying the assumption (61), the magnetic monopole is less suppressed in the trans-Planckian region with respect to the electric charge. This could confirm that the magnetic monopole is a problem related to the very early universe. On the other hand, when we are on the cis-Planckian region, the electric charge and the magnetic monopole are not suppressed by the Rainbow's functions and behave in the same way. For this reason, we can first study the electric charge setting $Q_m^2=0$. It is straightforward to see that if we fix

$$\beta=-\frac{\sqrt{\alpha\pi}}{2}, \tag{70}$$

then the linear divergent term of the asymptotic expansion (68) disappears and Eq. (63) vanishes for large x, namely when $r=r_t=2MG$ and $M\to 0$. Therefore on the throat r_t one gets

$$Q_e^2(\alpha,\beta,r_t)=-\frac{r_t^4E_P^4}{\pi^2}f\left(\alpha,-\frac{\sqrt{\alpha\pi}}{2};\frac{3}{2r_t^2E_P^2}\right). \tag{71}$$

By imposing that,

$$Q_e^2(\alpha,\beta,\bar{r}_t)=\frac{1}{137}=0.73\times 10^{-2}, \tag{72}$$

then we find

$$\bar{r}_t=0.295/E_P, \tag{73}$$

where we have fixed $\alpha=1/4$. The situation is not much different if we choose

$$\beta=-\frac{4}{3}\sqrt{\frac{\alpha}{\pi}}. \tag{74}$$

Indeed, Eq. (63) becomes

$$Q_e^2(\alpha,\beta,r_t)=-\frac{r_t^4E_P^4}{\pi^2}f\left(\alpha,-\frac{4}{3}\sqrt{\frac{\alpha}{\pi}};\frac{3}{2r_t^2E_P^2}\right) \tag{75}$$

and fixing again Q_e^2 like in Eq. (72), one finds

$$\bar{r}_t=0.571/E_P. \tag{76}$$

Both the solution require a sub-Planckian throat. It is clear that the comparison of the fine structure constant $1/137$ with Q_m^2 is not possible. However later we will discuss how the charge operator Q_Σ can give information about the magnetic monopole. If we move to the region where $5r_t/4>r>r_t$, we introduce a dependence on the radius r, which can be eliminated with the computation of

$$\frac{dQ_e^2}{dr} = 0. \tag{77}$$

However when we choose the parametrization (70), the solution of Eq. (77) is imaginary and therefore will be discarded. On the other hand, when we choose parametrization (74), we find that the expression (69) reduces to

$$Q_e^2 = -\frac{1}{4\pi^2}\ln\left(\frac{3MG}{4r^3E_P^2}\alpha e^{\gamma+11/6}\right)\frac{(3MG)^2}{r^2} + O\left((2MG)^{5/2}\right) \tag{78}$$

and the computation of Eq. (77) in this case leads to the following relationship

$$\bar{r}^3 = \frac{3r_t\alpha}{8E_P^2}\exp\left(\gamma + \frac{10}{3}\right). \tag{79}$$

Since $r \in [r_t, 5r_t/4]$, this implies that

$$r \in \left[\sqrt{\frac{3\alpha}{8E_P^2}\exp\left(\gamma + \frac{10}{3}\right)}, \frac{5}{4}\sqrt{\frac{3\alpha}{8E_P^2}\exp\left(\gamma + \frac{10}{3}\right)}\right], \tag{80}$$

then we find the following bound

$$2.9638 \times 10^{-2} = \frac{3^{\frac{8}{3}}}{64\pi^2} \geqslant Q_e^2(\alpha, \bar{r}) \geqslant \frac{3^{\frac{8}{3}}}{64\pi^2}\sqrt[3]{\left(\frac{5}{4}\right)^4} = 3.9908 \times 10^{-2}. \tag{81}$$

3.2. The de Sitter (Anti-de Sitter) case

Even in this case, we examine the classical constraint for the dS and AdS metric, respectively. For the AdS metric it is immediate to verify that the condition $\mathcal{H} = 0$ reduces to

$$\tilde{R} - \kappa\frac{Q^2_{eg_1;mg_2}}{4\pi r^4} = 0 \quad \Longrightarrow \quad G\left(\frac{Q_e^2}{r^4}g_1^2(E) + \frac{Q_m^2}{r^4}g_2^2(E)\right) = -\Lambda_{AdS}, \tag{82}$$

which is never satisfied, while for the dS metric, we find

$$\tilde{R} - \kappa\frac{Q^2_{eg_1;mg_2}}{4\pi r^4} = 0 \quad \Longrightarrow \quad G\left(\frac{Q_e^2}{r^4}g_1^2(E) + \frac{Q_m^2}{r^4}g_2^2(E)\right) = \Lambda_{dS}. \tag{83}$$

Moreover, if we fix the radius to the value $r = \sqrt{3/\Lambda_{dS}}$, we find

$$G\Lambda_{dS}\left(Q_e^2 g_1^2(E) + Q_m^2 g_2^2(E)\right) = 9, \tag{84}$$

which fixes the values of Q_e, Q_m and Λ_{dS} to values incompatible with observation. However, things can be different from the quantum point of view. Since in the dS and AdS cases, the condition (48) holds, Eq. (45) becomes

$$\frac{1}{2}\frac{g_1^2(E/E_P)}{g_2(E/E_P)}\frac{Q^2_{eg_1;mg_2}}{r^4} = -\frac{2}{3\pi^2}I_-, \tag{85}$$

where I_- is given by Eq. (52). Choosing the Rainbow's functions like in the Schwarzschild case, one finds

$$\frac{g_1^2(E/E_P)Q^2_{eg_1;mg_2}}{2E_P^4r^4} = -\frac{\beta}{4\alpha^{\frac{5}{2}}\pi^{\frac{3}{2}}}\left(3 + 2\alpha x^2\right)e^{-\alpha x^2} - \frac{x^2}{2\alpha\pi^2}e^{-\frac{\alpha x^2}{2}}K_1\left(\frac{\alpha x^2}{2}\right), \tag{86}$$

where x is expressed by Eq. (62), but with a different $m_0^2(r)$. Indeed, we have

$$x=\sqrt{\frac{m_0^2(r)}{E_P^2}}=\frac{1}{E_P r}\begin{cases}\sqrt{6-\Lambda_{dS}r^2} & \text{de Sitter } b(r)=\Lambda_{dS}r^3/3\\ \sqrt{6+\Lambda_{AdS}r^2} & \text{Anti-de Sitter } b(r)=-\Lambda_{AdS}r^3/3\end{cases}. \tag{87}$$

We can gain more information by evaluating the r.h.s. of Eq. (86) for small and large x. For large x, one gets

$$\begin{aligned}\frac{g_1^2(E/E_P)Q^2_{eg_1;mg_2}}{2E_P^4r^4}\simeq e^{-\alpha x^2}\bigg[&-\frac{\beta}{2\pi^{3/2}\alpha^{3/2}}x^2-\frac{1}{2\pi^{3/2}\alpha^{3/2}}x-\frac{3\beta}{4\pi^{3/2}\alpha^{5/2}}\\&-\frac{3}{8\pi^{3/2}\alpha^{5/2}}\frac{1}{x}+\frac{15}{64\pi^{3/2}\alpha^{7/2}}\frac{1}{x^3}+O\left(x^{-5}\right)\bigg],\end{aligned} \tag{88}$$

while for small x, we get

$$\begin{aligned}\frac{g_1^2(E/E_P)Q^2_{eg_1;mg_2}}{2E_P^4r^4}\simeq &-\frac{(4\sqrt{\alpha}+3\beta\sqrt{\pi})}{4\pi^2\alpha^{5/2}}+\frac{(2\sqrt{\alpha}+\beta\sqrt{\pi})}{4\pi^2\alpha^{3/2}}x^2\\&-\frac{[\sqrt{\alpha}\ln(\frac{x^2\alpha}{4}\sqrt{\alpha})+\gamma\sqrt{\alpha}-\beta\sqrt{\pi}]}{8\pi^2\sqrt{\alpha}}x^4+O\left(x^6\right).\end{aligned} \tag{89}$$

It is interesting to note that the expression is finite for every x. Beginning with the dS case, we observe that the range of the radius r is $[0,\sqrt{3/\Lambda_{dS}}\,]$ and when $r\to 0$, $x\to\infty$ which is vanishing because of behavior (88). On the other hand, when $r\to\sqrt{6/\Lambda_{dS}}$, $x\to 0$. However $r=\sqrt{6/\Lambda_{dS}}$ corresponds to a region external to the dS horizon which is unphysical and therefore will be discarded. Rather when

$$r=\sqrt{\frac{3}{\Lambda_{dS}}}\quad\Longrightarrow\quad x=\frac{\sqrt{\Lambda_{dS}}}{E_P}. \tag{90}$$

Therefore keeping the same parametrization that allows a vanishing contribution for small x, Eq. (86) becomes

$$\begin{aligned}Q_e^2\left(\alpha,-\frac{4}{3}\sqrt{\frac{\alpha}{\pi}},\frac{\sqrt{\Lambda_{dS}}}{E_P}\right)=\frac{9E_P^4}{\Lambda_{dS}^2}\bigg[&\frac{2}{3\alpha^2\pi^2}\left(3+\frac{2\alpha\Lambda_{dS}}{E_P^2}\right)\exp\left(-\frac{\alpha\Lambda_{dS}}{E_P^2}\right)\\&-\frac{\Lambda_{dS}}{\alpha\pi^2E_P^2}\exp\left(-\frac{\alpha\Lambda_{dS}}{2E_P^2}\right)K_1\left(\alpha\frac{\Lambda_{dS}}{2E_P^2}\right)\bigg],\end{aligned} \tag{91}$$

where we have excluded the trans-Planckian region which suppresses the charge contribution. By imposing that

$$Q_e^2\left(\frac{1}{4},-\frac{2}{3\sqrt{\pi}},\frac{\sqrt{\bar{\Lambda}_{dS}}}{E_P}\right)=\frac{1}{137}, \tag{92}$$

we find that

$$\bar{\Lambda}_{dS}\simeq 16E_P^2 \tag{93}$$

and the corresponding "*Cosmological radius*" becomes

$$\bar{r}_{\Lambda}^{Q_e}=\sqrt{\frac{3}{\bar{\Lambda}_{dS}}}=\frac{0.43301}{E_P}. \tag{94}$$

Concerning the AdS case, we observe that since $r \in [0, +\infty)$, when

$$r \to +\infty \ \to \ x = \frac{\sqrt{\Lambda_{AdS}}}{E_P}. \tag{95}$$

and $Q_e^2(1/4, -2/(3\sqrt{\pi}), \sqrt{\Lambda_{AdS}}/E_P) \to \infty$ and therefore will be discarded.

4. Magnetic monopoles

As introduced in Section 2, our calculation applies also to magnetic monopoles. However, since we have no experimental evidence in high energy physics, we need to use the Dirac proposal between the magnetic monopole and the electric charge described by the relationship (7) to fix numbers. Therefore, it is immediate to see that

$$Q_m = \frac{2\pi}{Q_e} = 73.543 \quad \Longrightarrow \quad Q_m^2 = 5408.6. \tag{96}$$

Since the value of Q_m^2 is quite large, for the Schwarzschild metric we can use parametrization (70) which keeps under control large values of x, while the parametrization (74) will be discarded. Setting

$$-\frac{r_t^4 E_P^4}{\pi^2} f\left(\frac{1}{4}, -\frac{\sqrt{\pi}}{4}; \frac{3}{2r_t^2 E_P^2}\right) = 5408.6, \tag{97}$$

we find

$$\bar{r}_t^M = \frac{6.6}{E_P}. \tag{98}$$

Note that when we compare $\bar{r}_t^{Q_m}$ with $\bar{r}_t^{Q_e}$, we find

$$\frac{\bar{r}_t^{Q_m}}{\bar{r}_t^{Q_e}} = \frac{6.6}{0.295} = 22.373. \tag{99}$$

On the other hand, if we use the dS metric, we find

$$Q_m^2\left(\frac{1}{4}, -\frac{2}{3\sqrt{\pi}}, \frac{\sqrt{\bar{\Lambda}_{dS}}}{E_P}\right) = 5408.6, \tag{100}$$

which implies

$$\bar{\Lambda}_{dS} \simeq 0.024 E_P^2 \tag{101}$$

and the corresponding "*Cosmological radius*" becomes

$$\bar{r}_\Lambda^{Q_m} = \sqrt{\frac{3}{\bar{\Lambda}_{dS}}} = \frac{11.18}{E_P}. \tag{102}$$

Once again, when we compare $\bar{r}_\Lambda^{Q_m}$ with $\bar{r}_\Lambda^{Q_e}$ we find

$$\frac{\bar{r}_\Lambda^{Q_m}}{\bar{r}_\Lambda^{Q_e}} = \frac{11.18}{0.43301} = 25.819. \tag{103}$$

5. Conclusions

In this paper we have explored the possibility that quantum fluctuations of the gravitational field be considered as a source for the electric/magnetic charge. The idea is not new, because it has its origin in the Wheeler's proposal of "*charge without charge*" and "*mass without mass*" arising from the *spacetime foam* picture [2]. Moreover, a first approach has been proposed by one of us in Ref. [7]. What is new in this paper is that the UV divergences are kept under control by Gravity's Rainbow which is a distortion of spacetime activating at the Planck's scale. This distortion avoids the introduction of any regularization/renormalization process, like in Noncommutative theory approaches [14]. Note that the *Rainbow's functions* $g_1(E/E_P)$ and $g_2(E/E_P)$ are constrained only by the low energy limit (1) and by the request that the one loop integrals be UV finite [5,6,15,16]. It is interesting to note that differently from the approach of Ref. [7], here there is not a renormalization scale μ_0 which is free to be fixed depending on the problem under consideration. In this approach, $\mu_0 = E_P$ since the beginning. Moreover, as shown in Section 3.1, not every choice of $g_1(E/E_P)$ and $g_2(E/E_P)$ is possible, otherwise the final result could be unphysical. The choice adopted in this paper has been borrowed by the result obtained on the estimation of the cosmological constant made in Ref. [5]. Of course we have not exhausted all the possible choices, but if one takes seriously the method of Refs. [5,6], an agreement also with the procedure of the present paper must be found. Indeed, in discussing an inflationary scenario governed by Gravity's Rainbow [17], it appears that a different proposal has been chosen. Nevertheless, the functions $g_1(E/E_P)$ and $g_2(E/E_P)$ in Ref. [17], are present under the form of a ratio and therefore a major freedom on their choice can be introduced. It is important to note that the electric and magnetic charges appear as a quantum effect of the gravitational field. Indeed, the classical contribution related to the specific geometries hitherto examined leads to $Q_e = Q_m = 0$. It is also important to remark that once the charge has been created, the only correct metric that can be used to discuss the solutions of Eq. (22) is the Reissner–Nordström metric. Three basic geometries have been examined. One of these, the AdS background leads to inconsistent solutions and therefore has been discarded. On the other hand, the Schwarzschild and the dS background show that the computed particle radius of the electron is of the Planckian order. This has been obtained by fixing the value of the electric charge to the fine structure constant that, in the units we have adopted, is coincident with the square of the electron charge. As regards the magnetic monopole, since no direct observation at very high energies has ever been announced, we have used the Dirac quantization rule to obtain information about the magnetic charge and therefore recover its own particle radius. It is interesting to note that the ratio between the magnetic monopole radius and the electron radius r^{Q_m}/r^{Q_e} is of the same order for both the Schwarzschild and the de Sitter background. It is also interesting to observe that the appearance of the electric charge and the magnetic monopole as a quantum gravitational effect in the cis-Planckian region is not affected by the Rainbow's functions at the classical level, namely the l.h.s. of Eqs. (50), (85) as it should be. However in the trans-Planckian region an asymmetry is present between the electric charge and the magnetic charge. Indeed, the electric charge is suppressed by a factor of $g_1^4(E/E_P)$, while the magnetic monopole is suppressed by a factor $g_1^2(E/E_P)$. I draw the reader's attention on the property that $g_1(E/E_P) \to 0$ when $E/E_P \to \infty$. Therefore, from the Gravity's Rainbow point of view, it seems that the magnetic monopole in the trans-Planckian region can survive more compared to the electric charge, or in other terms the quantum gravitational fluctuations begin to produce a magnetic monopole and when the energy decreases even the electric charge begins to be produced. Recently another result relating Gravity's Rainbow and its influence on topology change has been obtained [18]. This seems to suggest that Gravity's Rainbow can be considered as a good tool for probing the spacetime foam picture suggested by Wheeler.

Appendix A. The electromagnetic energy–momentum tensor in SI units

The electromagnetic energy–momentum tensor in free space and in SI units is defined as

$$T_{\mu\nu}=\frac{1}{\mu_0}\left[F_{\mu\gamma}F_{\nu}^{\gamma}-\frac{1}{4}g_{\mu\nu}F_{\gamma\delta}F^{\gamma\delta}\right],\tag{A.1}$$

where μ_0 is the vacuum permeability, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the electromagnetic field strength tensor. For a pure electric field, the electromagnetic potential A_μ assumes the form

$$A_\mu=\left(\frac{Q_e}{4\pi\epsilon_0 rc},0,0,0\right)\quad\Longrightarrow\quad F_{\mu\nu}=F_{01}=-\frac{Q_e}{4\pi\epsilon_0 r^2 c},\tag{A.2}$$

where ϵ_0 is the vacuum permittivity, c is the speed of light and Q_e is the electric charge. On the other hand, for the pure magnetic field, the form is

$$A_\mu=\left(0,0,0,-\mu_0\frac{Q_m}{4\pi}\cos\theta\right)\quad\Longrightarrow\quad F_{\mu\nu}=F_{23}=\mu_0\frac{Q_m}{4\pi}\sin\theta,\tag{A.3}$$

where Q_m is the magnetic charge measured in Ampère·meter (A m). Thus the T_{00} component of the energy–momentum tensor for the electromagnetic charges becomes

$$\begin{aligned}T_{00}&=\frac{1}{\mu_0}\left\{\frac{1}{2}g^{11}(F_{01})^2-\frac{1}{2}g_{00}\left(g^{22}g^{33}\right)(F_{23})^2\right\}\\&=\frac{1}{2\mu_0(4\pi r^2)^2}\left\{g^{11}\frac{Q_e^2}{\epsilon_0^2c^2}-g_{00}\mu_0^2Q_m^2\right\}\\&=\frac{1}{2(4\pi r^2)^2}\left\{g^{11}\frac{Q_e^2}{\epsilon_0}-g_{00}\mu_0Q_m^2\right\},\end{aligned}\tag{A.4}$$

where we have used the following relationship $c^2\epsilon_0\mu_0=1$. With the help of the time-like vector u^μ, we obtain

$$T_{\mu\nu}u^\mu u^\nu=\frac{1}{2(4\pi r^2)^2}\left\{\frac{Q_e^2}{\epsilon_0}+\mu_0Q_m^2\right\}.\tag{A.5}$$

For a spherically symmetric metric described by (2) with $g_1(E/E_P)=g_2(E/E_P)=1$, it is easy to recognize that the classical constraint (11) reduces to

$${}^3R=\frac{2\kappa}{c^4}T_{\alpha\beta}u^\alpha u^\beta\quad\Longrightarrow\quad b'(r)=\frac{G}{4\pi r^2c^4}\left\{\frac{Q_e^2}{\epsilon_0}+\mu_0Q_m^2\right\},\tag{A.6}$$

whose solution represents the Reissner–Nordström metric if

$$N^2(r)=\left[1-\frac{b(r)}{r}\right]^{-1}\tag{A.7}$$

and

$$b(r)=\frac{2MG}{c^2}-\frac{G}{4\pi rc^4}\left\{\frac{Q_e^2}{\epsilon_0}+\mu_0Q_m^2\right\}.\tag{A.8}$$

Note that in CGS units, one defines $\epsilon_0=(4\pi)^{-1}$ and $\mu_0=4\pi$ and the energy–momentum tensor is in agreement with the expression in (4).

References

[1] A.D. Sakharov, Dokl. Akad. Nauk SSSR, Ser. Fiz. 177 (1967) 70;
A.D. Sakharov, Sov. Phys. Dokl. 12 (1968) 1040;
A.D. Sakharov, Sov. Phys. Usp. 34 (1991) 394;
A.D. Sakharov, Gen. Relativ. Gravit. 32 (2000) 365.

[2] J.A. Wheeler, Phys. Rev. 97 (1955) 511.

[3] G. Amelino-Camelia, Int. J. Mod. Phys. D 11 (2002) 35, arXiv:gr-qc/0012051;
G. Amelino-Camelia, Phys. Lett. B 510 (2001) 255, arXiv:hep-th/0012238.

[4] J. Magueijo, L. Smolin, Class. Quantum Gravity 21 (2004) 1725, arXiv:gr-qc/0305055.

[5] R. Garattini, G. Mandanici, Phys. Rev. D 83 (2011) 084021, arXiv:1102.3803 [gr-qc];
R. Garattini, G. Mandanici, Phys. Rev. D 85 (2012) 023507, arXiv:1109.6563 [gr-qc].

[6] R. Garattini, J. Cosmol. Astropart. Phys. 1306 (2013) 017, arXiv:1210.7760 [gr-qc].

[7] R. Garattini, Phys. Lett. B 666 (2008) 189, arXiv:0807.0082.

[8] Y. Ling, J. Cosmol. Astropart. Phys. 0708 (2007) 017, arXiv:gr-qc/0609129;
Y. Ling, X. Li, H. Zhang, Mod. Phys. Lett. A 22 (2007) 2749, arXiv:gr-qc/0512084;
Y. Ling, B. Hu, X. Li, Phys. Rev. D 73 (2006) 087702, arXiv:gr-qc/0512083.

[9] C. Castelnovo, R. Moessner, S.L. Sondhi, Magnetic monopoles in spin ice, Nature 451 (2008) 42–45, http://dx.doi.org/10.1038/nature06433;
L.D.C. Jaubert, P.C.W. Holdsworth, Signature of magnetic monopole and Dirac string dynamics in spin ice, Nat. Phys. 5 (2009) 258–261, http://dx.doi.org/10.1038/NPHYS1227;
S.T. Bramwell, S.R. Giblin, S. Calder, R. Aldus, D. Prabhakaran, T. Fennell, Measurement of the charge and current of magnetic monopoles in spin ice, Nature 461 (2009) 956–959, http://dx.doi.org/10.1038/nature08500.

[10] P.A. Dirac, Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 133 (1931) 60.

[11] H. Georgi, S. Glashow, Phys. Rev. Lett. 32 (1974) 438;
G. 't Hooft, Nucl. Phys. B 79 (1974) 276;
A.M. Polyakov, JETP Lett. 20 (1974) 194.

[12] R. Garattini, Vestn. Tomsk. Gos. Pedagog. Univ. 44 (7) (2004) 72, arXiv:gr-qc/0409016;
R. Garattini, J. Phys. Conf. Ser. 33 (215) (2006), arXiv:gr-qc/0510062.

[13] B.S. DeWitt, Phys. Rev. 160 (1967) 1113.

[14] R. Garattini, P. Nicolini, Phys. Rev. D 83 (2011) 064021, arXiv:1006.5418 [gr-qc].

[15] R. Garattini, Phys. Lett. B 685 (2010) 329, arXiv:0902.3927 [gr-qc].

[16] R. Garattini, F.S.N. Lobo, Phys. Rev. D 85 (2012) 024043, arXiv:1111.5729 [gr-qc].

[17] R. Garattini, M. Sakellariadou, Does Gravity's Rainbow induce inflation without an inflaton? arXiv:1212.4987 [gr-qc].

[18] R. Garattini, F.S.N. Lobo, Gravity's Rainbow induces topology change, arXiv:1303.5566 [gr-qc].