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Convergence of Adomian's Method Applied to Differential Equations

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Abstract—This paper deals with a new proof of convergence of Adomian's method applied to differential equations. We also give new formulae and properties, and we suggest a simple computational form for Adomian's polynomials.

1. INTRODUCTION

In many papers [1–5], Adomian has presented a technique using special polynomials for solving nonlinear equations of various kind (algebraic, differential, partial differential, integral, etc.). The solution is found as an infinite series in which each term can be easily determined and that converges quickly towards an accurate solution. However, only few works have been done on the convergence of the method (for instance, [6–8]).

In this paper, we propose new formulae for calculating Adomian's polynomials, and we give a proof of convergence using the classical Taylor's method.

2. THE DECOMPOSITION METHOD APPLIED TO DIFFERENTIAL EQUATIONS

Let us consider a differential equation in the form:

$$\frac{du}{dt} = f(u) + g, \tag{2.1}$$

$$u(t)|_{t=0} = c_0, \tag{2.2}$$

where f is the nonlinear term and g is given.

Adomian's method consists in calculating the solution, in the series form:

$$\sum_{n=0}^{\infty} u_n. \tag{2.3}$$

The nonlinear term $f(u)$ becomes

$$f(u) = \sum_{n=0}^{\infty} A_n, \tag{2.4}$$

where the A_n's are polynomials depending on $u_0, u_1, \dots, u_n$, called Adomian's polynomials. They are obtained from the relationship

$$v = \sum_{i=0}^{\infty} \lambda^i u_i, \quad f\left(\sum_i \lambda^i u_i\right) = \sum_{i=0}^{\infty} \lambda^i A_i, \tag{2.5}$$

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX where λ is a parameter introduced for convenience. The A_n's are calculated from (2.5) by the formulae (see [1]):

$$n!\, A_n = \frac{d^n}{d\lambda^n} f\left(\sum_i^{\infty} \lambda^i u_i\right)\Bigg|_{\lambda=0}. \tag{2.6}$$

Inserting (2.3) and (2.4) into equation (2.1) leads to

$$\sum_n^{\infty} u_n = c_0 + L^{-1} g(t) + L^{-1} \sum_n^{\infty} A_n. \tag{2.7}$$

Each term of the series $\sum_{n=0}^{\infty} u_n$ can be identified by the formulae

$$\begin{aligned} u_0 &= c_0 + L^{-1} g, \\ u_1 &= L^{-1} A_0, \\ &\vdots \\ u_{n+1} &= L^{-1} A_n. \end{aligned} \tag{2.8}$$

The exact solution of equation (2.1) is now entirely determined. However, in practice, all terms of the series $\sum_{n=0}^{\infty} u_n$ cannot be determined; so we use an approximation of the solution from the truncated series:

$$\phi_n = \sum_{i=0}^{n-1} u_i, \qquad \text{with } \lim \phi_n = u. \tag{2.9}$$

3. CONVERGENCE OF THE TECHNIQUE

For every sequence $u_n(\lambda) = \sum_{i=0}^{n} \lambda^i u_i$, we define $f(u_n(\lambda))$ by [6]:

$$f(u_n(\lambda)) = \sum_{i=0}^{n} \lambda^i A_i. \tag{3.1}$$

Then, we have the following result.

THEOREM 3.1. *For a composed function $A(\lambda) = f(u_n(\lambda))$, where we suppose that $f(u)$ is differentiable up to the n^{th} order, A_n are given by*

$$\begin{aligned} A_0 &= f(u_0); \\ A_n &= \sum_{\substack{k_1+2k_2 \\ +3k_3+\cdots+nk_n=n}} \left(\frac{d^{(k_1+k_2+\cdots+k_n)} f}{du^{(k_1+k_2+\cdots+k_n)}}\right)_{u=u_0} \cdot \frac{u_1^{k_1}}{k_1!} \cdot \frac{u_2^{k_2}}{k_2!} \cdot \frac{u_3^{k_3}}{k_3!} \cdot \dots \cdot \frac{u_n^{k_n}}{k_n!}, \qquad n \neq 0. \end{aligned} \tag{3.2}$$

PROOF. Applying the classical formula [9] giving the n^{th} derivative of the function $A(\lambda) = f(u_n(\lambda))$, we obtain

$$\begin{aligned} &A_n(u_0, u_1, u_2, \dots, u_n) \\ &= \frac{1}{n!} \sum_{k_1+2k_2+3k_3+\cdots+nk_n=n} \frac{n!\,(u_1)^{k_1}\,(2!u_2)^{k_2}\,(3!u_3)^{k_3} \dots (n!u_n)^{k_n}}{(1!)^{k_1}\,(2!)^{k_2} \dots (n!)^{k_n}\, k_1!\, k_2! \dots k_n!} \cdot \left(\frac{d^{(k_1+k_2+\cdots+k_n)} f}{du^{(k_1+k_2+\cdots+k_n)}}\right)_{u=u_0} \\ &= \sum_{\substack{k_1+2k_2 \\ +3k_3+\cdots+nk_n=n}} \left(\frac{d^{(k_1+k_2+\cdots+k_n)} f}{du^{(k_1+k_2+\cdots+k_n)}}\right)_{u=u_0} \cdot \frac{u_1^{k_1}}{k_1!} \cdot \frac{u_2^{k_2}}{k_2!} \cdot \frac{u_3^{k_3}}{k_3!} \cdot \dots \cdot \frac{u_n^{k_n}}{k_n!}. \end{aligned}$$

∎

COROLLARY 3.2.

$$A_0 = f(u_0);$$

$$A_n(u_0, u_1, u_2, \dots, u_n) = \sum_{\substack{\alpha_1+\alpha_2 \\ +\dots+\alpha_n=n}} \left(\frac{d^{\alpha_1} f}{du^{\alpha_1}}\right)_{u=u_0} \frac{u_1^{(\alpha_1-\alpha_2)}}{(\alpha_1-\alpha_2)!} \cdot \frac{u_2^{(\alpha_2-\alpha_3)}}{(\alpha_2-\alpha_3)!} \cdot \dots \cdot \frac{u_{n-1}^{(\alpha_{n-1}-\alpha_n)}}{(\alpha_{n-1}-\alpha_n)!} \cdot \frac{u_n^{\alpha_n}}{\alpha_n!}, \qquad n \neq 0, \tag{3.3}$$

where $(\alpha_i)_{i=1,2,\dots,n}$ is a decreasing sequence.

PROOF. It is sufficient to choose

$$k_1 = \alpha_1 - \alpha_2,$$
$$k_2 = \alpha_2 - \alpha_3,$$
$$\vdots$$
$$k_{n-1} = \alpha_{n-1} - \alpha_n,$$
$$k_n = \alpha_n,$$

which leads to

$$k_1 + 2k_2 + 3k_3 + \dots + n\,k_n = \alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n = n, \qquad \text{and}$$
$$k_1 + k_2 + k_3 + \dots + k_n = \alpha_1.$$

∎

THEOREM 3.3. *If $u_i = a_i\, t^i$, then*

$$A_n(u_0, u_1, \dots, u_n) = t^n\, A_n(a_0, a_1, \dots, a_n). \tag{3.4}$$

PROOF. $A_n(u_0, u_1, \dots, u_n) = A_n(a_0, a_1 t, \dots, a_n t^n)$

$$= \sum_{k_1+2k_2+\dots+nk_n=n} \left(\frac{d^{(k_1+k_2+\dots+k_n)} f}{du^{(k_1+k_2+\dots+k_n)}}\right)_{u=u_0} \frac{(a_1\, t)^{k_1}}{k_1!} \cdot \frac{(a_2\, t^2)^{k_2}}{k_2!} \cdot \dots \cdot \frac{(a_n\, t^n)^{k_n}}{k_n!}$$

$$= \sum_{k_1+2k_2+\dots+nk_n=n} t^{k_1+2k_2+\dots+nk_n} \left(\frac{d^{(k_1+k_2+\dots+k_n)} f}{du^{(k_1+k_2+\dots+k_n)}}\right)_{u=u_0} \frac{(a_1)^{k_1}}{k_1!} \cdot \frac{(a_2)^{k_2}}{k_2!} \cdot \dots \cdot \frac{(a_n)^{k_n}}{k_n!}$$

$$= t^n \sum_{k_1+2k_2+\dots+nk_n=n} \left(\frac{d^{(k_1+k_2+\dots+k_n)} f}{du^{(k_1+k_2+\dots+k_n)}}\right)_{u=u_0} \frac{(a_1)^{k_1}}{k_1!} \cdot \frac{(a_2)^{k_2}}{k_2!} \cdot \dots \cdot \frac{(a_n)^{k_n}}{k_n!}$$

$$= t^n\, A_n(a_0, a_1, \dots, a_n).$$

∎

THEOREM 3.4. *In the differential system (2.1), (2.2), we suppose that $f(u)$ is infinitely differentiable and that g is expandable in entire series in the neighborhood of $t_0 = 0$; then the series solution of (2.1), (2.2) is given by the scheme*

$$u_0 = U(0);$$
$$u_{n+1} = L^{-1}\, A_n + L^{-1}\, \alpha_n\, t^n, \qquad \alpha_n = \frac{g^{(n)}(0)}{n!}, \tag{3.5}$$

which is a Taylor series.

PROOF. Using Theorems 3.1 and 3.3, with $a_i = \frac{u^{(i)}(0)}{i!}$, leads to

$$u_0 = u(0);$$

$$u_{n+1} = L^{-1}\, A_n + L^{-1}\, \alpha_n\, t^n = L^{-1}\, t^n\, A_n(a_0, a_1, \dots, a_n) + \alpha_n \frac{t^{n+1}}{(n+1)}$$

$$= \frac{t^{n+1}}{(n+1)!}\left(n!\, A_n(a_0, a_1, \dots, a_n) + n!\, \alpha_n\right) = u^{(n+1)}(0)\, \frac{t^{n+1}}{(n+1)!},$$

where $u^{(n+1)}(0)$ is the $(n+1)^{\text{th}}$ derivative of $u(t)$ evaluated at $t = t_0 = 0$. ∎

COROLLARY 3.5. *For $g = 0$,*

(1) $A_n = u^{(n+1)}(0)\, t^n/n!$;

(2) *if* $\left|\left(\frac{d^k f}{du^k}\right)_{u=u_0}\right| \le M$ *for any nonnegative integer k, then the series solution of (2.1), (2.2) obtained by the scheme (3.5) is absolutely convergent within the interval $(-1/M, 1/M)$ and, furthermore,*

$$|u_n| \le \frac{M^n\, t^n}{n}.$$

PROOF. (1) Theorem 3.4 implies

$$u_{n+1} = L^{-1} A_n + L^{-1} \alpha_n\, t^n = u^{(n+1)}(0)\, \frac{t^{n+1}}{(n+1)!},$$

so that, if $g = 0$, $u_{n+1} = L^{-1} A_n = u^{(n+1)}(0)\, t^{n+1}/(n+1)!$ and

$$A_n = u^{(n+1)}(0)\, \frac{t^n}{n!}.$$

(2) Since $u_n = u^{(n)}(0)\, t^n/n!$ and $\left|\left(\frac{d^k f}{du^k}\right)_{u=u_0}\right| \le M$, it is easy to see that

$$|u_n| = \left|u^{(n)}(0)\, \frac{t^n}{n!}\right| \le (n-1)!\, M^n\, \frac{t^n}{n!} = M^n\, \frac{t^n}{n}. \qquad \blacksquare$$

4. A SIMPLE COMPUTATIONAL FORM FOR ADOMIAN'S POLYNOMIALS

Using the decreasing sequence $(\alpha_i)_{i=1,2,\dots,n}$, Adomian's polynomials can be easily calculated. All possible nonnegative integer solutions of the equations

$$\alpha_1 + \alpha_2 + \cdots + \alpha_n = n, \qquad \text{with} \quad \alpha_1 \ge \alpha_2 \ge \cdots \ge \alpha_n \tag{4.1}$$

can be searched without any difficulties by using a simple software. For example, for $n = 10$ we have:

1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	0
2	2	1	1	1	1	1	1	0	0
2	2	2	1	1	1	1	0	0	0
2	2	2	2	1	1	0	0	0	0
2	2	2	2	2	0	0	0	0	0
3	1	1	1	1	1	1	1	0	0
3	2	1	1	1	1	1	0	0	0
3	2	2	1	1	1	0	0	0	0
3	2	2	2	1	0	0	0	0	0
3	3	1	1	1	1	0	0	0	0
3	3	2	1	1	0	0	0	0	0
3	3	2	2	0	0	0	0	0	0
3	3	3	1	0	0	0	0	0	0
4	1	1	1	1	1	1	0	0	0
4	2	1	1	1	1	0	0	0	0
4	2	2	1	1	0	0	0	0	0
4	2	2	2	0	0	0	0	0	0
4	3	1	1	1	0	0	0	0	0
4	3	2	1	0	0	0	0	0	0

4	3	3	0	0	0	0	0	0	0
4	4	1	1	0	0	0	0	0	0
4	4	2	0	0	0	0	0	0	0
5	1	1	1	1	1	0	0	0	0
5	2	1	1	1	0	0	0	0	0
5	2	2	1	0	0	0	0	0	0
5	3	1	1	0	0	0	0	0	0
5	3	2	0	0	0	0	0	0	0
5	4	1	0	0	0	0	0	0	0
5	5	0	0	0	0	0	0	0	0
6	1	1	1	1	0	0	0	0	0
6	2	1	1	0	0	0	0	0	0
6	2	2	0	0	0	0	0	0	0
6	3	1	0	0	0	0	0	0	0
6	4	0	0	0	0	0	0	0	0
7	1	1	1	0	0	0	0	0	0
7	2	1	0	0	0	0	0	0	0
7	3	0	0	0	0	0	0	0	0
8	1	1	0	0	0	0	0	0	0
8	2	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0

And we can list the expression of the A_n's for $n = 0, 1, 2, \ldots 10$:

$$
\begin{aligned}
A_0 &= f(u_0),\\
A_1 &= f^{(1)}(u_0)u_1,\\
A_2 &= f^{(1)}(u_0)u_2 + \frac{1}{2}\, f^{(2)}(u_0)\, u_1^2,\\
A_3 &= f^{(1)}(u_0)u_3 + f^{(2)}(u_0)u_1\, u_2 + \frac{1}{6}\, f^{(3)}(u_0)\, u_1^3,\\
A_4 &= f^{(1)}(u_0)u_4 + f^{(2)}(u_0)\left\{u_1\, u_3 + \frac{1}{2}\, u_2^2\right\} + \frac{1}{2}\, f^{(3)}(u_0)\, u_1^2\, u_2 + \frac{1}{24}\, f^{(4)}(u_0)\, u_1^4,\\
A_5 &= f^{(1)}(u_0)u_5 + f^{(2)}(u_0)\ \{u_1\, u_4 + u_2\, u_3\} + \frac{1}{2}\, f^{(3)}(u_0)\,(u_1^2\, u_3 + u_1\, u_2^2) + \frac{1}{6}\, f^{(4)}(u_0)\, u_1^3\, u_2\\
&\quad + \frac{1}{120}\, f^{(5)}(u_0)\, u_1^5,\\
A_6 &= f^{(1)}(u_0)u_6 + f^{(2)}(u_0)\left\{u_1\, u_5 + u_2\, u_4 + \frac{1}{2}\, u_3^2\right\}\\
&\quad + f^{(3)}(u_0)\left\{\frac{1}{2}u_1^2\, u_4 + u_1\, u_2\, u_3 + \frac{1}{6}u_2^3\right\} + f^{(4)}(u_0)\left\{\frac{1}{6}u_1^3\, u_3 + \frac{1}{4}u_1^2\, u_2^2\right\}\\
&\quad + \frac{1}{24}\, f^{(5)}(u_0)\, u_1^4\, u_2 + \frac{1}{720}\, f^{(6)}(u_0)\, u_1^6,\\
A_7 &= f^{(1)}(u_0)u_7 + f^{(2)}(u_0)\,(u_1\, u_6 + u_2\, u_5 + u_3\, u_4)\\
&\quad + f^{(3)}(u_0)\left\{\frac{1}{2}\, u_1^2\, u_5 + u_1\, u_2\, u_4 + \frac{1}{2}u_1\, u_3^2 + \frac{1}{2}u_2^2\, u_3\right\}\\
&\quad + f^{(4)}(u_0)\left\{\frac{1}{6}\, u_1^3\, u_4 + \frac{1}{2}\, u_1^2\, u_2\, u_3 + \frac{1}{6}\, u_1\, u_2^3\right\} + f^{(5)}(u_0)\left\{\frac{1}{24}\, u_1^4\, u_3 + \frac{1}{12}\, u_1^3\, u_2^2\right\}\\
&\quad + \frac{1}{120}\, f^{(6)}(u_0)u_1^5\, u_2 + \frac{1}{504}f^{(7)}(u_0)u_1^7,
\end{aligned}
$$

$$
\begin{aligned}
A_8 = {} & f^{(1)}(u_0)u_8 + f^{(2)}(u_0)\left\{u_1 u_7 + u_2 u_6 + u_3 u_5 + \frac{1}{2}u_4^2\right\} \\
& + f^{(3)}(u_0)\left\{\frac{1}{2}u_1^2 u_6 + u_1 u_2 u_5 + u_1 u_3 u_4 + \frac{1}{2}u_2 u_3^2 + \frac{1}{2}u_2^2 u_4\right\} \\
& + f^{(4)}(u_0)\left\{\frac{1}{6}u_1^3 u_5 + \frac{1}{2}u_1^2 u_2 u_4 + \frac{1}{4}u_1^2 u_3^2 + \frac{1}{2}u_1 u_2^2 u_3 + \frac{1}{24}u_2^4\right\} \\
& + f^{(5)}(u_0)\left\{\frac{1}{24}u_1^4 u_4 + \frac{1}{6}u_1^3 u_2 u_3 + \frac{1}{12}u_1^2 u_2^3\right\} + f^{(6)}(u_0)\left\{\frac{1}{120}u_1^5 u_3 + \frac{1}{48}u_1^4 u_2^2\right\} \\
& + \frac{1}{720}f^{(7)}(u_0)\, u_1^6 u_2 + \frac{1}{40320}f^{(8)}(u_0)\, u_1^8, \\
A_9 = {} & f^{(1)}(u_0)u_9 + f^{(2)}(u_0)\left(u_1 u_8 + u_2 u_7 + u_3 u_6 + u_4 u_5\right) \\
& + f^{(3)}(u_0)\left\{\frac{1}{2}u_1^2 u_7 + u_1 u_2 u_6 + u_1 u_3 u_5 + \frac{1}{2}u_1 u_4^2 + \frac{1}{2}u_2^2 u_5 + u_2 u_3 u_4 + \frac{1}{6}u_3^3\right\} \\
& + f^{(4)}(u_0)\left\{\frac{1}{6}u_1^3 u_6 + \frac{1}{2}u_1^2 u_2 u_5 + \frac{1}{2}u_1^2 u_3 u_4 + \frac{1}{2}u_1 u_2^2 u_4 + \frac{1}{2}u_1 u_2 u_3^2 + \frac{1}{6}u_2^3 u_3\right\} \\
& + f^{(5)}(u_0)\left\{\frac{1}{24}u_1^4 u_5 + \frac{1}{6}u_1^3 u_2 u_4 + \frac{1}{12}u_1^3 u_3^2 + \frac{1}{4}u_1^2 u_2^2 u_3 + \frac{1}{24}u_1 u_2^4\right\} \\
& + f^{(6)}(u_0)\left\{\frac{1}{120}u_1^5 u_4 + \frac{1}{24}u_1^4 u_2 u_3 + \frac{1}{36}u_1^3 u_2^3\right\} + f^{(7)}(u_0)\left\{\frac{1}{720}u_1^6 u_3 + \frac{1}{240}u_1^5 u_2^2\right\} \\
& + \frac{1}{5040}f^{(8)}(u_0)\, u_1^7 u_2 + \frac{1}{362880}f^{(9)}(u_0)\, u_1^9, \\
A_{10} = {} & f^{(1)}(u_0)u_{10} + f^{(2)}(u_0)\left\{u_1 u_9 + u_2 u_8 + u_3 u_7 + u_4 u_6 + \frac{1}{2}u_5^2\right\} \\
& + f^{(3)}(u_0)\left\{\frac{1}{2}u_1^2 u_8 + u_1 u_2 u_7 + u_1 u_3 u_6 + u_1 u_4 u_5 + \frac{1}{2}u_2^2 u_6 + u_2 u_3 u_5 + \frac{1}{2}u_2 u_4^2 \right. \\
& \qquad \left. + \frac{1}{2}u_3^2 u_4\right\} \\
& + f^{(4)}(u_0)\left\{\frac{1}{6}u_1^3 u_7 + \frac{1}{6}u_1 u_3^3 + \frac{1}{2}u_1^2 u_2 u_6 + \frac{1}{2}u_1^2 u_3 u_5 + \frac{1}{4}u_1^2 u_4^2 + \frac{1}{2}u_1 u_2^2 u_5 \right. \\
& \qquad \left. + u_1 u_2 u_3 u_4 + \frac{1}{6}u_2^3 u_4 + \frac{1}{4}u_2^2 u_3^2\right\} \\
& + f^{(5)}(u_0)\left\{\frac{1}{24}u_1^4 u_6 + \frac{1}{6}u_1^3 u_2 u_5 + \frac{1}{6}u_1^3 u_3 u_4 + \frac{1}{4}u_1^2 u_2^2 u_4 + \frac{1}{4}u_1^2 u_2 u_3^2 + \frac{1}{6}u_1 u_2^3 u_3 \right. \\
& \qquad \left. + \frac{1}{120}u_2^5\right\} \\
& + f^{(6)}(u_0)\left\{\frac{1}{120}u_1^5 u_5 + \frac{1}{24}u_1^4 u_2 u_4 + \frac{1}{48}u_1^4 u_3^2 + \frac{1}{12}u_1^3 u_2^2 u_3 + \frac{1}{48}u_1^2 u_2^4\right\} \\
& + f^{(7)}(u_0)\left\{\frac{1}{720}u_1^6 u_4 + \frac{1}{120}u_1^5 u_2 u_3 + \frac{1}{144}u_1^4 u_2^3\right\} + f^{(8)}(u_0)\left\{\frac{1}{5040}u_1^7 u_3 + \frac{1}{1440}u_1^6 u_2^2\right\} \\
& + \frac{1}{40320}f^{(9)}(u_0)\, u_1^8 u_2 + \frac{1}{3628800}f^{(10)}(u_0)\, u_1^{10}.
\end{aligned}
$$

REMARKS.

(1) The results given in Theorem 3.3 are also valid for functions of several variables $f(u, v, w, \dots)$.
(2) Our previous results can also be applied to systems of differential equations.

5. CONCLUSION

Our original formula (3.3) allows us to calculate quickly Adomian's polynomials. The number of all possible nonnegative solutions of equation (3.6) is easily obtained. The formula is very simple to use; by means of our TURBO-C software, for example, we have been able to list the A_n from 1 to 100. The computation becomes more troublesome in [8,10]; e.g., Yang [8] uses the coefficients of $f^{(k)}(u_0)$.

Furthermore, we have proved that Adomian's method is more general than a Taylor's technique, and that the two schemes (2.8) and (3.5) give series with different terms.

But the two methods are identical if $g = 0$. With these improvements Adomian's technique becomes a very powerful tool for solving nonlinear equations of functions of one or several unknowns.

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