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Abstract
In classical methods only average torque of the stepper motors is controlled which causes high speed and torque ripple. In order to control the torque instantaneously and improve the performance of the hybrid stepper motor, direct torque control strategy is used in this paper. Then by taking model reference adaptive system scheme, which uses the hybrid stepper motor itself as the reference model, speed of the motor is estimated. The sensorless control of hybrid stepper motor based on MRAS is built and simulated with Matlab software. The results show that this control technology is simple and effective and accuracy of the estimation is considerably high.

1. Introduction
Stepper motors are suitable for opened loop control with no need of speed and position sensors which make them one of the commonly used motors in industrial applications. According to operation principle of stepper motors, they can be classified into three types, variable reluctance, permanent magnet and hybrid. Among these types of stepper motors, hybrid stepper motors (HSM) are the most commonly used in industry because, they have higher efficiency and torque. Torque of the motor is produced by both reluctance and permanent magnet effects. The main parts of HSM are the stator and the rotor. The stator is made of magnetic poles shaped as teeth on which the winding are placed. The rotor teeth are magnetized by permanent magnet \cite{1}-\cite{2}. In precise motion and high dynamic requirements, performance of stepper motor with opened loop control is very poor so, closed loop control is required. Field oriented control (FOC) strategy is applied to HSM and improved its performance \cite{3}-\cite{4}. Because of the stepping magnetic field, the traditional control of stepping motor mainly controls the average torque \cite{10}. In order to improve the performance of hybrid stepping motor, the torque must be controlled instantaneously. At present, direct torque control (DTC) method is considered a more effective and useful method for controlling rotary motors, as compared with other methods. DTC of HSM has some advantages over vector control such as simplicity, fast dynamic response, low parameter dependency and lack of rotor position and coordinate transformation \cite{5}. Position sensors used in stepper motor are bulky and could be

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easy to fail in harsh environments. Many techniques have been introduced in order to estimate the rotor position of the motor [3]-[4]. Model reference adaptive system (MRAS) is an adaptive controller and can estimate the speed of the motor only by using stator voltages and currents. In this paper, model of hybrid stepper motor is given and with MRAS technique, sensorless speed and position control of hybrid stepper motor with direct torque control strategy is investigated. The results will show that the control system is simple, feasible and effective.

2. Model of hybrid stepper motor

The mathematical of the hybrid stepper motor is described by the following equations [1], [6]:

\[ u_a = R_i a + L \frac{di_a}{dt} - \omega K_m \sin(\theta) \] (1)

\[ u_b = R_i b + L \frac{di_b}{dt} + \omega K_m \cos(\theta) \] (2)

\[ T_e = K_m (i_b \cos(\theta) - i_a \sin(\theta)) \] (3)

\[ J \frac{d\omega}{dt} = T_e - T_L - B_m \omega \] (4)

Where \( R \) is winding phase resistance [\( \Omega \)], \( L \) is winding phase inductance [\( H \)], \( K_m \) is torque constant [\( V.s/rad \)], \( J \) is total inertial momentum [\( kgm^2 \)], \( B_m \) is friction coefficient [\( Nms \)], \( T_e \) is electromagnetic torque [\( Nm \)] and \( T_L \) is load torque [\( Nm \)].

3. Direct torque control

The basic idea of DTC is to control the torque and flux linkage by selecting the voltage vectors properly using the switching table. Stator magnetic flux can be calculated using equation:

\[ \overline{\Psi_s} = \int_{t}^{t+\Delta t} (\overline{u_s} - R_i \overline{i_s})dt \] (5)

In equation (5), the resistor \( R \) is small so stator flux linkage can be integral of stator voltage vectors. Control of flux and torque are by choosing proper voltage vectors. These voltage vectors are obtained from the switching table based on the flux and torque [5].

4. Model reference adaptive system

The MRAS estimator uses two models to calculate the speed of the hybrid stepper motor as shown in Fig. 1. One model is the reference model and the other an adaptive model. Outputs of these two models are compared in an adaptation mechanism to estimate the adjustable parameter. Finally it tunes the adaptive model in order to decrease the output error between two models to zero. The motor is the reference model and the current model of the HSM is the adjustable model. The estimated value of speed will converge to the true value through an appropriate adaptive mechanism.

Equations (1) and (2) are model of the HSM in stationary frame. By applying the d-q transformation to the voltage equations of hybrid stepper motor, the voltage equations on d-q frame which is rotating synchronously with the magnetic field is given below [8]:

\[ u_d = R_i d + L \frac{di_d}{dt} - L\omega i_q \] (6)
\[ u_q = L \omega i_d + R i_q + L \frac{d i_q}{dt} + K_m \omega \]  \hspace{1cm} (7)

The current model of HSM can be obtained from equation (6) and (7) as shown below:

\[ \frac{d i_d}{dt} = - \frac{R}{L} i_d + \omega i_q + \frac{u_d}{L} \]  \hspace{1cm} (8)

\[ \frac{d i_q}{dt} = - \omega i_d - \frac{R}{L} i_q - \frac{K_m}{L} \omega + \frac{u_q}{L} \]  \hspace{1cm} (9)

The equations (8) and (9) can be transformed to equation (10):

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
= \begin{bmatrix}
  \frac{R}{L} & \omega \\
  -\omega & \frac{R}{L}
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
+ \frac{1}{L} \begin{bmatrix}
  u_d \\
  u_q
\end{bmatrix}
\]  \hspace{1cm} (10)

In above equation, p means differentiate. According to equation (10), we can obtain equation (11):

\[
\begin{bmatrix}
  i_d + \frac{K_m}{L} \\
  i_q
\end{bmatrix}
= \begin{bmatrix}
  \frac{R}{L} & \omega \\
  -\omega & \frac{R}{L}
\end{bmatrix}
\begin{bmatrix}
  i_d + \frac{K_m}{L} \\
  i_q
\end{bmatrix}
+ \frac{1}{L} \begin{bmatrix}
  u_d + \frac{R K_m}{L} \\
  u_q
\end{bmatrix}
\]  \hspace{1cm} (11)

The system model can be simplified by using \( i_{d*}, i_{q*}, u_{d*} \) and \( u_{q*} \):

\[
i_{d*} = i_d + \frac{K_m}{L}, \quad i_{q*} = i_q, \quad u_{d*} = u_d + \frac{R K_m}{L} \quad \text{and} \quad u_{q*} = u_q
\]  \hspace{1cm} (12)

Putting equation (12) in equation (11), current model can be obtained:

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
= \begin{bmatrix}
  \frac{R}{L} & \omega \\
  -\omega & \frac{R}{L}
\end{bmatrix}
\begin{bmatrix}
  i_{d*} \\
  i_{q*}
\end{bmatrix}
+ \frac{1}{L} \begin{bmatrix}
  u_{d*} \\
  u_{q*}
\end{bmatrix}
\]  \hspace{1cm} (13)

Equation (14) is obtained by shortening equation (13) as shown below:

\[ p i^{*} = A i^{*} + B u^{*} \]  \hspace{1cm} (14)

According to equation (13), the adjustable model of HSM with speed angle as the adjustable parameter is obtained:

\[
\begin{bmatrix}
  \hat{i}_d \\
  \hat{i}_q
\end{bmatrix}
= \begin{bmatrix}
  \frac{R}{L} & \hat{\omega} \\
  -\hat{\omega} & \frac{R}{L}
\end{bmatrix}
\begin{bmatrix}
  \hat{i}_{d*} \\
  \hat{i}_{q*}
\end{bmatrix}
+ \frac{1}{L} \begin{bmatrix}
  u_{d*} \\
  u_{q*}
\end{bmatrix}
\]  \hspace{1cm} (15)

Where \( \hat{i}_{d*} \) and \( \hat{i}_{q*} \) are estimated values of d-q axis current and \( \hat{\omega} \) is estimated rotor speed. Equation (16) is obtained by shortening equation (15):

\[ p i^{*} = \hat{A} i^{*} + B u^{*} \]  \hspace{1cm} (16)

Consider error \( e = e^{*} - e^{*} \), and equation (17) is gotten by subtracting equation (14) from equation (16).
\[
\begin{bmatrix}
\mathbf{e}_d \\
\mathbf{e}_q
\end{bmatrix} =
\begin{bmatrix}
-\frac{R}{L} & \omega \\
-\omega & -\frac{R}{L}
\end{bmatrix}
\begin{bmatrix}
\mathbf{e}_d \\
\mathbf{e}_q
\end{bmatrix} + J(\omega - \hat{\omega})
\begin{bmatrix}
\hat{i}_d^* \\
\hat{i}_q^*
\end{bmatrix}
\]

Where \( \mathbf{e}_d = \mathbf{e}_d^* - \hat{\mathbf{e}}_d^* \), \( \mathbf{e}_q = \mathbf{e}_q^* - \hat{\mathbf{e}}_q^* \) and \( J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \). Consider \( W = J(\omega - \hat{\omega})\hat{i}^* \), and equation (18) is obtained by shortening equation (17)

\[
\mathbf{p} \mathbf{e} = \mathbf{A} \mathbf{e} - \mathbf{W}
\]

The system expressed by equation (18) is a standard nonlinear time-varying feedback system, so hyperstability theory based on feedback system can be used to design the adaptive mechanism. Equation (18) is changed to equation (19), In order to describe the system as a standard feedback system.

\[
\begin{cases}
\mathbf{p} \mathbf{e} = \mathbf{A} \mathbf{e} - \mathbf{W} \\
\mathbf{v} = \mathbf{C} \mathbf{e}
\end{cases}
\]

By considering \( \mathbf{C} = \mathbf{I} \), then \( \mathbf{v} = \mathbf{e} \). Hyperstability theory guarantees stability of the MARS if two conditions fulfill:

1) \( H(s) = (s\mathbf{I} - \mathbf{A})^{-1} \) must be a strictly positive matrix.

2) According to hyperstability theory, the Popov integral inequality must be satisfied as describe by equation (20).

\[
\eta(0, t_1) = \int_0^{t_1} \mathbf{v}^T \mathbf{W} \mathbf{d}t \geq -\gamma_0^2, \forall t_1 \geq 0
\]

In (20), \( \gamma_0 \) is a finite positive constant. Finally, the equation of \( \omega \) can be obtained as:

\[
\dot{\hat{\omega}} = (K_p + \frac{K_i}{s})(i_d \hat{i}_q - i_q \hat{i}_d - \frac{K_m}{L}(i_q - \hat{i}_q))
\]

(21)
5. Simulation Results

Fig. 1 shows the system schematic diagram of sensorless direct torque control of hybrid stepper motor based on model reference adaptive system. The drive system is simulated by Matlab software to confirm the analysis. In simulation, reference speed is set at 30rad/s. It drops to -30rad/s at 0.5s and then leaps to 30rad/s at 1s. Fig. 2 shows the motor torque. The load torque is 2N.m. Fig. 3 shows the output current of HSM. The waveforms of the real speed and the estimated speed of the motor are shown in Fig. 4 and Fig. 5. The error between the estimated speed and the real speed is illustrated in Fig. 6. The error is great when the motor is starting up but the steady state error is tiny. The waveforms of the real and the estimated position of the motor are shown in Fig.7 and Fig 8 and confirm that the precision of the system is high. Amplitude of stator flux linkage is maintained constantly about 0.5Wb as shown in Fig. 9.
6. Conclusion

In this paper, direct torque control strategy is applied to the hybrid stepper motor to control electromagnetic torque instantaneously and improve the dynamic response of the system. With model reference adaptive system, speed of the motor is estimated. The simulation results show that this method for HSM has strong stability, immunity and high steady state accuracy.

References


