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Physics Letters B 763 (2016) 416–421

Contents lists available at ScienceDirect

Physics Letters B

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Double scattering production of two positron–electron pairs in ultraperipheral heavy-ion collisions

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ARTICLE INFO

Article history:
Received 21 July 2016
Received in revised form 25 October 2016
Accepted 31 October 2016
Available online 4 November 2016
Editor: L. Rolandi

ABSTRACT

We present first measurable predictions for electromagnetic (two-photon) double scattering production of two positron–electron pairs in ultraperipheral heavy-ion collisions at LHC. Measurable cross sections are obtained with realistic cuts on electron/positron (pseudo)rapidities and transverse momenta for the ALICE and ATLAS or CMS experiments. The predictions for total and differential cross sections are presented. We show also two-dimensional distributions in rapidities of the opposite-sign (from the same or different subcollisions) and of the same-sign (e^+e^+ or e^-e^-) electrons and in rapidity distance between them. Expected number of events are presented and discussed. Our calculations strongly suggest that relevant measurements with the help of ATLAS, CMS and ALICE detectors are possible in a near future. We show and compare energy dependence of the cross sections for one-pair and two-pair production.

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1. Introduction

Multiple scattering effects are present in many reactions at high energies such as proton–nucleus (multiple nucleon–nucleon scatterings), proton–proton (double parton scatterings) and in ultraperipheral collisions (UPC) of heavy-ions. The double parton scattering effects in proton–proton collisions become increasingly important with steadily increasing energy in proton–proton collisions [1]. The cross section for double (multiple) scattering can be large provided the cross section for single parton scattering is large. The best example is double charm production in proton–proton collisions [2,3].

Not much attention was devoted to multiple scattering in UPC of heavy ions. In UPC of heavy ions, where in real experiments the integrated luminosity is rather small, in our opinion, only cross section for $AA \to AA\rho^0$ and $AA \to AAe^+e^-$ reactions is large enough [4] to potentially observe double scattering effects. The double-scattering (DS) mechanism for $\rho^0\rho^0$ production was studied e.g. in [5,6]. So far our prediction for four charged pion production was confronted only with the STAR data [7]. There the DS mechanism was not sufficient [6] to explain the existing STAR data [7]. Production of other meson combinations was discussed very recently in [8].

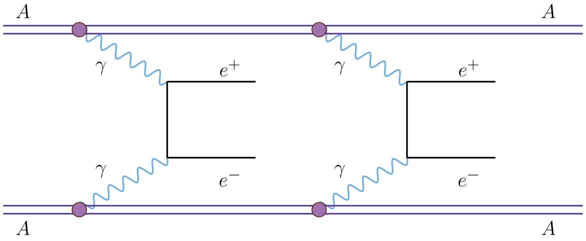

Fig. 1. Double-scattering mechanism for $e^+e^-e^+e^-$ production in ultrarelativistic UPC of heavy ions.

The double production of two dielectron pairs (see Fig. 1) was discussed e.g. in the context of bound-free production [9]. There rather total cross section is discussed. The total cross section is dominated by the very low transverse momenta of electrons. The low-transverse momentum electrons cannot be, however, measured at the LHC. Here we wish to make first predictions that have a chance to be verified experimentally at the LHC. Such a measurement would allow to verify (for the first time) our understanding of the underlying double scattering reaction mechanism in ultraperipheral heavy-ion collisions. We wish to emphasize that so far no double scattering mechanism in UPC was confirmed or unambiguously verified by experimental results on UPC of heavy ions. As we will show in the following the $PbPb \to PbPbe^+e^-e^+e^-$ (see Fig. 1) is a good candidate which has a good chance to be the first case in this context.

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http://dx.doi.org/10.1016/j.physletb.2016.10.079
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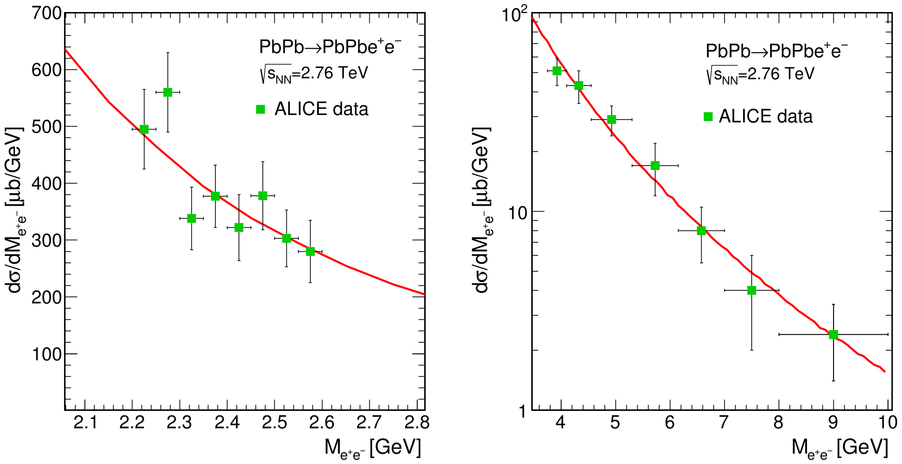

Fig. 2. Invariant mass distributions of dielectrons in UPC of heavy ions calculated within our approach [10] together with the recent ALICE data [22].

2. Sketch of the formalism

The cross section for single e^+e^- production is calculated as described in Ref. [10]. The total cross section can be written as:

$$\sigma_{A_1A_2\to A_1A_2e^+e^-}\left(\sqrt{s_{A_1A_2}}\right) = \int \sigma_{\gamma\gamma\to e^+e^-}\left(W_{\gamma\gamma}\right) N\left(\omega_1,\mathbf{b_1}\right) N\left(\omega_2,\mathbf{b_2}\right) S^2_{abs}\left(\mathbf{b}\right) \times 2\pi b\,\mathrm{d}b\,\mathrm{d}\overline{b}_x\,\mathrm{d}\overline{b}_y\,\frac{W_{\gamma\gamma}}{2}\mathrm{d}W_{\gamma\gamma}\,\mathrm{d}Y_{e^+e^-}\ , \tag{2.1}$$

where $N(\omega_i,\mathbf{b_i})$ are photon fluxes, $W_{\gamma\gamma} = M_{e^+e^-}$ and $Y_{e^+e^-} = \left(y_{e^+} + y_{e^-}\right)/2$ is a invariant mass and a rapidity of the outgoing e^+e^- system, respectively. Energy of photons is expressed through $\omega_{1/2} = W_{\gamma\gamma}/2\exp(\pm Y_{e^+e^-})$. $\mathbf{b}_1$ and $\mathbf{b}_2$ are impact parameters of the photon–photon collision point with respect to parent nuclei 1 and 2, respectively, and $\mathbf{b} = \mathbf{b_1} - \mathbf{b_2}$ is the standard impact parameter for the A_1A_2 collision. The quantities $\overline{b}_x$ and $\overline{b}_y$ are the components of the $(\mathbf{b}_1 + \mathbf{b}_2)/2$: $\overline{b}_x = (b_{1x} + b_{2x})/2$ and $\overline{b}_y = (b_{1y} + b_{2y})/2$. The five-fold integration is performed numerically. The gap survival factor $S^2_{abs}(\mathbf{b})$ of geometrical nature assures that we consider only ultraperipheral collisions as measured experimentally. For more details see [10]. Only in approximate case of simplified charge form factor the integration can be done analytically [4]. In both cases the integrated cross section can be then written formally as

$$\sigma_{A_1A_2\to A_1A_2e^+e^-} = \int P_{\gamma\gamma\to e^+e^-}(b)\,\mathrm{d}^2b\ . \tag{2.2}$$

Here $P_{\gamma\gamma\to e^+e^-}(b)$ has an interpretation of a probability to produce a single e^+e^- pair in the collision at the impact parameter b. This general formula is not very useful for practical calculation of double scattering. If the calculation is done naively $P_{\gamma\gamma\to e^+e^-}(b)$ can be larger than 1 in the region of low impact parameter. Then a unitarization procedure is needed [11].

If one wishes to impose some cuts on produced particles (electron, positron) which come from experimental requirements or to have distribution in some helpful and interesting kinematical variables of individual particles (here e^+ or e^-), more complicated calculations are required [12]. To have detailed information about rapidities of individual electrons an extra integration over a kinematical variable describing angular distribution for the $\gamma\gamma \to e^+e^-$ subprocess is required and the total $\sigma_{\gamma\gamma\to e^+e^-}$ cross section has to be replaced by relevant differential cross section. Then formula (2.2) can be written more differentially in kinematical variables of the produced leptons (rapidities and transverse momenta) as:

$$\frac{d\sigma_{A_1A_2\to A_1A_2e^+e^-}}{dy_+dy_-dp_t} = \int \frac{\mathrm{d}P_{\gamma\gamma\to e^+e^-}(b;y_+,y_-,p_t)}{dy_+dy_-dp_t}\,\mathrm{d}^2b\ . \tag{2.3}$$

Other choices of kinematical variables are possible as well. If one imposes cuts on transverse momenta of leptons the probabilities become small and no unitarization is needed.

The cross section for double scattering can be then written as:

$$\frac{\mathrm{d}\sigma_{AA\to AAe^+e^-e^+e^-}}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}y_3\mathrm{d}y_4} = \frac{1}{2}\int\left(\frac{\mathrm{d}P_{\gamma\gamma\to e^+e^-}\left(b,y_1,y_2;p_t>p_{t,cut}\right)}{\mathrm{d}y_1\mathrm{d}y_2} \times \frac{\mathrm{d}P_{\gamma\gamma\to e^+e^-}\left(b,y_3,y_4;p_t>p_{t,cut}\right)}{\mathrm{d}y_3\mathrm{d}y_4}\right) \times 2\pi b\,\mathrm{d}b\ . \tag{2.4}$$

The combinatorial factor 1/2 takes into account identity of the two pairs. We shall use the formula above to estimate the double scattering cross sections.

In our calculations here we use both realistic fluxes of photons calculated with charge form factors of a nucleus, being Fourier transform of realistic charge distributions or a more simplified formula from [10] is used.

From the technical point of view, first $\frac{dP(b,y_1,y_2;p_t>p_{t,cut})}{dy_1dy_2}$ are calculated on the three-dimensional grid in b, y_1 and y_2. Then in the next step those grids are used to calculate the cross sections corresponding to double scattering. We use the MC-based numerical integration program VEGAS [13]. For test we use also a grid-type integration.

In the present paper the calculation for both one-pair and two-pair production is done in the lowest-order QED approach. As will be shown in the Result section for one-pair production it is sufficient to reasonably describe the so-far available data for one-pair production (see our Fig. 2). The effect of higher-order corrections is very interesting. For example the effect of Coulomb corrections to the total (phase-space integrated) cross section for one-pair creation was discussed in a series of papers [14–21]. However, we do not know about calculations that can easily include experimental cuts (in rapidity or transverse momentum). The situation for two-pair production is even more complicated. The issue is very interesting but clearly goes beyond the scope of our paper where we wish to present only first estimation of the cross section for experimental cuts. One should return to the problem once higher-statistics experimental data are available.

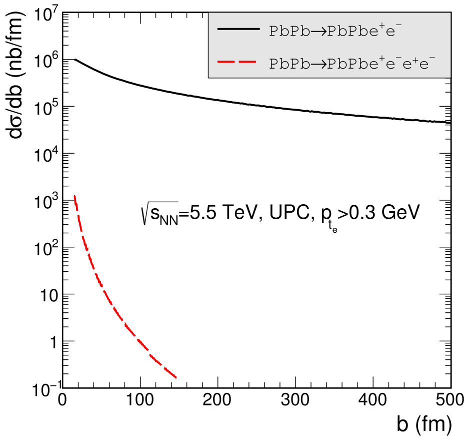

Fig. 3. Differential cross section as a function of impact parameter (distance between two colliding nuclei). The upper line denotes result for the PbPb → PbPbe$^+e^-$ reaction and the lower line shows result for the PbPb → PbPbe$^+e^-e^+e^-$ reaction.

Table I
Nuclear cross section for $PbPb \to PbPbe^+e^-e^+e^-$ at $\sqrt{s_{NN}} = 5.5$ TeV for different cuts specified in the table.

Cut set	σ_{UPC}	N$_{\text{events}}$ for L = 1 nb^{-1}
$p_{t_e} > 0.2$ GeV	52.525 μb	52 525
$p_{t_e} > 0.2$ GeV, $\lvert y_e\rvert < 2.5$	10.636 μb	10 636
$p_{t_e} > 0.2$ GeV, $\lvert y_e\rvert < 1$	0.649 μb	649
$p_{t_e} > 0.3$ GeV, $\lvert y_e\rvert < 4.9$	7.447 μb	7 447
$p_{t_e} > 0.3$ GeV, $\lvert y_e\rvert < 2.5$	2.052 μb	2 052
$p_{t_e} > 0.5$ GeV, $\lvert y_e\rvert < 4.9$	0.704 μb	704
$p_{t_e} > 0.5$ GeV, $\lvert y_e\rvert < 2.5$	0.235 μb	235
$p_{t_e} > 1$ GeV	25.2 nb	25
$p_{t_e} > 1$ GeV, $\lvert y_e\rvert < 4.9$	22.6 nb	23
$p_{t_e} > 1$ GeV, $\lvert y_e\rvert < 2.5$	9.8 nb	10
$p_{t_e} > 1$ GeV, $\lvert y_e\rvert < 1$	0.6 nb	1

3. First results

Before we present our results for $e^+e^-e^+e^-$ production let us compare our result with existing experimental data for single e^+e^- pair production. In Fig. 2 our results are compared with recent ALICE data [22]. Here we consider lead–lead UPC at $\sqrt{s_{NN}} =$ 2.76 TeV with $|y_e| < 0.9$. The left panel shows the ALICE data [22] for 2.2 GeV $< M_{ee} <$ 2.6 GeV and the right panel shows their results for 3.7 GeV $< M_{ee} <$ 10 GeV. Our results for single-scattering mechanism almost coincide with the experimental data.

Having shown that our approach allows to describe single pair production we can go to our predictions for two e^+e^- pair production. Now we are going to discuss briefly a purely theoretical distribution. Fig. 3 shows differential cross section as a function of impact parameter (distance between two nuclei) for lead–lead UPC at $\sqrt{s_{NN}} = 5.5$ TeV and $p_{t,e} > 0.3$ GeV. One can see that the cross section for $e^+e^-e^+e^-$ production drops off much faster than in the case of single e^+e^- production. The probability for the production of four particles is of course much lower than the probability for production of one electron–positron pair.

The shape in a somewhat theoretical distribution in the impact parameter is in our case (with cuts on transverse momenta) rather different from that shown in Fig. 10 in Ref. [23] for the total cross section. There is a clear correlation between the invariant mass of the produced system and the impact parameter.

In Table I we have collected integrated cross sections for different experimental cuts corresponding to ALICE and ATLAS or CMS experiments. In the ATLAS case we show result for the tracking detectors ($|\eta| < 2.5$) as well as including forward calorimeters ($|\eta| < 4.9$). The rapidity coverage of the CMS calorimeters is very similar. In the later case particle identification (PID) is much worse than for the tracker. However, the cross sections are then much larger than when using the tracker only. The main ALICE detector allows for the particle identification practically down to transverse momenta of 0.2 GeV, which makes it rather special. The number for full rapidity coverage and $p_t > 0.2$ GeV given in the table is much (three orders of magnitude) smaller than the total cross section for two pair production [9], where it was estimated to be about 10 mb.

In Fig. 4 we show our predictions for the opposite-sign $d\sigma/dy_1dy_2$ (left panel) and the same-sign $d\sigma/dy_1dy_3$ (right panel) electrons. We omit here trivial (experimental) factor 2 (two possibilities: two-scatterings for opposite sign and two signs of electrons for the same sign case). While the e^+e^- are correlated by the matrix element for the $\gamma\gamma \to e^+e^-$ subprocess the e^+e^+ (or e^-e^-) are not correlated. As a consequence the two-dimensional distributions in rapidities are broader for the case of the same-sign electrons.

In Fig. 5 we compare results for $d\sigma/dy_{diff}$ as a function of rapidity difference between the same-sign (solid line) and, from the same subcollision opposite-sign (dashed line) electrons assuming each of the electrons/positrons to be within the ATLAS main detector ($-2.5 < \eta_+, \eta_- < 2.5$) for transverse momenta $p_t > 0.5$ GeV (left panel) and for $p_t > 1$ GeV (right panel). Such distributions can, in our opinion, be measured at the LHC and could allow for a first verification of the double scattering mechanism in UPC of heavy ions. We wish to remind here that such a verification

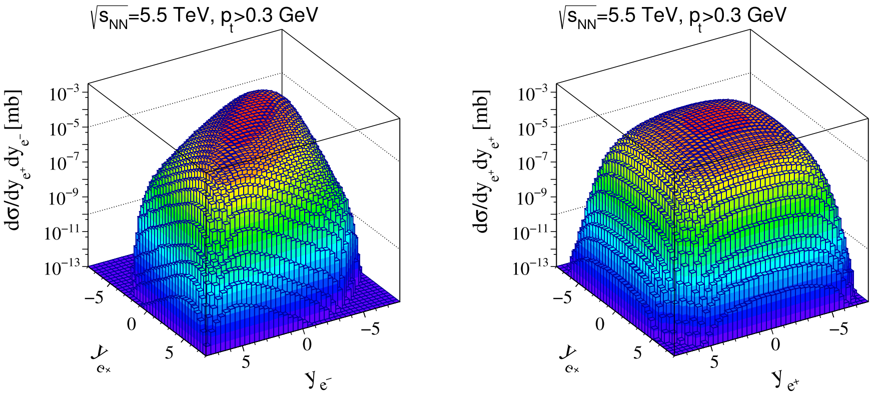

Fig. 4. Two-dimensional distribution in rapidities of the opposite-sign leptons from the same collision (left panel) and for the same-sign leptons (right panel). The cross section for the $e^+e^-e^+e^-$ production is calculated for lead–lead UPC at $\sqrt{s_{NN}} = 5.5$ TeV and $p_t > 0.3$ GeV.

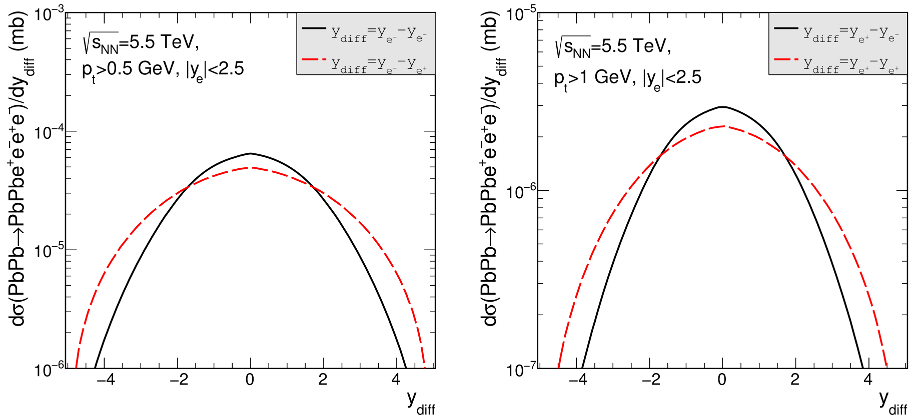

Fig. 5. Distributions in rapidity difference between the opposite-sign electrons (solid line) and between the same-sign electrons (or positrons) from the same subcollision (dashed line) for two different lower cuts on lepton transverse momenta: 0.5 GeV (left panel) and 1.0 GeV (right panel). This calculation is done assuming that electrons/positrons are measured by the ATLAS main tracker.

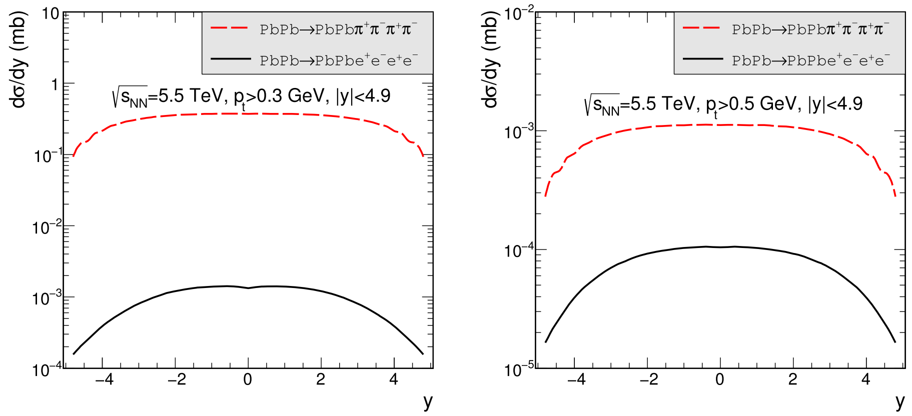

Fig. 6. Rapidity distribution of electron/positron (solid line) and charged pion (dashed line) for lead–lead collisions at the LHC ($\sqrt{s_{NN}} = 5.5$ TeV) together with the limitation on rapidity and transverse momenta of each single outgoing particle. The left panel shows results with limitation on $p_t > 0.3$ GeV and the right panel corresponds to $p_t > 0.5$ GeV.

Table II
Nuclear cross section for the $PbPb \to PbPb\pi^+\pi^-\pi^+\pi^-$ and $PbPb \to PbPbe^+e^-e^+e^-$ reactions at $\sqrt{s_{NN}} = 5.5$ TeV with $|y| < 4.9$ and for different cuts on transverse momenta of pions or electrons.

Reaction	$p_{t,min} = 0.3$ GeV	$p_{t,min} = 0.5$ GeV
$PbPb \to PbPb\pi^+\pi^-\pi^+\pi^-$	2.954 mb	8.862 µb
$PbPb \to PbPbe^+e^-e^+e^-$	7.447 µb	0.704 µb

was not possible for the double scattering production of two ρ^0 mesons [6] where other, at the moment not well understood, mechanisms probably play the dominant role [6].

Finally we wish to discuss briefly potential background(s). The $\mathrm{PbPb} \to \mathrm{PbPb}\pi^+\pi^-\pi^+\pi^-$ reaction discussed in Ref. [6] is a possibility. Here we include only double scattering production of two ρ^0 mesons which decay into four pions. In Fig. 6 we show a comparison of the cross sections for the $e^+e^-e^+e^-$ and $\pi^+\pi^-\pi^+\pi^-$ final states for two different lower cuts on transverse momenta. The cross section for four pions is much bigger than the cross section for four electrons. The situation improves when increasing the lower cut.

In Table II we show the cross section for the signal ($e^+e^-e^+e^-$) and the reducible background ($\pi^+\pi^-\pi^+\pi^-$) for broader range of pseudorapidities including not only main tracker but also calorimeters. The problem of PID in the calorimeter is not clear to us.

It is very interesting how the double scattering contribution depends on center-of-mass energy. In Fig. 7 we show and compare the cross sections for e^+e^- and $e^+e^-e^+e^-$ production for two different cuts on lepton transverse momenta. Both corresponding cross sections quickly grow with energy. The double scattering grows, however, much faster. Similar effect was observed e.g. for $c\bar{c}c\bar{c}$ production in proton–proton collisions [2].

In Fig. 8 we present the ratio of the $e^+e^-e^+e^-$ to e^+e^- cross sections as a function of the energy, again for two different cuts on p_t. The ratio strongly depends not only on the energy but also on the lower cut. The lower the lower cut, the larger the ratio is. In both cases (two different cuts) the ratio is much smaller than the ratio which can be inferred from Fig. 11 in Ref. [23]. The situation with cuts is therefore very different from that for total (unfortunately not accessible experimentally) cross section. This together with Table I shows that at the LHC the production of two pairs can be identified. At the FCC one could hopefully perform more detailed studies. May be only then one should worry about the QED higher orders.

4. Conclusions

In this paper we have presented first predictions for the production of two pairs of e^+e^- in ultraperipheral collisions for leptons with transverse momenta larger than some fixed values characteristic for specific detectors at the LHC [22]. We have presented results for the full range of rapidities as well as the results taking

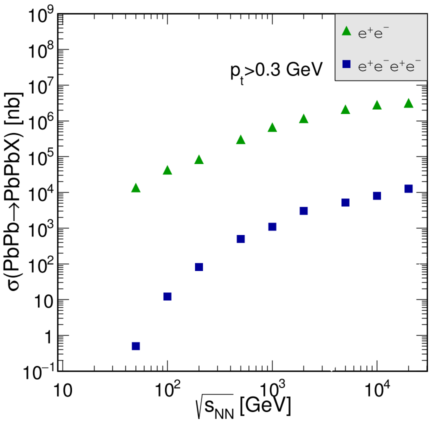

Fig. 7. Cross section for $AA \to AAe^+e^-$ and $AA \to AAe^-e^+e^-$ as a function of center of mass energy $\sqrt{s_{NN}}$ for two different cuts on lepton transverse momenta: $p_t > 0.3$ GeV (left panel) and $p_t > 2$ GeV (right panel).

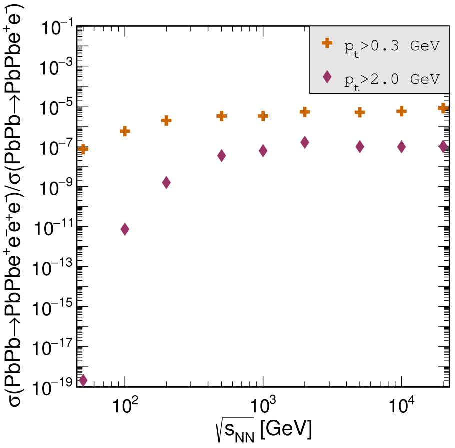

Fig. 8. The ratio of the integrated cross sections for $AA \to AAe^+e^-e^+e^-$ and $AA \to AAe^+e^-$ as a function of the center-of-mass energy for $p_t > 0.3$ GeV (upper points) and $p_t > 2$ GeV (lower points).

into account experimental cuts on rapidities characteristic for different experiments.

Before presenting our results for $e^+e^-e^+e^-$ production we have checked whether our approach describes the production of a single e^+e^- pair. A good agreement with the ALICE invariant mass distribution has been obtained.

Even imposing the experimental cuts relevant for different experiments we obtain cross sections that could be measured at the LHC even with relatively low luminosity required for UPC of heavy ions of the order of 1 nb^{-1}. For instance, assuming the integrated luminosity of 1 nb^{-1} for the main ATLAS detector angular coverage and transverse momentum cut on each electron/positron $p_t > 0.5$ GeV we predict 235 events.

Measurements of two electrons of the same sign would be already a clear signal of the double scattering mechanism. In addition, one could measure also two dimensional distributions or distributions in rapidity distance between two out of four produced electrons. The electron and positron from the same scattering have well balanced transverse momenta. They are also back-to-back in azimuthal angle. Excluding such cases by imposing exclusion cuts in transverse momentum balance and/or azimuthal angle, one could measure in coincidence electrons/positrons from different scatterings.

The distribution in relative azimuthal angle between two electrons or two positrons is another interesting observable. Assuming dominance of double scattering mechanism such a distribution should be flat (constant when assuming no azimuthal correlation in lepton production with respect to the nuclear scattering plane). Another, presented here, possibility is to measure distribution in relative rapidity distance between the same-sign and opposite-sign electrons. One could also measure corresponding invariant mass distributions (not discussed here) that are more difficult to calculate, however, from purely technical reasons.

Here we have calculated cross section for production of two pairs. The calculations for three pair production can be done easily. However, corresponding cross section is very small (further suppression by a few orders). Again the effect depends on center of mass energy. The situation is very different from for total cross section where the relative damping is only by one order of magnitude (see Fig. 11 in [23]). This means that at present the production of three pairs is no accessible experimentally.

Here we have considered production of two pairs of electrons and positrons. Similar calculation can be done for muons. If cuts are included the results for muons are not very different from those for electrons (for $\mu^+\mu^-$ production in UPC see e.g. [10]). For total cross section the situation would be, however, very different. Then we expect significantly smaller ratio than for electrons.

In future, for exact comparison to the measured cross sections a calculation of the single scattering $\gamma\gamma \to e^+e^-e^+e^-$ contribution may be also necessary. This computation goes, however, beyond the scope of the present study, where we have concentrated exclusively on double scattering mechanism. We leave such a study for a future.

In summary, our analysis shows that first measurement(s) of the double scattering in the $e^+e^-e^+e^-$ channel should be feasible. We expect therefore a clear response to our proposal of all experimental groups at the LHC.

Acknowledgements

We are indebted to Janusz Chwastowski and Rafał Staszewski for a discussion on possibilities of measuring the discussed here double scattering processes by the ATLAS Collaboration. This work was partially supported by the Polish grant No. DEC-2014/15/B/ST2/02528 (OPUS) as well as by the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge in Rzeszów.

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