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Natural supersymmetry and $b \to s\gamma$ constraints

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A R T I C L E   I N F O

A B S T R A C T

We investigate constraints from the observed branching ratio for $b \to s\gamma$ and fine-tuning in the framework of natural supersymmetry. The natural supersymmetry requires the large trilinear coupling of the stop sector, light higgsinos (a small μ parameter) and light stops, in order to reduce the fine-tuning in the Higgs sector while avoiding the LEP constraint. It is found that in such a scenario 5% (10%) level of fine-tuning is inevitable due to the $b \to s\gamma$ constraint even if the messenger scale is as low as 10^5 GeV (10^4 GeV), provided that the gaugino masses satisfy the GUT relation.

1. Introduction

The Standard Model (SM) of particle physics has achieved a remarkable success in describing physical phenomena accessible so far. In the SM, Higgs boson plays an important role since it breaks the electroweak symmetry and becomes the origin of the masses of the SM particles. The Higgs mass itself, however, is not protected by any symmetry, and thus receives disastrously large radiative corrections. It is so-called the hierarchy problem, or the naturalness problem, which may indicate that the SM is just a low-energy effective theory. One of the most attractive extensions of the SM is the Minimal Supersymmetric Standard Model (MSSM). Supersymmetry (SUSY) protects SM-like Higgs mass from quadratic-divergent correction, and then it provides a good solution to the hierarchy problem when superparticles exist just above the electroweak scale.

The ATLAS and CMS Collaborations are now searching the superparticles at the Large Hadron Collider (LHC). Since there has been no signal of superparticles so far, the collaborations provide stringent limits on the masses of colored superparticles [1,2]. However, these limits are directly applicable only to gluino and the first generation squarks in the general context. Naturalness in the supersymmetric models, on the other hand, requires the third generation squarks, especially stops, to have their masses around the electroweak scale. Thus, it is worth studying the case where only the third generation squarks are relatively light and the other superparticles are beyond the reach of the current collider experiments.

When considering the light stop scenario, one has to evaluate the Higgs mass carefully. This is because the prediction of the Higgs mass in the scenario might be lower than the LEP-II bound, 114.4 GeV [3]. In the evaluation of the Higgs mass in the MSSM, radiative corrections from top/stop loops are important. Since the tree-level SM-like Higgs boson mass is lighter than the Z boson mass, the stop masses cannot be too light. As listed in Refs. [4,5], naturalness argument calls for several conditions in addition to light stops; an adequate value of $\tan\beta$ in the Higgs sector, the large stop trilinear coupling A_t, the small Higgsino mass parameter μ in the superpotential and the small messenger scale. (Definition of those parameters are given in the next section.) The first two conditions are related to the Higgs mass bound [6–8], while the others are to improve naturalness, which is discussed in the subsequent section. Although such a light stop scenario is favored in terms of the naturalness argument, contributions of superparticles to the inclusive decay rate $B \to X_s\gamma$ are enhanced [9], and it should be carefully taken care of.

In this Letter we study the level of fine-tuning by taking into account the constraints from the branching ratio for $b \to s\gamma$, as well as the LEP-II bound for the mass of the SM-like Higgs boson. Assuming the GUT relation among gaugino masses, it is found that such region is severely constrained, and consequently at least about 5% fine-tuning is required within the framework of the MSSM.[1] Constraints on natural supersymmetry is also discussed based on the recent LHC results in Ref. [10]. (See also the earlier

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[1] In the following analysis, we set the messenger scale to be 10^5 GeV. When the messenger scale is 10^4 GeV, the minimum requirement of fine-tuning is relaxed, up to 10%.

work [11].) While the constraints given there would be relaxed by *R*-parity violation our result is applicable even for such a case.

2. Naturalness

In the MSSM, there are two Higgs fields, H_u and H_d, with their vacuum expectation values (VEVs), v_u and v_d, breaking the electroweak symmetry. The VEVs are corresponding to the minimum of the Higgs potential, and in the radial direction to the minimum, the potential is simply written as

$$V = m^2|h|^2 + \frac{\lambda}{4}|h|^4, \tag{1}$$

where h is a linear combination of the Higgs fields. The mass of the physical Higgs boson m_h is determined by the curvature of the potential in the direction around the minimum. A brief calculation leads to the relation

$$m_h^2 = -2m^2. \tag{2}$$

At tree-level, the physical Higgs mass is bounded above in the MSSM:

$$m_h \leqslant m_Z\big|\cos(2\beta)\big|, \tag{3}$$

where m_Z is the mass of Z boson, and $\tan\beta \equiv v_u/v_d$. The relation originates from the fact that the quartic coupling λ in Eq. (1) is written in terms of the electroweak gauge couplings. The upper limit is satisfied in the so-called decoupling limit, where the mass of the CP-odd neutral scalar m_A is much larger than m_Z. In this case, the lighter Higgs mass eigenstate behaves like the SM Higgs boson. However, even in the decoupling limit with a large value of $\tan\beta$, the tree-level mass is not sufficient to exceed the LEP bound. It is accomplished with the help of radiative corrections [12]. In the region with (moderately) large $\tan\beta$, m^2 in Eq. (1) is expressed as

$$m^2 \simeq |\mu|^2 + m_{H_u}^2\big|_{M_{\mathrm{mess}}} + \delta m_{H_u}^2, \tag{4}$$

where μ is the mass for the Higgs superfields and $m_{H_u}\big|_{M_{\mathrm{mess}}}$ is the soft SUSY breaking mass parameter for the up-type Higgs at a scale M_{mess} where the soft SUSY breaking terms are generated. The third term on the right-hand side is radiative correction from the renormalization group running between M_{mess} and the stop mass scale $m_{\tilde{t}}$, which can be estimated as[2]

$$\begin{aligned}
\delta m_{H_u}^2 &\simeq m_{H_u}^2\big|_{m_{\tilde{t}}} - m_{H_u}^2\big|_{M_{\mathrm{mess}}} \\
&\simeq -\frac{3y_t^2}{8\pi^2}\big(m_{\tilde{Q}_3}^2 + m_{\tilde{t}_R}^2 + |A_t|^2\big)\ln\frac{M_{\mathrm{mess}}}{m_{\tilde{t}}}, \tag{5}
\end{aligned}$$

where y_t is the top Yukawa coupling, and A_t is the trilinear scalar coupling of the top sector. $m_{\tilde{Q}_3}^2$ and $m_{\tilde{t}_R}^2$ denote the soft SUSY breaking mass parameters of third generation squark doublet and singlet, respectively. Since the radiative correction is controlled by the mass scale of the stops, the realization of the electroweak symmetry breaking requires fine-tuning among three terms in Eq. (4), unless M_{mess} is sufficiently low and/or the soft masses for the stops are small. In order to quantify the level of fine-tuning, we define a measure of the fine-tuning as [4]

$$\Delta^{-1} = \frac{m_h^2}{2(-\delta m_{H_u}^2)}. \tag{6}$$

It is obvious that larger values of the stop mass parameters lead to further fine-tuning. Note that a large μ parameter also requires the cancellation, therefore the maximum value is constrained by

$$|\mu| \lesssim (210\,\mathrm{GeV})\left(\frac{15\%}{\Delta^{-1}}\right)^{1/2}\left(\frac{m_h}{115\,\mathrm{GeV}}\right). \tag{7}$$

The Higgs mass bound given by the LEP-II experiment, $m_h > 114.4$ GeV, can be satisfied if one sets A_t instead of the stop soft masses to be large. Therefore it is expected that the fine-tuning is reduced when the stops are light and A_t is large. However, we will show that with such light stops and large A_t, the chargino contribution to the $b \to s\gamma$ process becomes large and then wide range of the parameter space where $\Delta^{-1} \gtrsim$ a few % is excluded.

3. The constraint from $b \to s\gamma$

The process $b \to s\gamma$ is suppressed by a loop-factor and the off-diagonal element of the CKM matrix in the SM. The experimental value of $b \to s\gamma$ roughly agrees with the prediction in the SM:

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{exp}} = (3.55 \pm 0.26) \times 10^{-4}, \tag{8}$$

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{SM}} = (3.15 \pm 0.23) \times 10^{-4}. \tag{9}$$

Here "exp" and "SM" mean the experimental value [13] and the SM prediction including next-to-next-to-leading order QCD corrections [14] of the branching ratio for the inclusive radiative decay $B \to X_s\gamma$. The deviation of the SM prediction from the observation is

$$-0.3 \times 10^{-4} < \Delta\,\mathrm{Br}(B \to X_s\gamma) < 1.1 \times 10^{-4}, \tag{10}$$

at 2σ level. Generally, in supersymmetric models, the loop diagrams in which superparticles run could induce comparable or even larger contributions to $b \to s\gamma$ [9], resulting in significant deviation from the SM prediction. For example, it is known that the light sparticles with large $\tan\beta$ are severely constrained from the experimental data.
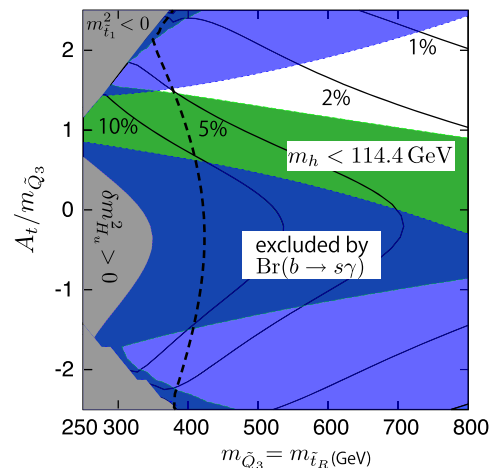

Fig. 1. Contours of the fine-tuning measure Δ^{-1}. The SUSY parameters are set to $m_A = 1$ TeV, $\tan\beta = 10$ and $\mu = 200$ GeV. The gluino mass is taken as $M_3 = 750$ GeV, and the GUT relation among gaugino masses is assumed so that $M_1 : M_2 : M_3 = 1 : 2 : 6$. The input parameters are defined at the scale of $m_{\tilde{Q}_3} = m_{\tilde{t}_R}$. The messenger scale is set to be 10^5 GeV. The blue region is excluded by $b \to s\gamma$ and the green region is excluded by the LEP bound. In the left region of the black dashed line, $m_{\tilde{Q}_3}^2$ is negative at the messenger scale, $m_{\tilde{Q}_3}^2(M_{\mathrm{mess}}) < 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)
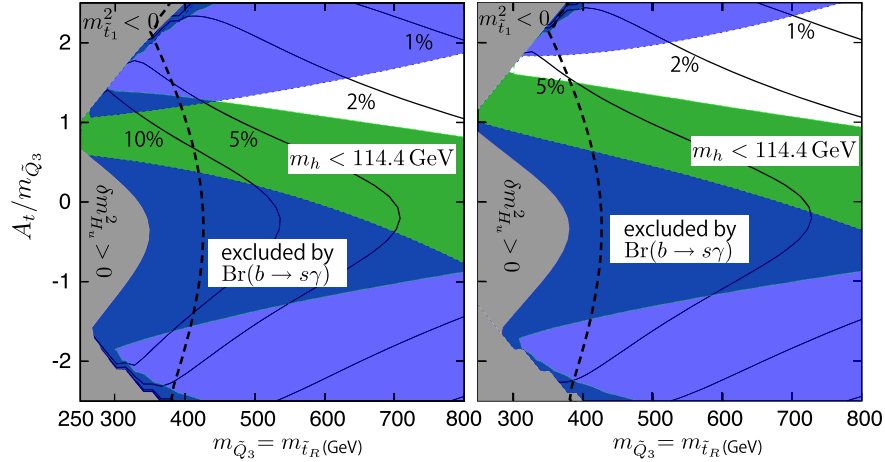
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[2] Although this estimation is a leading log approximation, in numerical calculations, we evaluate $\delta m_{H_u}^2$ by solving renormalization group equations, with gluino contributions included.

Fig. 2. Contours of the fine-tuning measure Δ^{-1}. The μ parameters are taken as $\mu = -200$ GeV in the left panel, and $\mu = 300$ GeV in the right panel. Other parameters are same as in Fig. 1.

Important contributions to $b \rightarrow s\gamma$ arise from the charged Higgs and chargino loop diagrams within the framework of the minimal flavor violation in which the only source for flavor/CP violation arises through the CKM matrix elements. (For SUSY contributions to CP/flavor violating processes, see Ref. [15].) The charged Higgs contribution always increases the branching ratio since it interferes constructively with the SM contribution. On the other hand, the chargino contributions can be either constructive or destructive, depending on the sign and the size of the μ parameter, the wino mass, M_2, and the trilinear coupling A_t. When both M_2 and μ are positive, a negative value of A_t decreases the branching ratio while a (sufficiently large) positive A_t increases it.[3] In a nutshell, contributions from superparticles in the loops to $b \rightarrow s\gamma$ are enhanced by a small μ parameter, a large A_t, light stops and a large $\tan\beta$. However, avoiding the enhancement of $b \rightarrow s\gamma$, a certain level of fine-tuning is expected in such a parameter region, as discussed in the previous section.

In our numerical calculation, we demand that contributions of superparticle loops to $b \rightarrow s\gamma$ should not exceed the experimental value. Namely, we constrain parameter region in the MSSM by imposing

$$-0.3 \times 10^{-4} < \Delta'\,\mathrm{Br}(B \rightarrow X_s\gamma) < 1.1 \times 10^{-4}, \qquad (11)$$

where $\Delta'\,\mathrm{Br}(B \rightarrow X_s\gamma) = \mathrm{Br}'(B \rightarrow X_s\gamma)_{\mathrm{MSSM}} - \mathrm{Br}'(B \rightarrow X_s\gamma)_{\mathrm{SM}}$. (Primes indicate results from our numerical calculation.)

In Fig. 1, the contours of Δ^{-1} and the constraint from the $b \rightarrow s\gamma$ are shown. The blue and green regions are excluded by $b \rightarrow s\gamma$ and the LEP bound, respectively. Here we take the CP-odd Higgs mass parameter as $m_A = 1$ TeV, $\mu = 200$ GeV and $\tan\beta = 10$. As for gaugino masses, gluino mass M_3 is taken to be 750 GeV and the GUT relation is assumed for the rest. The messenger scale is set to be 10^5 GeV. The top quark mass is taken as $m_t = 173.2$ GeV. The Higgs pole mass is calculated by using FeynHiggs [17] and the $\delta m_{H_u}^2$ is evaluated by solving renormalization group equations in SOFTSUSY package [18]. The branching ration of $b \rightarrow s\gamma$ is calculated by SusyBSG [19]. In the region where stops are light and A_t is large, chargino contributions are large and the SUSY contributions exceed the constraint given in Eq. (11). Therefore the large part of parameter space in which the fine-tuning is relaxed, is excluded. It is found that the maximum value of the fine-tuning parameter in the allowed region is $\sim 5\%$.
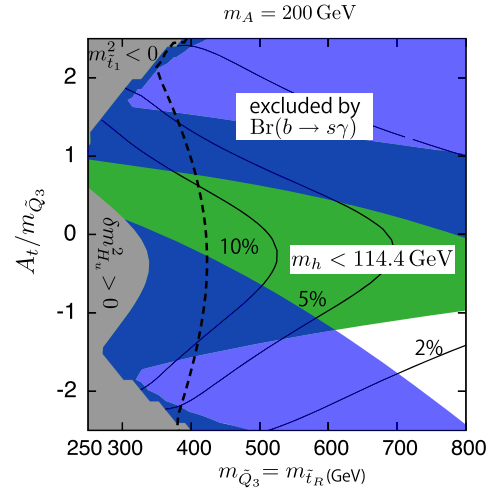

Fig. 3. Contours of the fine-tuning measure Δ^{-1}. The CP-odd Higgs mass is taken as $m_A = 200$ GeV. Other parameters are same as in Fig. 1.

In Fig. 2, we also show contours of Δ^{-1} with different values of μ parameter. In the left panel, we take $\mu = -200$ GeV.[4] In this case, the allowed region is slightly shifted to the direction in which A_t is small. However, the result does not change significantly and the level of fine-tuning becomes slightly worse than the case with positive a μ. In the right panel, we take $\mu = 300$ GeV, where the maximum value of Δ^{-1} is limited to $\sim 7\%$ (see Eq. (7)). In this case, the allowed region turns out to be merely wider than the case with $\mu = 200$ GeV and the region with $\Delta^{-1} \sim 7\%$ is allowed. However, such a region is likely to be exclude because the lightest stop mass is less than 90 GeV. Moreover the stop mass squared $m_{\tilde{Q}_3}^2$ is negative at the messenger scale; there exist a color breaking minimum, which tends to be deeper than the electroweak symmetry breaking minimum as $V \sim m_{\mathrm{soft}}^2 M_{\mathrm{mess}}^2$ (m_{soft} is a typical scale of soft SUSY breaking parameters).

We have also checked the cases where (i) $\tan\beta$ is larger ($\tan\beta = 15$), (ii) $\tan\beta$ is smaller ($\tan\beta = 5$) and (iii) m_A is smaller (e.g., $m_A = 200$ GeV (see Fig. 3) and 400 GeV), while keeping other parameters unchanged. In the case (i), it is obvious that the chargino contributions to $b \rightarrow s\gamma$ become larger, resulting in

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[3] We follow the convention in the SLHA [16].

[4] Although a negative μ parameter is not favored in terms of the muon $g-2$, the SUSY contributions can be suppressed with sufficiently heavy sleptons.
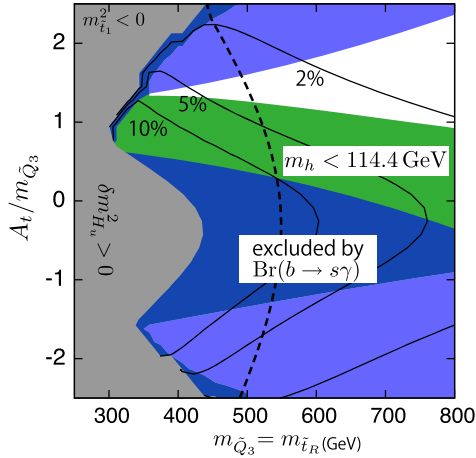
Fig. 4. Contours of the fine-tuning measure Δ^{-1}. The parameters are same as in Fig. 1 except for gaugino masses as $M_3 = 1$ TeV. The Bino and Wino masses are set with the GUT relation.

narrower allowed region. Therefore the level of fine-tuning is not relaxed at all. On the other hand, in the case (ii), although the allowed region by $b \to s\gamma$ becomes wider, larger amount of the radiative corrections to the Higgs mass is required to avoid the LEP constraint. This is because the tree-level Higgs mass is smaller than that with $\tan\beta = 10$. In fact, the maximum value of Δ^{-1} is smaller compared to the case with $\tan\beta = 10$. In the case (iii) the allowed region is shifted to the direction with small A_t (see Fig. 3 for example). The fine-tuning is not relaxed, even worse in this case.

In Fig. 4 the result for larger gaugino masses is shown. The gaugino mass is taken as $M_3 = 1$ TeV with satisfying the GUT relation. The fine-tuning is not ameliorated (becomes worse) unless the stops are tachyonic at the messenger scale.

It is noted that we assume the GUT relation among the gaugino masses in this work. Although we could not find a region with larger Δ^{-1} due to $b \to s\gamma$ constraints,[5] it is possible to find a "less" fine-tuned region with an elaborate choice of m_A and M_2, once the GUT relation is relaxed; the different contributions to $b \to s\gamma$ can cancel each other, which is yet another tuning.

Finally we comment on Higgs phenomenology at the LHC. In the allowed region, we also calculated Higgs production rate at the LHC and its branching ratio of each decay mode. At the LHC, gluon fusion is the main production process. It turns out that the Higgs production rate is almost unchanged compared to the SM value. The Higgs decay property is also similar to that in the SM. The decay mode $h \to \gamma\gamma$ is especially important for the discovery of Higgs boson, as well as the determination of its mass, in the light Higgs scenario. We found a few % difference in the decay rate. The channels in which Higgs decaying to WW and ZZ, on the other hand, are reduced by up to about 10%.

4. Conclusion

In this Letter we have studied the level of fine-tuning in the Higgs sector, considering the constraints from the observation of

the branching ratio for $b \to s\gamma$ in the MSSM. While light stops are favored by the relaxation of the fine-tuning, it would predict the large branching ratio which significantly deviates from the experimental value. It is found that the parameter region where the fine-tuning measure is larger than 5% (10%) is excluded by $b \to s\gamma$ constraints even for low messenger scale as 10^5 GeV (10^4 GeV), assuming the GUT relation among gaugino masses. Therefore, being consistent with the present experiments, realization of the natural supersymmetry is difficult. Note that although the light stops may avoid the constraints from SUSY searches in the cases that R-parity is violated, our result can be applied even for such a case.

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[5] In mirage mediation [20,21] scenario where gaugino masses are unified at TeV scale, it is shown that $\Delta^{-1} > 20\%$ is possible in the parameter region which is consistent with $b \to s\gamma$ observation [22].