

Available online at www.sciencedirect.com**SciVerse ScienceDirect**

Procedia Engineering 29 (2012) 375 – 380

**Procedia
Engineering**www.elsevier.com/locate/procedia

2012 International Workshop on Information and Electronics Engineering

A Review on the Numerical Inversion Methods of Relative Permeability Curves

Jian Hou^{a*}, Dai-gang Wang^a, Fu-quan Luo^a, Yan-hui Zhang^a^aCollege of Petroleum Engineering, China University of Petroleum, Qingdao 266555, China

Abstract

In general, relative permeability data can be obtained from laboratory coreflooding experiments. Such experimental data can be interpreted analytically or numerically. Compared to analytical methods, when the numerical inversion methods are applied to interpret the coreflooding experimental data, the reservoir performance obtained prior to and after breakthrough can be utilized comprehensively, the capillary effects and the heterogeneity of core samples can also be taken into account, so the estimated result is not only accurate but also complete. Moreover, the numerical inversion methods can be applied to large-scale reservoirs. This article introduces systematically the methodology of numerical inversion methods, and then reviews the present research status. Finally, several proposals of implicitly estimating relative permeability data are put forward from aspects of optimization algorithms' properties, estimation of endpoint saturations and treatment scale by automatic history matching.

© 2011 Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](http://creativecommons.org/licenses/by-nc-nd/3.0/).

Keywords: relative permeability curve; numerical inversion method; optimization algorithm; research status

1. Introduction

In general, relative permeability data can be obtained from laboratory coreflooding experiments. However, owing to the limitation of assumptions, the accuracy of relative permeability data obtained by analytical methods is relatively low. In order to improve reliability of the estimated results, a novel technique, history matching, where relative permeabilities as well as absolute permeability, and porosity are adapted using a reservoir simulator was introduced to achieve a reservoir representation in an agreement with the observed reservoir performance after the 1960s. Sigmund and McCaffery^[1] were the

*Jian Hou. Tel.: 13954670741.

E-mail address: houjian@hdpu.edu.cn.

first to apply nonlinear regression to solve the problem of history matching laboratory coreflooding data. Moreover, Chavent et al.^[2] also made thorough studies on the estimation of relative permeability curves using numerical inversion methods. Recently, Chen et al.^[3-4] did a lot of work in the aspects of relative permeability representation models and optimization methods. Li et al.^[5-6] estimated relative permeability curves by matching pressure and water cut data using optimization methods, e.g., the conjugate gradient algorithm. In addition, Yao et al.^[7] made a thorough study on optimization methods such as the EnKF algorithm and the gradient-based algorithms. This article introduces systematically the methodology of numerical inversion methods, and then reviews the present research status. Finally, several proposals of implicitly estimating relative permeability data are put forward from aspects of optimization algorithms' properties, estimation of endpoint saturations and treatment scale by automatic history matching.

2. Methodology

The basic idea of implicitly estimating relative permeability curves is depicted as the following: select reservoir performance to establish a least-squares objective function; determine a relative permeability representation model; with the assistance of a simulator, use an optimization algorithm to subsequently adjust the controlling parameters of the representation model to minimize the objective function, and thus estimate relative permeability curves implicitly.

2.1. Establishment of objective function

Based on the theory that the predicted data should be in agreement with the observed, a least-squares objective function must be established to implicitly estimate relative permeability curves, which can be expressed as:

$$J = \left[\vec{Y} - \vec{F}(\vec{\beta}) \right]^T W \left[\vec{Y} - \vec{F}(\vec{\beta}) \right] \quad (1)$$

Where $\vec{\beta}$ is a $(n \times 1)$ vector of the unknown parameters; \vec{Y} is a $(n \times 1)$ vector of the observed data; \vec{F} is a $(n \times 1)$ vector of the predicted data; W is a $(n \times n)$ covariance matrix. For the inversion history matching problems, the objective function J is usually nonlinear, and the controlling vector $\vec{\beta}$ should be limited to certain rational range according to actual situation.

As to the estimation of relative permeability curves, most researchers utilize both pressure data and production data to establish the objective function; a few of researchers discuss the feasibility of using pressure or production data only. For example, Watson et al.^[8] estimated oil-water relative permeability curves by matching water cut data of a linear flow system only. They found that there is considerable uncertainty in the estimated result when only water cut data is matched, but that the uncertainty is less when both pressure and water cut data are matched. Chen et al.^[9] also made a thorough study on the uncertainty of using different reservoir performance to estimate relative permeability curves. They found that it is easier to obtain an accurate estimate of the sensitivity of cumulative oil production to parameters than to obtain a good estimate of the sensitivity of the instantaneous oil (or water) rate to model parameters. Barroeta and Thompson^[10] made a preliminary study on the feasibility of matching pressure data prior to and after water breakthrough only. They found that the results obtained while matching pressure only were better than those from matching volumetric data only. After the 1990s, Chardaire et al.^[11] proposed a new method to estimate oil-water relative permeability curves using pressure drop, local saturation profile and cumulative production data as observation data.

2.2. Selection of optimization algorithms

Among the inverse history matching problems, the commonly used optimization algorithms include gradient-based algorithms, stochastic algorithms and hybrid algorithms.

When using the gradient-based algorithms for optimization, the search direction is determined by computing the Hessian matrix or gradient of the objective function. Due to their efficiency and favorable convergence, the gradient-based algorithms have been widely used in automatic history matching. Moreover, the time spent in computing the gradient is longest, so how to obtain the gradient rapidly and accurately is the key to improve convergence speed. For history matching of large-scale reservoirs, the objective function is so complex, even implicit, that numerical methods must be utilized to compute the gradient. Once the gradient is obtained, optimization algorithms, e.g., the LBFGS algorithm can be used to adjust the controlling parameters automatically.

Stochastic algorithms are heuristic algorithms constructed by certain intuitive basis, which are suitable for the complex problems whose objective function is difficult to express explicitly. The commonly methods include genetic algorithm^[12], particle swarm optimization algorithm^[13], and so on. However, the application of stochastic algorithms relies on properties of practical problems and researchers' experience, and it is difficult to summarize the rules. The global optimal solution can be found using stochastic algorithms, but it may take hundreds or thousands of simulation runs for the calculation to converge, which makes such methods infeasible in field-case history matching.

Hybrid algorithms are combinations of several optimization algorithms, which make up for their own deficiencies. Compared with the gradient-based and stochastic algorithms, both the computation accuracy and the convergence speed can be preferably guaranteed, which makes them hopeful to be applied for history matching on large-scale fields. The main methods include the EnKF algorithm^[14], the EnOpt algorithm^[15], and so on. However, they are still at exploring stage.

It is usually according to petrophysical properties of reservoirs to select the optimization algorithms. Moreover, whether it is easy to implement must also be taken into account. In general, the gradient-based algorithms which have high efficiency and rapid convergence speed should be considered in priority. When the objective function is so complex that it is difficult to compute its gradient, it is feasible to select the stochastic algorithms for optimization. Furthermore, the hybrid algorithms or parallel computation techniques can also be utilized to improve convergence speed.

2.3. Construction of relative permeability representation model

When implicitly estimating relative permeability data, an accurate relative permeability representation model is required. The following two requirements are necessary in regard to the representation model: (1) the model must be capable of representing the actual shape of the relative permeability curves; (2) the model must be controlled by a limited number of controlling parameters. According to whether it is required to assume the shape of the relative permeability curves, there are two main categories in the representation models: the parametric model and the non-parametric model.

The parametric model uses explicit equations to generate the relative permeability curves, assuming the relative permeability curves fit into the shape of a certain type of functional model. Due to its simplicity, the power law model^[16] has been widely used to depict relative permeability curves, the basic idea of which is to approximate the actual relative permeability curves using power-law representations. In the exponential model^[17], an exponential function is utilized to approximate the true curves. Similar to the power-law model, the exponential model is also controlled by six parameters. It is found that the accuracy of the exponential model is higher than that of the power-law model. However, the model has not been used widely due to its complex structure.

The non-parametric model is far more general and flexible as there is no assumption regarding the shape of the relative permeability curves (e.g., the cubic spline model and B-spline model). In the cubic

spline model^[18], the cubic splines is used to generate the relative permeability curves. The B-spline model^[19] divides the study area into several intervals, and utilize a piecewise continuous cubic polynomial to represent the true curves of every interval. Figure 1(a) reflects its local fitting feature. Chen et al^[20] further developed the B-spline model, and proposed a cubic uniform B-spline model. Compared with classical B-spline model, the model is far more general and flexible. Figure 1(b) is the relative permeability curves generated from the uniform B-spline model. Recently, the uniform B-spline model has been the most commonly used representation model.

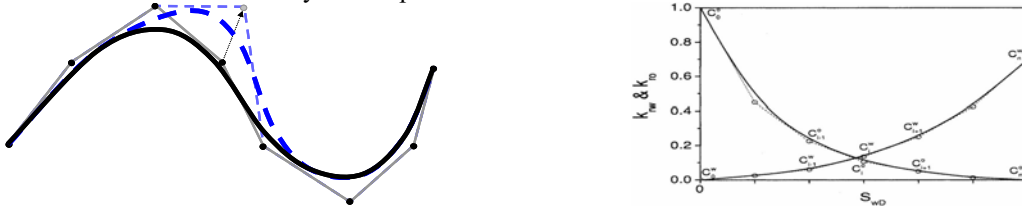


Fig. 1. (a) feature of B-spline model's local fitting; (b) relative permeability curves generated from the uniform B-spline model

When implicitly estimating three-phase relative permeabilities, the two-phase relative permeability curves for an oil-gas system and oil-water system must be depicted by representation models. Moreover, it is necessary to establish a relationship to calculate the three-phase relative permeabilities. Stone^[21] made a thorough study on percolation characteristics of multiphase fluid, and developed a modified Stone II model. At present, the modified Stone II model has been widely used to determine three-phase relative permeability curves.

3. Estimation of relative permeability curves at different scales

Relative permeability data can be obtained from laboratory coreflooding experiments or the actual production data. The research status of estimating relative permeability curves implicitly at different scales will be reviewed in the following.

3.1. Estimation of relative permeability curves from laboratory experiments

Compared to analytical methods, when numerical inversion methods are applied to laboratory experiments, the reservoir performance obtained prior to and after breakthrough can be utilized comprehensively, the capillary effects and the heterogeneity can also be considered, so the estimated result is not only accurate but also complete.

Similar to Sigmund and McCaffer's^[1] research, Kerig^[18] also considered a problem of history matching laboratory coreflooding data. They used cubic splines to parameterize relative permeability curves and compared the estimates obtained with the power-law estimates. Watson et al.^[19] also did a lot of work in aspects of estimating relative permeability curves from laboratory experiments, while the capillary effects are considered. Moreover, the heterogeneity can also be taken into account. Li Heng et al.^[22] proposed a novel technique to implicitly estimate the absolute and relative permeability by history matching laboratory coreflooding data with the EnKF algorithm.

3.2. Estimation of relative permeability curves on coarse-scale reservoirs

Due to the scale difference between the core samples and the reservoir, and the difference between the reservoir condition and the laboratory condition, the relative permeability curves obtained from laboratory

experiments may not be representative of the flow features at the field-scale. Therefore, many researchers studied on estimation of relative permeability curves on coarse-scale reservoirs.

Lee and Seinfeld^[23] considered the simultaneous estimation of the absolute and relative permeability for a two-dimensional, two-phase flow system. Yang and Watson^[24] implicitly estimated the relative permeability curves using a Bayesian approach with relative permeability functions modeled as a linear combination of B-splines. Okano et al.^[25] proposed a methodology to estimate relative permeability at the coarse scale using the neighborhood approximation for algorithm. Li et al^[26] implemented a procedure to implicitly estimate absolute and relative permeability using Bayesian estimation to generate estimates.

Reynolds and Oliver^[27] presented a procedure to estimate relative permeability curves together with the absolute permeability by history matching three-phase production data. The power law model was used to generate two-phase relative permeability curves, and the three-phase relative permeability was calculated from the modified Stone II model. Eydinov et al^[28] further developed the technique, and estimated relative permeability curves with gridblock porosities and permeabilities simultaneously.

4. Conclusion and recommendations

With the advent of modern computer technology, the numerical inversion methods have been widely used to estimate relative permeability curves implicitly at different scales. As to their shortages, several proposals are put forward in the following.

Further development of optimization algorithms' properties Properties of optimization algorithms have great influence on treatment scale of history matching. The traditional optimization algorithms can not satisfy the convergence speed. Recently, with the development of the gradient-based and hybrid algorithms, the treatment scale has been improved to a certain extent. But when dealing with inversion problems with large variables, it may take thousands or millions of simulation runs to converge. Therefore, how to consider the convergence speed and computation accuracy simultaneously still needs to be studied thoroughly. It is suggested that the research should focus on further development of hybrid optimization algorithms and parallel computation techniques.

Estimation accurately of the endpoint saturations The initial conditions are so sensitive to endpoint saturations that the endpoint saturations were often assumed to be known in previous studies. If the prior information for endpoint saturations is not enough, the computation accuracy can not be guaranteed. Therefore, when lacking of the prior information, estimation accurately of the endpoint saturations remains a challenge. If endpoint saturations are included as parameters, we advise to consider the fact that initial conditions are sensitive to endpoint saturations by modifying the optimization algorithms.

Further enlargement of treatment scale by automatic history matching It is manifested as three-dimensional, three-phase flow in actual reservoirs. However, the related researches mainly focus on estimating relative permeability curves from laboratory experiments or 2D synthetic cases. Relatively fewer researches have been carried out on estimation of relative permeability curves by automatic history matching three-dimensional, three-phase production data. With the advent of efficient optimization algorithms, estimation of relative permeability curves by history matching three-dimensional production data should be studied deeply.

Acknowledgements

The writers greatly appreciate the financial support of the Important National Science & Technology Specific Projects of China (Grant No. 2011ZX05011).

References

- [1] Sigmund PM, McCaffery F. An improved unsteady-state procedure for determining the relative-permeability characteristics of heterogeneous porous media[J]. SPE Journal, 1979, 19(1): 15-28.
- [2] Chavent G, Cohen G. Determination of relative permeability and capillary pressure by automatic adjustment method[C]. SPE Journal, 1980: 21-24.
- [3] Chen WH, Gavalas GR, Seinfeld JH. A new algorithm for automatic history matching[J]. SPE Journal, 1974, 14(6): 593-608.
- [4] Evensen G. Data assimilation: the ensemble kalman filter[M]. Berlin: Springer, 2006.
- [5] Li TJ, Chen MQ. An automatic history matching method of black oil model[J]. Petroleum Geology & Oilfield Development in DaQing, 1999, 18(2): 27-29.
- [6] Deng BR, Yuan SY, Li JF. Applying computer-aided history matching to the numerical simulation of reservoirs[J]. Petroleum Exploration and Development. 2003, 30(1): 71-74.
- [7] Yan Xia, Zhang Kai, Yao Jun. Review on automatic history matching methods for reservoir simulation[J]. Petroleum Geology and Recovery Efficiency, 2010, 17(4): 69-73.
- [8] Watson AT, Gavalas GR, Seinfeld JH. Identifiability of estimates of two-phase reservoir properties in history matching[J]. SPE Journal, 1984, 24(6): 697-706.
- [9] Chen S, Li G, Peres A. A well test for in-situ determination of relative permeability curves[J]. SPE Journal, 2005: 9-12.
- [10] Barroeta RG, Thompson LG. Estimation of relative permeability from displacement pressure data[J]. SPE Journal, 2006: 22-26.
- [11] Chardaire RC, Chavent G, Jaffre J. Simultaneous estimation of relative permeabilities and capillary pressure[J]. SPE Formation Evaluation, 1992, 7(4): 283-289.
- [12] Akin S, Demiral B. Genetic algorithm for estimating multiphase flow functions from unsteady-state displacement experiments[J]. Computer & Geosciences, 1998, 24(3): 251-258.
- [13] Kennedy J, Eberhart R. Particle swarm optimization[J]. 1995.
- [14] Aanonsen SI, Naevdal G. The ensemble kalman filter in reservoir engineering—a review[J]. SPE Journal, 2009: 393-412.
- [15] Chen Y, Oliver DS, Zh GM. Efficient ensemble-based closed-loop production optimization[J]. SPE Journal, 2008: 20-23.
- [16] Brooks RH, Corey AT. Hydraulic properties of porous media[J]. Hydrology papers, 1964, 3(3): 27.
- [17] Chierici GL. Novel relations for drainage and imbibition relative permeabilities[J]. SPE Journal, 1984, 24(3): 275-276.
- [18] Kerig PD, Watson AT. Relative-permeability estimation from displacement experiments: an error analysis[J]. Society of Petroleum Engineering Reservoir Engineering, 1986, 1(2): 175-182.
- [19] Watson AT, Richmond PC, Kerig PD. A regression-based method for estimating relative permeabilities from displacement experiments[J]. SPE Reservoir Engineering, 1988, 3(3): 953-958.
- [20] Chen S, Li G, Peres A M. A well test for in situ determination of relative permeability curves[J]. SPE Reservoir Evaluation & Engineering, 2008, 11(1): 95-107.
- [21] Stone HL. Estimation of three phase relative permeability and residual oil data[J]. JCPT, 1973, 12(4).
- [22] Li H, Yang DY, Mohamed A. Relative permeability estimation from displacement experiments using EnKF method[C]. Beijing: International Oil and Gas Conference and Exhibition in China, 2010: 8-10.
- [23] Lee TY, Seinfeld J H. Estimation of absolute and relative permeabilities in petroleum reservoirs[J]. Inverse Problems, 1987.
- [24] Yang PH, Watson AT. A Bayesian methodology for estimating relative permeability curves[J]. SPE Reservoir Engineering, 1991, 6(2): 259-265.
- [25] Okano H, Pickup GE. Quantification of uncertainty in relative permeability for coarse-scale reservoir simulation[J]. SPE Journal, 2005: 13-16.
- [26] Li R, Reynolds AC. History matching of three-phase flow production data[J]. SPE Journal, 2003, 8(4): 328-340.
- [27] Reynolds AC, Li R, Oliver DD. Simultaneous estimation of absolute and relative permeability by automatic history matching of three-phase flow production data[J]. Journal of Canadian Petroleum Technology, 2004, 43(3): 37-46.
- [28] Eyidinov D, Gao G, Li G. Simultaneous estimation of relative permeability and porosity/permeability fields by history matching production data[J]. Journal of Canadian Petroleum Technology, 2009, 48(12): 13-25.