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## Multi-objective optimization model for a green vehicle routing problem

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### Abstract

The concept of green logistics stems from green economic concepts which are inherently driven by the environmental sustainability challenges. In this work, measures of carbon dioxide (CO<sub>2</sub>) emission are added to the canonical capacitated vehicle routing problem. The proposed multi-objective optimization model tackles the conflicting objectives of the emission reduction while holding-off the economic cost uplift, leading to a set of Pareto optimal solutions. A biologically inspired Ant Colony Optimization (ACO) based evolutionary constructive heuristic is used to obtain routing plans with minimum financial impact. A Variable Neighborhood Search (VNS) algorithm is designed to obtain low emission routes by exploring the neighborhood of the ant foraging paths. The hybrid ACO-VNS heuristic will provide a set of non-dominated solutions leading to the Pareto optimal solution frontier. For consistency of solutions and solution convergence, the algorithm is tested on randomly generated problem instances.

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*Keywords:* Environmental sustainability; Multi-objective optimization; Pareto optimality; Hybrid ACO-VNS heuristic.

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### 1. Introduction

Green intelligent transportation systems are designed and deployed as a part of gradual change in the industrial focus in production and distribution networks. The sustainability challenges are urging a drastic shift from the merely profit based economy to a more responsible and environmental friendly business world. The dire consequences of the greenhouse gas emissions are more evident and long lasting on the mother earth. The concepts

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of green logistics stems from the green economic concepts, which are inherently driven by the sustainability challenges all over the world. Carbon emissions from the transportation sector contribute a major sum of total greenhouse gas emissions. Green Logistics activities include measuring the environmental impact of different distribution strategies, reducing the energy usage in logistics activities, reducing waste and managing its treatment (Sbihi & Eglese, 2010). Implementing green logistic ideas in vehicle routing problems and its variants give rise to the Green Vehicle Routing Problems (GVRP). GVRP are concerned about weaving routes that meet ever growing environmental concerns and are financially sound. GVRP have just arisen in the literature in recent years, there is a continuing need to enrich the related studies either through theoretical contributions or by real applications (Lin et al., 2014).

In this work, a multi depot capacitated vehicle routing problem with CO<sub>2</sub> emission is analyzed. The CO<sub>2</sub> emission measures are added to the scenario of canonical capacitated vehicle routing problem. In this study, the CO<sub>2</sub> emission due to the fossil fuel consumption is only considered. The optimization model tackles the conflicting objectives of emission reduction while holding off the economic cost uplift, leading to a set of Pareto optimal solutions. A mathematical model is formulated for the problem. Economic and environmental factors are considered in the analysis of the model. The economic factors considered in this study include depot opening fixed costs, route operating costs, fuel consumption costs and other operational costs. The environmental impact is measured in the monetary terms of carbon emission which is calculated based on the load distribution pattern in a route, the vehicle type and efficiency of the engine. The environmental impact is measured in terms of cost of tons of CO<sub>2</sub> emission. A hybrid algorithm combining an Ant Colony Optimization (ACO) algorithm with a Variable Neighborhood Search (ACO-VNS) is proposed to solve the model. The algorithms are tested on problem instances generated by mimicking the real-life product delivery of carboys. The environmental analysis of the model can be used as a guideline for managerial decisions and governmental regulations.

The rest of the paper is structured as follows. In section 2, problem is mathematically formulated. Solution methodology is discussed in section 3. In Section 4, the computational study is presented. Finally, conclusions and suggestions for future research are given in Section 5.

## 2. Green vehicle routing problem : Problem formulation

A Multi Depot Vehicle Routing Problem (MDVRP) is considered with the following objectives

- Allocate the clients or demand centres to the distribution points.
- Determine the routes from the depots to its allocated customers.

All the above solutions are obtained so that the total economic cost – emission cost provides a Pareto optimal curve for the non- dominant vehicle routing solutions. The problem assumptions are as follows:

- The supply chain network consists of a number of fixed-capacity distribution points which serves multi-customers with known demand.
- Homogeneous Vehicles are used. Capacity, speed and emission parameters are same.
- The location of the depots and the customers are known.
- Individual demand of each customer cannot exceed the capacity of a vehicle;
- Each vehicle starts and ends the route at the same depot and each customer is visited by a vehicle exactly once, i.e. the entire demand of the depot or the customer is met by a single vehicle.

*Notations*

$I$  Set of depots ( $i=1, 2, \dots, d$ ), where  $d$  being the upper bound on the number of depots.

$J$  Set of customers ( $j=1, 2, \dots, c$ ), where  $c$  being the upper bound on the number of customers

$$V = I \cup J$$

**Decision variable**

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ is travelling from depot } i \text{ to customer } j, \forall i \in I, j \in J, k \in K \\ 0, & \text{otherwise} \dots \end{cases}$$

$$f_{ij} = \begin{cases} 1, & \text{if customer } j \text{ is assigned to depot } i, \forall i \in I, j \in J \\ 0, & \text{otherwise} \end{cases}$$

**Parameters**

- $l_{ij}$  Euclidian distance from node  $i$  to node  $j$ , for all  $i \in V, j \in V$
- $C_v$  Variable vehicle operating cost per unit distance
- $O_i$  Fixed cost of opening a depot  $i$ , for all  $i \in I$
- $T$  Fixed depot vehicle cost
- $C_{fuel}$  Average fuel consumption cost per unit distance per unit vehicle weight
- $F_{CO2}$  CO<sub>2</sub> emission cost per unit weight of vehicle per unit distance
- $P_f$  Diesel fuel price per unit volume
- $V_f$  Volume of fuel consumption per unit distance per unit vehicle weight
- $W_p$  Weight of each delivered product (the weight of recycled products is neglected)
- $W_{CO2}$  Weight of CO<sub>2</sub> emission per liter consumption of diesel
- $P_{CO2}$  Average price per unit weight of CO<sub>2</sub>
- $k$  Ratio of vehicle volume versus curb weight
- $W_{cargo}$  Average cargo weight per vehicle through travelling on each route
- $W_v$  Average gross weight per vehicle through travelling on each route
- $d_j$  Demand of customer  $j$ , for all  $j \in J$
- $Q_v$  Capacity of the depot vehicle

**Objective function**

Minimise

$$f_1(x) = \sum_{i \in I} o_i + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} C \times l_{ij} \times x_{ijk} + \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} T \times x_{ijk} \tag{1}$$

$$f_2(x, d) = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} l_{ij} \times x_{ijk} \times P_{CO2} \times W_{CO2} \times V_f \times \left( W_{cargo} + \frac{W_{curb}}{k} \right) \tag{2}$$

Subject to:

$$\sum_{j \in J} \sum_{i \in V} d_j \times x_{ijk} \leq Q_v, \quad \forall k \in K \tag{3}$$

$$\sum_{j \in J} d_j f_{ij} \leq W_i y_i, \quad \forall i \in I \tag{4}$$

$$\sum_{j \in J} \sum_{i \in V} x_{ijk} = 1, \quad \forall k \in K \tag{5}$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0, \quad \forall i \in V, k \in K \tag{6}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1, \quad \forall k \in K \tag{7}$$

$$x_{ijk} \in \{0, 1\}, \quad \forall i \in I, \forall j \in V, \forall k \in K \tag{8}$$

$$f_{ij} \in \{0, 1\}, \quad \forall i \in I, j \in V \tag{9}$$

The objective function (1) represents the economic side of the supply chain costs. The cost fragments are the depot opening costs and the routing costs, including the travel costs and the fixed costs associated with

vehicle uses. The objective function (2) is the emission counter part of the network, which is measured in the monetary terms of the carbon emission. This depicts the environmental impact of the supply chain considered. The emission cost is defined as a function of the distance as well as the demand pattern inside a route. Constraints (3) and (4) are the capacity constraints associated with the routes and the depots, respectively. Constraint (5) ensures that each customer belongs to exactly one route, and that each customer has only one predecessor in the route. Constraint (6) guarantees the continuity of each route, and that each route terminates at the depot where the route starts. . Constraints (7) ensure that a customer must be assigned to a depot if there is a route connecting them. Constraints (8) and (9) specify the binary variables.

**3. Solution methodology**

The multi-objective problem is solved using a hybrid meta- heuristic algorithm combining an Ant Colony Optimization (ACO) algorithm and a Variable Neighborhood Solution (ACO-VNS). The details of the solution methodology are provided in the following subsection.

*3.1. Ant Colony Optimization (ACO) algorithm*

An Ant Colony Optimization (ACO) based heuristic is proposed to solve the multi-objective green MDVRP. Ants mimic the vehicles or trucks in the problem. Every ant makes a trail with a chemical substance called pheromone. Ants travel from a depot to a customer or from a customer to another customer. Once an ant travels from one node to another node, it updates the pheromone. This pheromone information will direct the search of the ants. The ant selects the customer depending on the probability function. The capacities of vehicle and depot are updated before the next customer is selected. The ant returns to the depot when the vehicle capacity constraint is met or when all the customers are served or when the depot capacity constraint is met. The heuristic constructs a complete tour for the first ant prior to the second ant starting its tour. The algorithmic parameters are (i) Magnitude of the pheromone intensity ( $\alpha$ ) (ii) Magnitude of visibility ( $\beta$ ) (iii) Evaporation rate of pheromone ( $\rho$ ) (iv) Pheromone increment amount ( $Q$ ) (v) Number of ants and number of iterations.

All the elements in the initial pheromone matrix are set to  $\alpha$  value. Probability matrices between the depot to customer ( $P_{dc}$ ) and customer to customer ( $P_{cc}$ ) are calculated based on the initial values. The probability between the two nodes  $i$  and  $j$  is calculated using equation (10).

$$P_{ij}^k = \begin{cases} \frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{l \in N_i^k} \tau_{il}^\alpha \eta_{il}^\beta}, & \text{if } j \in N_i^k \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

where  $\tau_{ij}$ : The pheromone intensity between the nodes  $i$  and  $j$ ,  $N_i^k$ : The feasible neighborhood of ant  $k$ ,  $\eta_{ij}$ : The visibility value between the edges  $i$  and  $j$ .

The visibility is calculated using equation (11) as follows

$$\text{Visibility, } \eta_{ij} = \frac{1}{\text{function value between node } i \text{ and } j} \tag{11}$$

As an ant moves from one node to another, the pheromone content is updated using equation (12).

$$\tau_{ij} = \tau_{ij} + \Delta\tau_{ij}^k \tag{12}$$

where  $\Delta\tau_{ij}^k$  is the increment in the pheromone value. The increment in the pheromone trail is determined by

equation (13).

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{L_k}, & \text{if ant } k \text{ uses edge}(i, j) \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where,  $Q$  is the pheromone increment amount and  $L_k$  is the function value by the ant  $k$  on the edge  $(i, j)$ . In this problem,  $L_k$  is the distance between the nodes. The pheromone evaporation on edge  $(i, j)$  is updated using equation (14).

$$\tau_{ij} = (1 - \rho)\tau_{ij} \tag{14}$$

where,  $\rho$  is the parameter that controls the pheromone evaporation rate.

### 3.2. ACO-VNS based heuristic

A Variable Neighborhood Search is applied on to the Pareto solution set generated from ACO. The  $k^{th}$  neighborhood in the  $n^{th}$  iteration,  $N_n^k$  is generated by swapping the  $i^{th}$  element of the solution with  $j^{th}$  element, where  $j \neq i$ . A sample solution representation is shown in Figure 1. Elements  $i=1, 2, \dots, n$  are used for customers and  $n+1$  to  $n+m$  are used for representing depots. Zeros indicate a different route from the same depot.

22	13	14	18	20	16	17	19	11	12	21	1	6	4	2	5	8	15	7	23	10	9	3	0
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Fig. 1. The solution representation.

## 4. Computational experiments

The developed heuristics are coded in MATLAB and implemented on a Core i3 processor at 2.13 GHz PC with 3 GB RAM to solve the problem. The algorithm is tested on a set of randomly generated problem instances. The data for the MDVRP has been randomly generated. The problem instances are generated by varying different parameters such as the number of customers,  $n$ , number of depots  $m$  and vehicle capacity  $Q_v$ . The number of depots considered is 3 and 5. The vehicle capacity considered is 70 or 150. The number of customers,  $n \in \{20, 50\}$ . The other data (demands, depot capacities, fixed costs) are assumed to be integers. The problem instances are randomly generated with the following characteristics namely; Demand follows a uniform distribution [11, 20]. Data pertaining to carbon emission and carbon costs are assumed based on Forbes (2009) and the report of the Automotive Research Association of India (2008).

The values for the various problem parameters are given by:  $C_{fuel} = \$1.0338$ ,  $T = \$15/\text{vehicle}$ ,  $P_{CO2} = \$20$ ,  $W_{CO2} = 0.027 \text{ Ton}$ ,  $W_{prod} = 0.0189 \text{ Ton}$ ,  $V_f = 0.01653 \text{ L/Ton/km}$ ,  $F_{CO2} = \$0.89262/\text{Ton/km}$ ,  $W_{curb} = 16.2 \text{ Ton}$ , and  $K = 5$ .

The algorithmic parameters are fixed using Taguchi method of experimental design on a randomly generated problem instance and are adopted for the computational study. The values for the algorithmic parameters are given by: Magnitude of pheromone intensity ( $\alpha$ ) = 0.4, Magnitude of visibility ( $\beta$ ) = 2, Evaporation rate of pheromone ( $\rho$ ) = 0.2, Pheromone increment amount ( $Q$ ) = 4, Number of ants = 100 and Number of iterations = 40.

### 4.1. Results and discussion

The developed algorithm is tested on a number of randomly generated instances. The generated problem instances are characterized by the following criteria (i) The number of depots,  $m$  (ii) The number of customers,  $n$  (iii) The vehicle capacity,  $Q_v$ . The results are tabulated in Table 1. The Pareto set of solutions are given for the problem instances. The Pareto optimal curves are plotted between two objective functions and are shown in Figures 2(a) – 2(b).

From the results, it is evident that the vehicle routes that provide minimum economic cost is not necessarily an environment friendly route with less emission or vice versa. Hence, the Pareto curve explains possible trade-offs between these two objectives of different dimensions. Further more, the results imply that the CO<sub>2</sub> emission varies with the travelling distance and gross vehicle weight. Under this managerial perspective, decision makers have to be more focused on the economic cost reduction side. The governmental agencies will be assimilating the realistic scenario of emission reduction and possibly will consider the financial impact of the greening measures on the supply chain while drawing environmental guidelines and tax fixation for the implementation of environmental governance principles.

**5. Conclusions**

This paper presented a multi-objective green multi depot capacitated vehicle routing optimization problem. A mathematical model has been formulated for the problem. The economic and environmental factors are considered in the analysis of the model. A hybrid meta- heuristic algorithm combining an Ant Colony Optimization algorithm and a Variable Neighborhood Solution (ACO-VNS) is developed for finding the non-dominant Pareto optimal solutions for the proposed problem. The algorithm is tested on randomly generated problem instances. The environmental analysis of the model can be used as a guideline for managerial decisions and governmental regulations. The main contribution of this study is that according to the business scenario the decision maker has the flexibility to choose an appropriate solution from the set of solutions.

Table 1. Experimental results.

Problem instance number	Number of depots, <i>m</i>	Number of customers, <i>n</i>	Vehicle capacity, <i>Q<sub>v</sub></i>	Economic cost in monetary units (x 10 <sup>4</sup> )	Emission cost in monetary units
1	3	20	70	2.7197	4.5757
				2.5720	4.6398
				2.5719	4.6471
				2.5710	4.8055
				2.1331	5.5722
				2.1330	5.5725
2	3	20	150	2.1429	4.3330
				2.0928	4.5746
				1.9927	4.8016
				1.9171	4.8259
				1.9169	4.8319
				1.5645	4.8736
3	3	20	70	2.4340	3.7403
				2.2917	4.0401
4	3	20	150	3.3187	3.7434
				1.4038	3.2126
5	5	50	70	2.9142	0.1199
				2.5860	0.1264
				2.3814	0.1301
				2.0230	0.1314
				1.5234	0.1642

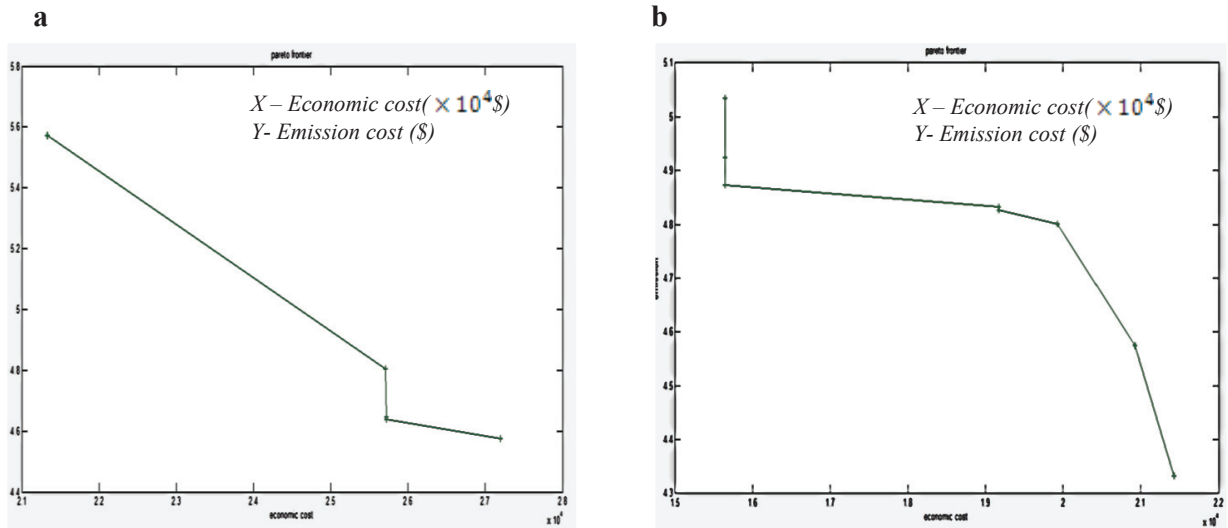


Fig. 2. (a) Pareto curve for Problem Instance 1; (b) Pareto curve for Problem Instance 2.

Future work can be conducted by considering the vehicle routing problem variants and with different supply chain structure. The stochastic demand scenarios will be an interesting extension since the demand variability and the ambiguity in the demand pattern will result in a challenging route optimization problem. The dynamic vehicle routing variant will be more helpful while designing real-time business applications.

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