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Comment

Comment to: "Corrections to the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle" [Phys. Lett. B 632 (2006) 151]

Geusa de A. Marques

Departamento de Física, Universidade Federal de Campina Grande, 58109-790 Campina Grande, Pb, Brazil

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Abstract

In the paper [F. Nasseri, Phys. Lett. B 632 (2006) 151–154], F. Nasseri supposed that the value of the angular momentum for the Bohr's atom in the presence of the cosmic string is quantized in units of $\hbar$. Using this assumption it was obtained an incorrect expression for Bohr radius in this scenario. In this Comment I want to point out that this assumption is not correct and present a corrected expression for the Bohr radius in this background.

In a recent paper F. Nasseri [1] analyzed the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle. The author claims that the Bohr's radius in this system increases by a factor of order $\pi/4 \times 10^{-6}$. Such a result was obtained by the author considering that the stationary orbits in the spacetime of a cosmic string have an integer number of wavelengths in the interval from 0 to 2π. This is not correct once for stationary orbits we have

$$\oint \frac{dS}{\lambda} = n; \quad n = 1, 2, 3, \ldots \tag{1}$$

where S is the length of the orbit and λ the wavelength. As the metric of a cosmic string is given by

$$ds^2 = c^2\,dt^2 - dz^2 - d\rho^2 - \rho^2\,d\varphi'^2, \tag{2}$$

where $\varphi' = (1 - \frac{4G\mu}{c^2})\varphi$, which implies that φ' varies from 0 to $2\pi b$, where $b = 1 - \frac{4G\mu}{c^2}$. Therefore $dS = \rho\,d\varphi'$, leads to a correction in the condition of quantization of the angular momentum. Thus, instead of $L_n = n\hbar$ as considered by the author in [1], we have $L_{n(b)} = \frac{n}{b}\hbar$, where b is the deficit angle parameter [2]. With this consideration, the radius of the nth Bohr orbit of the hydrogen atom in the presence of a cosmic string becomes

$$\rho_n = \frac{4\pi\epsilon n^2\hbar^2}{me^2(1 - \frac{\pi}{4}\frac{G\mu}{c^2})(1 - 4\frac{G\mu}{c^2})^2}. \tag{3}$$

The Eq. (17) of [1] is correct only for flat spacetime. In the presence of a cosmic string, the radius of the nth Bohr orbit of the hydrogen atom is given by

$$\hat{a}_B = \frac{4\pi\epsilon\hbar^2}{me^2(1 - \frac{\pi}{4}\frac{G\mu}{c^2})(1 - 4\frac{G\mu}{c^2})^2}. \tag{4}$$

In the absence of a cosmic string, the lowest orbit ($n = 1$) of the Bohr orbit has the following expression

$$a_B = \frac{4\pi\epsilon\hbar^2}{me^2} = 5.29 \times 10^{-11}\ \text{m}. \tag{5}$$

Thus combining Eqs. (4) and (5), we obtain

$$\frac{a_B}{\hat{a}_B} = \left(1 - \frac{\pi}{4}\frac{G\mu}{c^2}\right)\left(1 - 4\frac{G\mu}{c^2}\right)^2. \tag{6}$$

In the weak field approximation, Eq. (6) turns into

$$\frac{a_B}{\hat{a}_B} \approx 1 - \frac{G\mu}{c^2}\left(8 + \frac{\pi}{4}\right). \tag{7}$$

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E-mail addresses: gmarques@df.ufcg.edu.br, geusaphysics@gmail.com (G. de A. Marques).

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In the limit $\mu \to 0$, i.e., in the absence of a cosmic string, $\frac{a_B}{\hat{a}_B} \to 1$. It is worth noticing that there is a difference between Eq. (22) of [1] and Eq. (6) of this Comment. Taking into account the correction given by Eq. (6) and inserting $\frac{G\mu}{c^2} \simeq 10^{-6}$ we get

$$\hat{a}_B = \frac{a_B}{\left(1 - \left(8 + \frac{\pi}{4}\right) \times 10^{-6}\right)}. \tag{8}$$

From the above equation, we conclude that the numerical factor which corrects the Bohr radius is different from the one obtained in Eq. (23) of Ref. [1].

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References

[1] F. Nasseri, Phys. Lett. B 632 (2006) 151.
[2] A. Vilenkin, E.P.S. Shellard, Cosmic String and Other Topological Defects, Cambridge Univ. Press, Cambridge, 1994.