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Nonlinear Effects and Multisolitons in Metamaterials

B. Mandal[a], A Biswas[b], K. Choudhary[c], S.J. Mandal[b], A. K. Bandyopadhyay[d], A. K. Bhattacharjee[e] and D. Mandal[e]

[a]*Faculty of Engg. & Technology, NSHM Knowledge Campus, Durgapur-713212, India*
[b] *Dumkal Institute of Engineering & Technology, Dumkal, Dumkal-742406, Dist. Murshidabad, India*
[c] Department of Materials Science and Engg, University of Florida, Gainesville, FL 32611, USA
[d]Ideal Institute of Engineering, Kalyani, Nadia, West Bengal, India
[e]*Dept. of Electronics and Communication Engineering, N. I. T., Durgapur, India*

Abstract

In this paper, we outline a novel class of materials, called metamaterials, with negative refractive index and a high degree of nonlinearity. A brief summary is given on the basic theory of optics to show how this condition arrives for metamaterials to be designed into an antenna with split-ring-resonator (SRR). An example is given on the modeling of such SRR-based metamaterials. On a continuum Hamiltonian, a Klein-Gordon equation was derived which gave rise to both dark and bright solitons that showed interesting behavior against nondimensional time, even to the extent of revealing bi- and tri-breathers.

Keywords: Metamaterials, Ferroelectrics, Continuum and Discrete Solitons;

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1. Introduction.

The electrodynamics of substances with simultaneous negative values of dielectric permittivity (ε) and magnetic permeability (μ) has been a subject of much study, with current interests in possible technological applications [1]. Substances with both negative ε and μ are predicted to possess a negative refractive index and, consequently, to exhibit a variety of optical properties not found in positive indexed materials. Negative index materials, however, do not occur in nature, and only recently has it been shown that they can be artificially fabricated. The experimental realization of such materials was demonstrated by Smith, et al [2] based on theoretical work of Pendry et al [3, 4]. Smith, et al. made a type of "metamaterial" (MM) as an artificial structure with negative refractive properties. The structure consists of metallic wires responsible for the negative permittivity and metallic "split ring resonators" (SRR) responsible for the negative permeability. The optical and electrical properties of the MM are modulated by the proper use of SRRs to give $\varepsilon, \mu < 0$ within a region of frequencies, and it is the SRRs that are key

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in setting negative μ within a material with $\varepsilon < 0$. In particular, unlike naturally occurring materials, the designed MMs show a relatively large magnetic response at THz frequencies. This, in combination with its negative permittivity in the THz, is responsible for an effective negative index in this range of frequencies.

From the standpoint of theory, linear and nonlinear SRRs have been shown to be described by equivalent resistor-inductor-capacitor (RLC) circuits **[5, 6]** featuring a self-inductance L from the ring, a ring ohmic resistance R, and a capacitance C from the split in the ring. MMs with negative refractive properties are then formed as a periodic array of SRR which are coupled by mutual inductance and arrayed in a material of dielectric constant ε. From the standpoint of experiment, the requirements for the effective electromagnetic application of metamaterials in the THz region introduces the necessity of very high accuracy in the fabrication of SRR based MMs to produce materials of uniform and consistent properties.

In this paper we will look on metamaterials, with negative refractive refractive index and a high degree of nonlinearity. A brief description is given on the basic theory of optics to show the condition arrives for metamaterials to be designed into an antenna with split-ring-resonator (SRR).

2. Split –Ring –Resonator Structure*:*

The structure of Single-Ring-Resonator based on “metamaterial” (MM), i.e., an artificial structure consisting of metallic wires responsible for the negative permittivity is shown in Fig. 1. The SRRs may have different shapes. The optical and electrical properties of a metamaterial can be harnessed by the proper use of SRRs that has application in antennas. However, very high accuracy is required for the fabrication of SRRs. Unlike natural materials, MMs also shows relatively large magnetic response at THz frequency and hence their THz application assumes more significance. Basically, in normal materials the wave vector and pointing vector are parallel, whereas in MMs they are anti-parallel

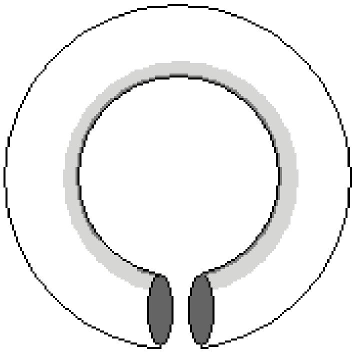

Fig. 1. A typical assembly of SRRs.

3. Theoretical Development:

Within good approximation, SRR can be realized as equivalent to a nonlinear resistor-inductor-capacitor (RLC) circuit [7] featuring a self-inductance *L*, Ohmic resistance *R*, and capacitance *C*. SRRs being the building blocks of MMs, large number of loosely coupled SRRs may be taken for a study of the dynamical behavior of a MM system [7]. The circuits become nonlinear due to Kerr-type medium in which SRRs are placed or by the introduction of certain nonlinear elements (e.g. diode).

Our mother ‘nature’ is nonlinear and nonlinear science is an emerging field in the field of science and technology. It has revolutionized the area of science in the last three decades. So, it is interesting to study the behavior of metamaterials in the realm of Nonlinear Science. Since the first days of the extensive studies of metamaterials and their properties, attention of most of the researchers was focused on linear properties of LHMs, when the parameters of the composites do not depend on the intensity of the electromagnetic field. However, the future efforts in creating *tunable structures* where, e.g., the field intensity can be used for dynamic control of the transmission properties of the composite structure would require the study of nonlinear properties of such metamaterials, which may be quite unusual.

Metamaterials behave as capacitive loaded loops [8]. These capacitive loaded loops support wave-propagation. Since the coupling is due to induced voltages, the waves are referred to as magneto-inductive waves (MI waves) [9-11]. MI waves represent a vast area of active research in different fields: (a) artificial delay lines and filters, (b) dielectric Bragg reflectors, (c) slow-wave structures in microwave tubes, and (d) coupled cavities in accelerators, modulators, antenna array application, etc.

Based on a Hamiltonian developed by Lazarides [7], where the nondimensional time is taken as $t' = \tau\omega$ (ω is the frequency), a nonlinear Klein-Gordon equation was developed as [12]:

$$\frac{\partial^2 q}{\partial \tau^2} - ab\frac{\partial^2 q}{\partial x^2} + b\left[q - \frac{\alpha}{3\varepsilon_l}q^3\right] - b\Lambda\Omega\sin(\Omega\tau) + \gamma\frac{\partial q}{\partial \tau} = 0 \tag{1}$$

Where q is the charge of the capacitor, x is rescaled to be expressed in units of d (i.e., $x/d \rightarrow x$), a term in γ is added to include the possibility of dissipation in the system, and Λ is an emf term that is generated within the system. Here, $a = \lambda/(1+2\lambda)$ and $b = 1/(1+2\lambda)$, where λ is an interaction parameter between the SRR sites in different positions in the antenna assembly. Following the methods developed in Ref. [13-15], the solutions of Eq. (1) for $\Lambda = 0$ and $\gamma = 0$ are obtained in two forms, i.e. in the forms of *tanh* and *sech*. A first solution is a dark soliton-type given by:

$$q(x,\tau) = A\tanh(Bx - C\tau) \tag{2}$$

Where A, B and C are different important parameters for antenna design. The velocity, $\frac{C}{B}$, of the excitation is, consequently, related to its width, $\frac{1}{B}$. A second solution is a bright soliton solution given by:

$$q(x,\tau) = A\,\mathrm{sec}\,h(Bx - C\tau) \tag{3}$$

4. Results and Discussion:

Here we focus our study of nonlinear dynamics and computer simulation work to develop systems with multisolionic features where more than one wave for a particular condition of damping and interaction coupling evolve in both focused and defocused nonlinear conditions under different non-dimensional values of time. A mild dissipation (0.01) is considered, which is usual in SRR based MMs [7], and for a value of the interaction term as 0.05 for focusing nonlinearity, i.e. α is +1, we get four curves at four different non-dimensional values of time. This is shown in Fig. 2.

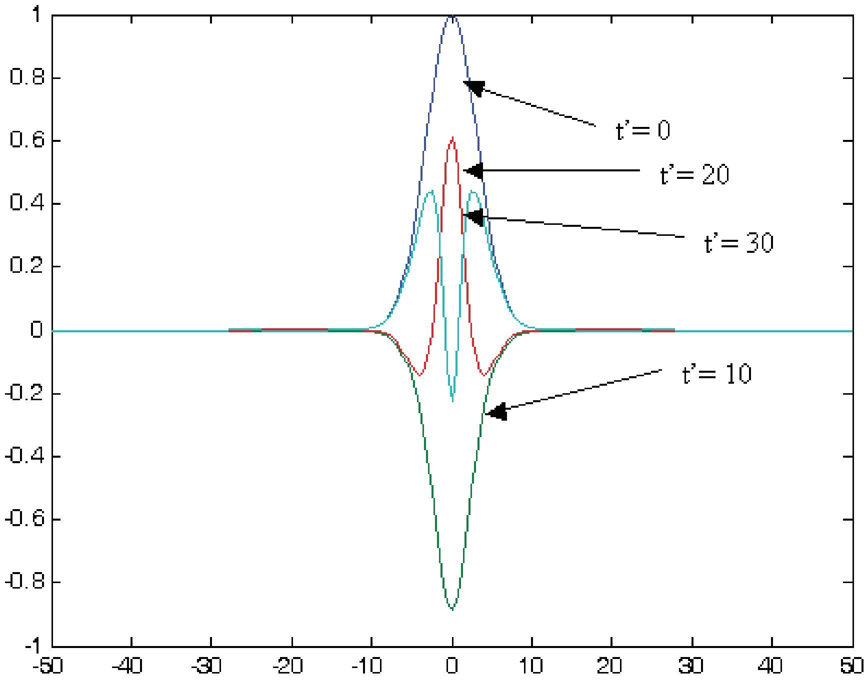

Fig. 2: Multisolitonic features for γ=0.01, λ=0.05, α=1

It is seen that at *t'*=0, the curve is purely Gaussian and symmetric. As the value of *t'* increases to 10, the curve gets inverted and the same feature has been observed in defocusing nonlinearity, but bi-solitons have been observed, which seems to be a promising condition for controlling the nature of the solitonic behavior. For still higher time at 30, it always shows bi-solitonic behavior in focusing and defocusing conditions. The bi-solitonic behavior may arise due to cross-phase modulation mechanism and/or vector soliton formation mechanism, which is mainly attributed to corresponding eigenvalues of the equation [16] (see references therein).

5. Conclusion

We have outlined the most important theoretical aspects of negative index materials (NIMs) in a concise manner. Numerous research works in defense for "invisible cloak", as well as THz optics foundation is going on around the globe. Currently, the NIM fabrication is much more demanding than the NIM design and optimization, namely at optical frequencies. The range of available materials with desired electromagnetic properties may be widened by the design of new artificial structures, i.e., metamaterials composed of different natural materials (dielectrics, metals, semiconductors, etc.) with different shapes. This significantly improves the chance of finding high quality NIMs. At the same time, the nonlinearity of metamaterials requires a lot of activities in material research with a focus on electromagnetic properties of composite materials. The multisonic behaviour of such metamaterials are being studied with computer simulation that already shows interesting features at higher nondimensional time.

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