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Learning Mathematics Using Heuristic Approach

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Abstract

The heart of mathematical problem solving is to explore and establish relationships between the different branches of knowledge in mathematics. Heuristics approaches are capable of illustrating how these knowledge branches are united by a few basic, universal principles during the mathematical knowledge transfer in the process of mathematical problem solving. This paper focuses on the application of heuristics approaches in an action research. The objectives of the study were (1) to investigate students' response in applying heuristics approach in solving mathematical tasks, (2) to examine students' abilities in applying the heuristics approach. This study involved a group of 26 prospective mathematics teachers who are studying in a college (aged 22 – 24). They had been exposed to the heuristics approaches in solving mathematical problems. Data were collected from classroom observations and the participants' reflective journals. The findings showed that the students were able to engage heuristic approaches in solving mathematical problems. Their reflections in the reflective journals indicated that they had applied the relevant strategies of the approach to solve mathematics problems. The strategies included giving representation, making a calculated guess and going through the process.

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Keywords: Heuristic approach; Mathematical problem; reflective writing

1. Background

In the mathematical classrooms students' creativities and abilities are fostered through a variety of teaching methods and learning strategies. The role of teachers as the facilitators in a mathematical classroom is to instill students' confidence in presenting their mathematical solutions. Chavez (2007) observed that students' mathematical problem solving abilities increase after the use of heuristic approaches. They were reported to engage profoundly in their learning and such active learning environment improved students' interest in solving mathematical problems as well as enabled them to respond creatively. Also, Akinsola (2007) found that students' mathematics performance and their attitudes towards mathematics were highly affected by their teachers' abilities

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to synthesize mathematical ideas in the teaching process. The teaching methods help to develop the students' cognitive abilities that ultimately challenge them to think mathematically. As such, teachers' guidance is important in facilitating their students' thinking. On the other hand, according to Tarnvik (2002) students gain advantage if they use multiple of ways to stimulate their thinking. The reason is solving problems is a creative cognition process and applying multiple methods to solve mathematical problems aided by knowledge from the past experience and with the skills of observing solution patterns can encourage the formation of new ideas from the subconscious cognition.

There are research findings that reported the advantages of technology in doing mathematics for example the computer games (Akinsola, 2007; Parmjit, Nor Aziah Alias, Teoh, & Ros Aizan Yahaya, 2010; Teoh, Parmjit, Nor Aziah Alias & Ros Aizan Yahaya, 2011). The focus of the mathematical games especially is to develop mathematical thinking towards comprehension and understanding of mathematical concepts. Undoubtedly the application of heuristics approaches cultivates students' skills in thinking in solving mathematical problem. It is known that these skills induce the subconscious cognition to spark ideas towards stimulating creativity without using fixed algorithms.

Heuristic approach encourages the communication of mathematical thoughts through discovery by drawing a diagram, examining special cases, specializing the solution, generalizing the solution. Polya (1980) emphasized the importance of induction and plausible reasoning in solving mathematical problem. Judgement based on inducation and plausible reasoning rely on the application of heuristic approaches (Polya, 1954; Dudczak,1995). Heuristic approach is a mathematical thinking tool to facilitate participants in solving mathematical problem. The approach involves the application of the following practices (Polya, 1954):

- To make a calculated guess, such as guess and check, look for patterns, make suppositions
- To go through the process, such as act it out, work backwards.
- To change the problem, such as restate the problem, simplify the problem, and solve part of the problem.

It is undeniable that heuristics approach is difficult to implement. Therefore, appropriate practice of the heuristic approach is important to ensure participants' readiness in applying the approach. This study aims to investigate participants' abilities in using heuristics approaches to present their mathematical solutions. The activities assigned in this research were guided by Polya's (1957, 1973) belief that there are alternatives ways in solving a mathematical problem. Communicating the mathematical thought will help to access more accessible related problems in finding and trying to solve the problems (Kahneman, 2002).

2. Methodology

This study involved a group of 26 prospective mathematics teachers aged 22 to 24 studying in a college. The participants had learned heuristics approaches in solving mathematical problems in one of their mathematics courses prior to this research. Data were collected from classroom observations and participants' reflective journals. Reflective journals for three lessons were analyzed. The three lessons focused on three different mathematical problems (See Table 1).

Table 1. Mathematical problems

Topics	Question
Whole numbers	A printer uses 999 digits to number the pages of a book. How many numbered pages are there in the books? Generalized.
Simultaneous equations	A freight train and a passenger train come to a halt on adjacent tracks. The engine of the freight train is 1000 yards ahead of the engine of the passenger train. The end of the caboose of the freight train is 400 yards ahead of the end of the caboose of the passengers train. The freight train is three times as long as the passenger train. What is length of each train?
Sequence	Joseph was offered two jobs. The starting pay of Job A is RM50 on the first day. It is increased by RM50 for each subsequent day. Job B was offered with a starting pay of RM0.50. The subsequent increment is doubled the previous day. He was offered to work for 14 days. After 14 days, which job gives more pay?

The participants were divided into eight groups. During each lesson, they participated in the group discussion activities. The classroom observations participants focused on participants' communication and their response to the activities during the discussion. After each lesson, the participants recorded their reflection of their learning in the reflective journals.

3. Findings

3.1. Analysis of the observation

The following lessons were observed:

Observation of Lesson One
The observation was recorded in Table 2. Participants were confused with the 999 digits. They discussed among themselves the meaning of the 999 digits. The numbers on pages and the 999 digits in the question had confused them.

Table 2. Observation 1

Topics	Question	Observation
Whole number	A printer uses 999 digits to number the pages of a book. How many numbered pages are there in the books? Generalized.	• All participants engaged in the group discussion. • Papers and pencils were used during the discussion. • Books were used as samples to count page numbers.

The participants presented the following interpretation on papers (Fig. 1). They gradually worked out the relationship between the page numbers and the number of digits. They proceeded to guess the numbers by checking the total number of digits which did not exceed 999. At page 999, the total number of digits was (9+180+270) = 459, which did not reach 999, then they continued to find the page number at the number of digits 999 by calculating 999- 459=540 digits. Finally they concluded that from page 1000 to 1539, there were 540 digits. Then, the answer is 1539 pages.

Page	Number of digits	Total number of digits
1	1	
2	1	
3	1	
.		
9	.	9
10	2	
11	2	
12	2	
	.	
	.	
99		(99-9)x2=180
100	3	
.		
.		
999		(999-99)x3=270

Fig.1. Number of pages and number of digits

Observation of Lesson Two

From the observation, participants' reactions in solving the mathematical problem showed that they chose to construct diagrams to understand the question. The observation (as indicated in Table 3) showed that different groups perceived the problem differently. The question was interpreted and represented in two different diagrams by two different groups (see Fig. 2 and Fig. 3).

Table 3. Observation 2

Topics	Question	Observation
Solving equations simultaneously	A freight train and a passenger train come to a halt on adjacent tracks. The engine of the freight train is 1000 yards ahead of the engine of the passenger train. The end of the caboose of the freight train is 400 yards ahead of the end of the caboose of the passengers train. The freight train is three times as long as the passengertrain. What is length of each train?	• Participants in the groups presented a diagram. • Participants presented the diagram in a few ways. • They were discussing based on the diagram they drew.

The participants highlighted the word 'engine of the freight train is 1000 yards ahead of the engine of the passenger train', they drew a diagram (Fig. 2) showing the indication of 1000 yards. But, the mathematical expression was not written.

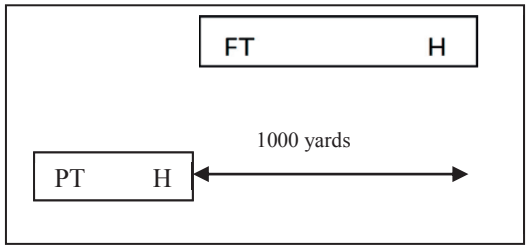

Fig. 2. The first attempt in solving the problem

Further highlighting on the second information about 'The end of the caboose of the freight train is 400 yards ahead of the end of the caboose of the passengers train', the diagram in Fig. 2 was modified to Fig. 3.

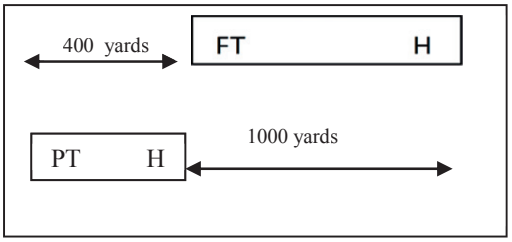

Fig. 3. The second attempt in solving the problem

Lastly, the participants highlighted the information 'The freight train is three times as long as the passenger train', then they present the following solutions based on the above diagrams. Some of the participants showed solution 1, some of them presented the solutions as in solution 2 (in Fig. 4).

Solution 1:
3PT + gap=1000 --- (1)
PT + gap = 400--- (2)
From (2), gap = 400 - PT
Substitute in (1),
3PT+ 400 - PT =1000
2PT + 400 = 1000
2PT = 600
PT = 300 yards, thus, FT =900 yards

Solution 2:
400 + FT=1000 + PT
400 + 3PT = 1000 + PT
2PT = 600
PT =300 yards, thus, FT = 900 yards

Fig. 4. The solutions for lesson 2

Observation of Lesson Three

Participants tended to apply their existing knowledge in solving mathematical problem. The question in Lesson 3 is about mathematical problems in the topic of sequence. Some of the participants used a table to present the given information and illustrated their understanding in a table. The observation about their strategies in solving the problem was described in Table 4.

Table 4. Observation 3: mathematical problems

Topics	Question	Observation
Sequence	Joseph was offered two jobs. The starting pay of Job A is RM50 on the first day. It is increased by RM50 for each subsequent day. Job B was offered with a starting pay of RM0.50. The subsequent increment is doubled the previous day. He was offered to work for 14 days. After 14 days, which job gives more pay?	• Some of the participants constructed a table. • Few participants attempted to solve the problem using the information directly from the sequence.

3.2 Analysis of Reflective Writing

The participants' reflective writing collected indicated that they were able to express mathematical ideas and mathematical thoughts during the activities. They acknowledged that the questions posed sharpened their mathematical thinking. The findings are summarized in the following. In general their problem solving strategies are:

- To give a representation, such as draw a diagram, make a list, use equations.
- To make a calculated guess, such as guess and check, look for patterns, make suppositions
- To go through the process, such as act it out, work backwards.

The findings showed the participants practically went through the above steps to work out the solutions, but strategy 'to change the problem, such as restate the problem, simplify the problem, and solve part of the problem was not applied during the use of the approaches in solving the mathematical problems.

Other than the above strategies, the participants also expressed that the group work had helped them to work out the strategies in solving the mathematical problems. The following results are the participants' reflections about the application of heuristics approach.

Give a representation

In the reflective journals, the participants emphasized that the diagrams they used really help them to solve the mathematical problems. The following statements describe the use of drawing and sketching during the process of problem solving.

> "*firstly, we try to understand the question. Then, we take out all the information given, drafts and draw based on the question…*" (G2)

> "*.. I can't think how to solve it... But, I'm still trying out the question by read it many times and sketch …*" (G7)

> "*…I need to get out all the information given and then I have to make the draft. From that I was get the idea how to solve the problem and I feel confident…*"(G8)

Make calculated guess

When participants looked for patterns of the solution, they made a calculated guess based on their previous knowledge. In doing this they had engaged the use of heuristic approaches. They expressed their words in the reflective writing that:

> "*..taking out the information, then understands the question, draw and recall and relate or have a strategy*." (G2)

> "*… suspect and relate, ...suspect and guessing to get clear*.." (G2)

Go through the process

When the participants went through the process of getting the solutions, they thought further. Creativity was developed from these processes of thinking by working or going backward to check the solutions. The participants indicated that they were encouraged to think further.

"*At first, I think it is impossible for me to find a solution with too little information on it… I learned that it is not easy to fully understand the question ... must be creative also ... with various method of Mathematics*…." (G1)

"*…the question is not difficult to solve but we have to think more creative to find a correct answer...*" (G4)

Group discussion

In the first lesson, participants were aware of the importance of group discussion. They acknowledged that they were more confident to carry out their solutions in the discussion. They raised a few comments in their reflective writings during the process of finding the right solution. For example,

"*..I get the answer, based on the discussion with my group member. From my opinion, when we discussed in group, we can get more opinion to solve problem because every person has their own way on thinking and solve the question. Besides that, it is actually good working and discuss in group because it is better if we ask other's opinion before we solve it. We also can get a bit confidence when answering the question*." (G3)

"*... I need to do many times until I get the correct answer. I refer to my friend on this question because I can't understand about the question. After that, we continue our class and discuss about ...*" (G5)

4. Discussion

From the observation, the participants worked in groups. When they were arranged in groups for solving mathematical problem solving questions, they involved themselves to experience approaches for the implementation of heuristics. They communicated their mathematical thinking through visualization in the forms of diagram or tables. Visualization is indeed an important aspect of mathematical reasoning since many studies have discovered that activities which inspire the representation of images and diagram show enhancement in mathematical learning (Bishop, 1988; Bodner & Goldin, 1991; Wheatley & Brown, 1994). In addition Shin (1994) described that diagrams can be a good heuristic aid to validate mathematical reasoning. Thus, solving mathematical problems with diagrams can help participants to understand better and help them to plan for the solutions.

It was also observed that although some participants were capable of using formal knowledge but not all of them were able to generate arguments. Studies by Garii (2002), Kramrski, Mevarech and Arami (2002) as well as Stein (2001) revealed that activities that involve individual cognition and social interactions promote thinking skills and hence improve mathematical understanding and achievement. Creativity in discussing the solution is equally important. The environment and social factor play an important part in participants thinking process and meaning making when engaging in heuristic approach. In addition, basic mathematical knowledge is needed for the participants to plan and design the solutions. Likewise, student solves problems according what he understands based on his own learning experiences. Nonetheless, the participants did not further stimulating their thinking to manipulate any related problems during the process of applying heuristics in solving the questions.

5. Conclusion

Results from the reflective writing indicated that diagram and basic knowledge are means of solving mathematical problems. Participants were also seen to revise repeatedly their solutions and to obtain alternative solutions. The students had shown their ability in using the heuristic approach which they had learned in the previous class. These students also acquired the ability to apply basic knowledge in their thinking process. Meanwhile, learning in groups and engaging class discussions are encouraged in students' learning process. In this study it was observed that the thinking skills are dependent on the ability of students to use their constructed knowledge based on prior learning (numbers) through communicating, processing and finally to interpret information needed in solving a mathematical problem. These thinking skills were built from heuristics process which provides a way of contextualizing the communication process and hence assisting students to solve mathematical problems. Since the problem in the third lesson is simple and directly involve prior knowledge, many students could answer through logical thinking or thinking deeply based on their previous knowledge. Also, presenting information using a table is useful for visualizing the schemas of getting the answer. Obviously, group as well as class discussions have inspired students' thinking, and hence to heuristic thinking. Nonetheless, the students still need more practices to further improve their skills in using the heuristic approaches.

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