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Slope Sensitivity Analysis using Spencer's Method in Comparison with General Limit Equilibrium Method

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Abstract

This paper analyses the sensitivity analysis of a natural-unreinforced slope in Kepong, Kuala Lumpur. In order to conduct a sensitivity analysis, several ranges of data, based on borehole information and figures and also typical range of earth materials geotechnical values are used to set the minimum and maximum value of parameters in Slide 6.0 software. Under Mohr-Coulomb failure criterion, Spencer's and General Limit Equilibrium methods of slices were used in the analysis to determine the influence of varying parameters values towards the change in safety factor. In addition, the percentage differences in safety factors obtained by both methods based on General Limit Equilibrium method are also determined in the analysis. In the analysis, water table location has the highest influence to the change in safety factors of the slope. Besides, the percentage differences in safety factors obtained by both methods are very nominal and showed a good agreement to each other.

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Keywords: Slope sensitivity analysis; Spencer's method; General limit equilibrium method

1. Introduction

The case study area is located on the west side of Taman Ehsan residential area in Kepong, Kuala Lumpur and is about 2.5 km to the southwest of Forest Research Institute Malaysia (FRIM). The studied slope is characterised by a

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hilly terrain, 125 m above sea level with an overall slope angle 34° from toe to crest. There were three boreholes dug at three points of different elevations along the slope[1]. Borehole 1 was located at the toe, borehole 3 at the top crest and borehole 2 at the middle bench between crest and toe of the slope. Based on the data of boreholes, the slope consists of two earth material layers, at which the top layer comprises of residual soil that covers a slightly fractured and weathered sandstone bedrock. Most of the soil layer depths were recorded between 11 m to 22 m and increase from crest to toe. From the soil testing data, majority of the soil is SM group and a few categorised as SC group. This classification is according to Unified Soil Classification System[2]. Overall, the overburden consists of coarse-grained soil with non-plastic silty fines.

Nomenclature

FRIM	Forest Research Institute Malaysia
F	safety factor
LEM	Limit Equilibrium Method
GLE	General Limit Equilibrium
S, S_m	total strength available and total shear strength mobilised, respectively
Q	resultant of pair of interslice forces
b, h	width and mean height of slice, respectively
α	slope of base of slice
X, E	vertical interslice shear forces and horizontal interslice normal forces, respectively
λ	a constant representing the percentage of portion of the interslice function
θ	slope of resultant Q of pair of interslice forces
F_f	value of safety factor obtained using force equilibrium equation
F_m	value of safety factor obtained using moment equilibrium equation
F_{mo}	value of safety factor which satisfies moment equation when $\theta = 0$
F_i	value of factor of safety which satisfies both force and moment equations
θ_i	value of θ which satisfies both force and moment equations
β	slope of embankment
r_u	pore-pressure coefficient
γ	bulk density
H	height of embankment
ϕ'	angle of shearing resistance with respect to effective stress
c'	cohesion with respect to effective stress
ϕ'_m	mobilised angle of shearing resistance
$f(x)$	a function that describes the manner in which X/E varies across the slope
θ_R, θ_L	angle of right and left interslice forces, respectively
Z_R,Z_L	right and left interslice forces, respectively
ICU	Isotropically Consolidated Undrained Triaxial Compression
SPT	Standard Penetration Test
CSS	critical slip surface
WTL	water table location

2. Literature Review

2.1. Limit Equilibrium Methods

Slope stability analysis can be carried out by using various methods. There are four main methods that can be used to determine the safety factor F of a slope; the Limit Equilibrium Method (LEM), Limit Analysis Method, Finite

Element Method and Finite Difference Method. It worth highlighting that each of the method possesses its own advantages and criteria. However, this paper focuses on LEM, specifically Spencer's and General Limit Equilibrium (GLE) methods of slices. Basically, LEM is a method that divided a soil mass above the slip surface into a finite number of slices at which the slices may be cut vertically or horizontally. It should be noted that the way the soil mass at the slip surface being cut does not contribute to major importance to the methods[3,4]. Besides, the slices also do not necessarily have a constant width[4]. Methods of slices are categorised into two categories; simplified and rigorous methods. Simplified methods satisfy only either force or moment equilibrium but cannot satisfy both at the same time while rigorous methods satisfy both force and moment equilibriums.

2.1.1. Spencer's Method

In early development of limit equilibrium methods of slices, the slip surface is assumed to be cylindrical with the earth mass within this slip surface is divided into a few vertical slices and in each slice calculation, the resultant forces and the sum of moments must be equal to zero[5]. However, the result of each of the method of slices is dependent upon several factors; definition of F and assumptions made relating to the interslice force between the vertical slices. The Spencer's method is basically a modified and extended version of the Bishop's simplified method. The Bishop's F is defined as the ratio of total strength available S on the slip surface to the total shear strength mobilised S_m[5]:

$$F = S/S_m \tag{1}$$

Besides, to apply this definition, the value of F must be constant for all slices. This indicates that there is an existence of interslice forces. The Bishop's rigorous method uses the initial value of F that obtained by using the simplified expression which assumed that the interslice forces between slice were horizontal[5]. This simplified expression will only satisfy the moment equilibrium while the other equilibrium is not satisfied. Furthermore, the value of F given by Bishop's simplified method was very near to the final value obtained using the rigorous method. In Spencer's analysis, the derived resultant Q of pair of interslice forces:

$$Q = \gamma H b \left[\frac{\dfrac{c'}{F\gamma H} + \dfrac{h \tan\phi'}{2HF}\left(1 - 2r_u + \cos 2\alpha\right) - \dfrac{h \sin 2\alpha}{2H}}{\cos\alpha \cos(\alpha - \theta)\left[1 + \dfrac{\tan\phi'}{F}\tan(\alpha - \theta)\right]} \right] \tag{2}$$

And if the external forces on the embankment are in equilibrium, the vectorial sum of the interslice forces must be zero. Hence:

$$\Sigma\left[Q\cos\theta\right] = 0 \tag{3}$$

$$\Sigma\left[Q\sin\theta\right] = 0 \tag{4}$$

Besides, if the sum of moments about the external forces about the centre of rotation is zero, the sum of moments of the interslice forces about the centre of rotation must also become zero. The sum of moment equation will also be simplified since the slip surface is assumed to be circular and the radius of curvature is constant. Therefore, the sum of moment equation will be:

$$\Sigma\left[Q\cos(\alpha - \theta)\right] = 0 \tag{5}$$

With the assumption of a constant (θ is constant) relationship between the magnitude of the interslice shear and normal forces which caused the interslice forces to be parallel to each other:

$$X/E = \tan\theta \tag{6}$$

Consequently, Eq. (3) and (4) above will be identical:

$$\Sigma Q = 0 \tag{7}$$

Thus, there are only two equations need to be solved; i.e. Eq. (5) and (7). In solving Eq. (5) and (7), several values of θ were chosen and for each, the value of F was found which would satisfy both Eq. (5) and (7). Three types of F can be derived; F_f is obtained using Eq. (7), F_m is obtained using Eq. (5), F_{mo} is obtained using Eq. (5) by taking θ as zero. After all these F have been calculated based on several θ values, a graph of curves F_f against θ and F_m against θ were plotted. The intersection of these two curves give a value of safety factor, F_i that satisfies both force and moment equilibriums and the corresponding slope, θ_i of the interslice forces. Next, the values of F_i and θ_i were substituted into Eq. (2) to obtain the values of the resultants of the interslice forces. Then, the points of action of the interslice forces were determined by taking moments about the middle of the base of each slice. It was found that there are several points can be concluded from Spencer's method[5]:

- The value of θ_i was always less than the slope of embankment β;
- F_f was affected to a much greater extent than those F_m with a varying value of θ; even when θ was less than θ_i, the variation in F_m was very small; in consequence there was only a small difference between the values of F_{mo} and F_i; this indicates that the accuracy of the simplified method lies in the insensitivity of moment to the varying value of θ when $\theta \leq \theta_i$; and
- The line passing through the points of action of the interslice forces passed very closed to the lower "third point" on each of the interslice boundaries.

Furthermore, in Spencer's method, the value of F is directly proportional to the number of slices but there is only a little gain in accuracy for the number of slices that is greater than 32[5]. Besides, the number of slices around 25 is adequate to acquire an accurate solution for most problems since a large value, e.g. 100 could lead to a higher computation time with the solution computed is not necessarily improved[6]. It was found that in Spencer's method, the locations of slip surface were influenced by β, pore-pressure coefficient r_u, and parameter $(\gamma H \tan ø')/c'$. For stability charts, they were influenced by r_u, mobilised angle of shearing resistance $ø'_m$, stability number $c'/(F\gamma H)$, and β.

2.1.2. General Limit Equilibrium Method

GLE method is an extension of Spencer's method that has been generalised to suit functions which describe the variations of interslice force angles to satisfy complete equilibriums[7]. GLE is a powerful method as it can formulate other methods of slices, e.g. Morgenstern-Price's, Spencer's, Bishop's simplified, Janbu's simplified, Janbu's generalised, Lowe and Karafiath, and Corps of Engineers' methods to be its special cases, based on the relations between interslice shear and normal forces[8]. However, the Ordinary/Fellenius' method was excluded from general formulation because it did not satisfy Newtonian force principle at interslice[8].

Eq. (8) shows the function that is adopted by GLE method. The function $f(x)$ represents the shape of the distribution that has been used to describe the variation of the interslice force angles and it ranges between 0 and 1. The discrete form of the continuous function, $f(x)$ was used to calculate the function at each interslice boundary using the angles labelled θ_L and θ_R for the left and right vertical sides of the slice, respectively. The distribution is usually implemented with a function that is normalised with respect to the lateral extent of the failure surface[7]. This lateral extent is assumed to range between the first and the last interslice boundary with an initial assumption that the interslice force angle for the left side of the first slice and the right side of the last slice are zero.

$$\theta = \lambda . f(x) \tag{8}$$

For force equilibrium, the interslice resultant forces, Z_L and Z_R, are inclined at θ_L and θ_R. The interslice forces are considered as the total forces and those hydrostatic components are not considered. In this case, Z_R for the last slice will be equal to the boundary force for which if there exist a tension crack filled with water at the crest, Z_R will be equal to the hydrostatic water force. However, if there is no water-filled cracks exist, zero boundary force is used. These conditions are also applied to moment equilibrium to determine the location of the interslice forces, for which moments of all slice forces are taken about the midpoint of the base of slice.

3. Methodology

The methodology involved four stages; boundary setup, data input, model computation and result interpretation. The first stage which is boundary setup was based on the topography of cross-section of the studied area and boreholes data for different earth material layers positions. For data input, the data in this analysis was based on the previous research conducted at FRIM[1]. The third stage which is the model computation in this paper utilised only Spencer's and GLE methods of slices in geotechnical analysis software called Slide 6.0 from Rocscience Inc. Spencer's and GLE methods were selected for the analysis because both of them satisfy force and moment equilibriums which entitled them to be comparable. As for the setting of the convergence option, the number of slices, tolerance and number of iterations were set to 32, 0.005 and 100 respectively.

The tolerance is the difference in *F*, between two successive iterations of the limit equilibrium analysis procedure, at which the solution is considered to have converged and the iteration process is stopped[6]. The value was set to be default 0.005 because a much smaller value will lengthen the computation time and also might lead to convergence problem. If the value is set to be greater than 0.01, the solution convergence will be obtained at a nominal time, however it might cause a less accurate value of *F*. The maximum iterations were set to be 100 which are 50 iterations more than the default. This value was set in order to better establish a constant solution achieved for which if the maximum number of iterations is reached, the iteration process is terminated for that slip surface and the last computed value of *F* is recorded.

Groundwater method chosen was the water surface with the pore fluid unit weight of 9.81 kN/m^3. Water surface method was selected because water table can be used to define the pore pressure conditions for each soil type[6]. In order to define the upper and lower limits of a water table for the sensitivity analysis, the maximum and minimum water table boundaries must be defined first. The maximum water table was set along the top external boundary of the slope whereas for minimum water table, the location was set along the bottom external boundary. After these maximum and minimum water tables have been set, the third boundary which is the mean water table would be automatically calculated. These water tables boundaries were determined at such condition to represent the best and the worst case scenarios. The mean water table was calculated based on the normalised mean parameter. Normalised mean is defined as the relative elevation of the mean water table, along any vertical line between the minimum and maximum water table boundaries and has a value between 0 and 1[6]. The normalised mean in this analysis was set to default value of 0.5 with a normal distribution.

The probabilistic analysis inputs for sensitivity analysis of the model were selected from both site investigation data and also typical range for the similar soil or rock type. For soil, the cohesion values were set according to the site investigation data which ranges from 0-15.5 kN/m^2. Zero value of cohesion denoted that soil comprises of entirely sand without the existence of clay to contribute cohesive strength. These ranges include both of the effective and total cohesion obtained through an Isotropically Consolidated Undrained Triaxial Compression (ICU) test. However, the analysis conducted was using effective stress behaviour under Mohr-Coulomb failure criterion. For soil friction angles, the value ranges from 28° to 45° was selected based on the typical friction angles on clean medium sand[9] and also on the uncorrected Standard Penetration Test (SPT) values from site investigation at which the lowest is 8 and the highest is greater than 50. On the other hand, both cohesions and friction angles for sandstone was defined according to rock shear strengths and friction angles based on geologic origin. It has been highlighted that the typical cohesions and friction angles of an intact fresh to slightly weathered clastic arenaceous sedimentary rocks are about 15 MPa and 35°, respectively[9].

For the last parameter, the range of bulk unit weight of soil was set according to the lowest initial bulk unit weight and highest final bulk unit weight after ICU test. Based on the test, the lowest and the highest bulk unit weights are 18.865 kN/m^3 and 23.220 kN/m^3, respectively. In opposition, unit weight of sandstone was defined from the representative range of dry unit weight from sedimentary rock[9] which states that sandstone has unit weight ranges from 18 - 26 kN/m^3. Table 1 shows the summary of probabilistic analysis inputs for the slope model.

Table 1. The summary of probabilistic analysis inputs.

Material	Strength type	Water Surface	Property	Distribution	Mean	Minimum	Maximum
Soil	Mohr-Coulomb	Water table	Cohesion (kN/m^2)	Normal	7.75	0	15.5
Soil	Mohr-Coulomb	Water table	Friction angle (°)	Normal	36.5	28	45
Soil	Mohr-Coulomb	Water table	Unit weight (kN/m^3)	Normal	21.0425	18.865	23.220
Sandstone	Mohr-Coulomb	Water table	Cohesion (kN/m^2)	Normal	15000	10000	20000
Sandstone	Mohr-Coulomb	Water table	Friction angle (°)	Normal	35	25	45
Sandstone	Mohr-Coulomb	Water table	Unit weight (kN/m^3)	Normal	22	18	26

There are two slope limits have been defined which cover the top crest and toe boundaries. These limits were defined at such condition in order to locate only the critical slip surface (CSS) at the main body of the slope. Due to some restrictions, this paper only locates the circular CSS of the slope using grid search method (grid spacing 50x50). After the model has been computed, the fourth stage which is the result interpretation was carried out to analyze the sensitivity of *F* towards the varying parameters values and also to compare the differences between both *F* obtained by Spencer's and GLE methods.

4. Results and Discussion

In sensitivity analysis, each of the parameter is varied in uniform increments, between the minimum and the maximum values and the slip surface is determined at each value[6]. During the analysis, all other input parameters are held constant at their mean values while a parameter is being varied[6]. Based on Fig. 1, the sensitivity plots of Spencer and GLE are very similar to each other. The percent of range at 50 % represents the mean value of each variable. These mean values are used for the deterministic analysis which computed before the sensitivity analysis is conducted. The deterministic analysis is compulsory to be conducted in order to determine CSS. The CSS of the slope is shown in Fig. 2.

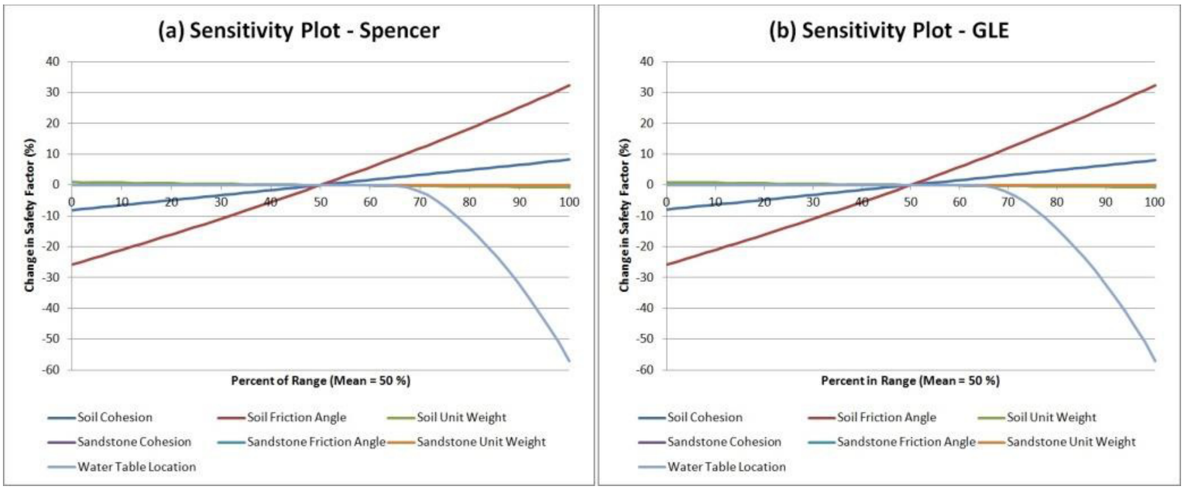

Fig. 1. (a) Spencer's sensitivity plot; (b) GLE sensitivity plot.

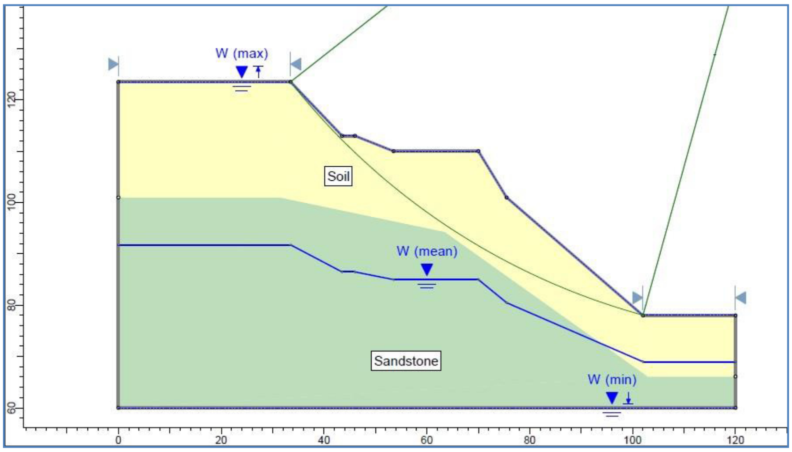

Fig. 2. CSS of the slope. Both of the methods of slices generated similar location of CSS.

From the sensitivity plots of Spencer and GLE, all of the parameters curve lines for sandstone are overlapping each other in a flat trend. This indicates that the varying sandstone cohesion, friction angle and unit weight have no influence to *F*. The insensitivity of *F* towards varying sandstone parameters is due to fact that CSS does not passing through the sandstone (see Fig. 2). It is worth to note that the location of CSS in this sensitivity analysis is always unchanged when sandstone parameters are varied during the analysis. Thus, only soil parameters and water table location (WTL) are the factors that influence the change in *F*. *F* obtained by using deterministic analysis (only mean value of each parameters are used) are 1.389 and 1.391 for both Spencer's and GLE methods, respectively.

Based on Fig. 1, it was noticed that the varying soil unit weight has caused insignificant influence to the change in *F*. The minimum soil unit weight gives the highest change in *F* which is 0.9 % for both methods of slices. Besides, *F* decreases with the increasing of soil unit weight. The increase in unit weight caused the total vertical force due to the mass of a slice to increase. Consequently, the increased in total vertical force has a higher chance to overcome the friction resistance of the soil, which makes it more susceptible to fail at CSS. The soil cohesion parameter in the analysis has a more influenced to the change in *F* than the soil unit weight. Both the minimum and maximum value of cohesion gives about 8 % change in *F* with the curve line increases proportionally to the increases in soil cohesion. This relationship is according to the definition of *F* in terms of shear strengths, and also according to the Mohr-Coulomb failure criterion. However, soil cohesion in this analysis has low effect to the change in *F* which might be due to the low value of cohesion.

In this analysis, soil friction angle gives the second highest influence to change in *F*. Based on Fig. 1, the minimum and maximum friction angles result in 26 % and 36 % change in *F*, respectively. According to the Mohr-Coulomb failure criterion, the shear strength in soil is described in two components, i.e. cohesive and frictional components. Based on the criterion, the magnitude of frictional component is dependent on the stress acting normal to the soil slice base. However, this is not the case for cohesive component for which the magnitude is independent to normal stress. In this analysis, *F* is more responsive to the varying value of soil friction angle than the soil cohesion because the value of soil cohesion is very small to contribute to the shear strength of the soil.

The last parameter, which is WTL has the highest influence towards the change in *F*. However, the sensitivity curve line has a very unique trend. There is a very insignificant change in *F* when WTL is varied below CSS (percent of range < 63 %) for both methods of slices. *F* started to decrease drastically once WTL coincide with CSS (see Fig. 1) and moves towards the maximum location. If the water table is located at maximum position, it will results in *F* of 0.597. This caused 57 % change in *F*. The reason to this high changes is because when WTL is higher, the existing pore pressure become higher and pushes the soil grain apart. This consequently reduces the normal forces and friction resistance at the base of CSS, thus results in weakening of CSS. Based on Table 2, the percentage difference of *F* based on GLE for all input parameters do not exceed 2 %. The very little differences in *F* between those two methods is due to similar interslice forces assumptions. In Spencer's method, the interslice forces are assumed to be parallel to each other while in GLE method, a function is adopted to describe a variation in interslice force angles in order to calculate θ_L and θ_R. Despite of the difference in interslice forces assumption, F obtained in this analysis shows a good agreement between the Spencer's and GLE methods.

Table 2. Percentage Difference of F Based on GLE (%) of various parameters.

Parameters	Percentage Difference of F Based on GLE (%)
Soil Cohesion	0.1664
Soil Friction Angle	0.1604
Soil Unit Weight	0.1571
Sandstone Cohesion	0.1581
Sandstone Friction Angle	0.1581
Sandstone Unit Weight	0.1581
Water Table Location	0.1720

5. Conclusion

A slope sensitivity analysis towards varying values of cohesion, friction angle, unit weight, and water table location has been presented. Among the four parameters concerned, the parameter that has the highest influence to the change in safety factor is water table location. This was followed by friction angle, cohesion and unit weight. These findings are in accordance to the Mohr-Coulomb failure criterion. However, this condition only applies to the earth layer at which the critical slip surface is passing. For the earth layer where the critical slip surface does not pass, the parameters have insignificant influence towards the change in safety factor. In addition, the differences in safety factor obtained by Spencer’s and General Limit Equilibrium methods are very nominal and showed a good agreement to each other.

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