Rough center mining algorithm of rough cognitive map

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Abstract

For the large-scale systems with complex causal connection in practical problems, fusion rough cognitive maps and graph mining methods, designed a rough central area mining algorithm of rough cognitive maps to complete the pruning problem of rough cognitive map, simplified the structure of rough cognitive map to facilitate the further relations. First of all, from the perspective of the arc, give another definition of rough cognitive map, introduce the concept of rough centrality $R$ accuracy; second, define certainty -possible class road, according to the given threshold to determine the rough central area. Finally, design an effective rough central area mining algorithm for further analysis and forecast the rough cognitive map.

Keywords Rough cognitive maps; central area; rough central area mining algorithm

1. Introduction

The expression of causal reasoning is an important research in artificial intelligence field. In the past 10 years, fuzzy cognitive maps[1] have been widely used, For the problem that fuzzy cognitive map and probabilistic fuzzy cognitive map consider that the relation between the concepts is unique, and often reflected by a fixed fuzzy measure which subjectively determined by experts, In the text of the[2], the author introduce rough sets [3] ideas into the cognitive map, and propose rough cognitive maps (Rough Cognitive Map, RCM) model to indicate the rough causal relation in uncertain problems.

Rough cognitive map can solve the edge rough problems in cognitive map, however, in solving practical problems, we often encountered in large-scale complex systems. Rough cognitive maps have a very large scale and complex causal connections [4]. Design rough cognitive map and express the dynamic information only by observation and intuition is not enough. In general, rough cognitive maps

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and cognitive maps are the same, not all nodes are equal important in graphs. In the analysis of problems, some nodes are often irrelevant to or low important to the analysis of the problem. Therefore, when analysis causal relations, in order to simplify the rough cognitive maps, we can remove those nodes that not very important, in order to simplify the problem. Can also be said that in the rough cognitive maps, mining an important nodes and causal relations between nodes are very important for complete analysis and forecast of rough cognitive map. It can simplify the relations reasoning in rough cognitive map, it is a very worthwhile study.

This article give another definition of rough cognitive map from the perspective of the arc,, introduce the concept of rough centrality \( R \) accuracy from the perspective of the node. By calculating rough accuracy and the given threshold to determine central area, thus, cutting off the lower approximation class out-degree of node \( v \) represent directed edges that node \( v_i \) to node \( v_j \). But in this representation it is difficult to calculate in-degree, out-degree and centrality of nodes. Therefore, from the perspective of the node, we express the edge starting from node \( v_i \) to the node \( v_j \) with arc, give another definition of rough cognitive map:

\[
\text{Definition 2.1 Given discourse cognitive map} \; U = (V, E, W), U = \{e_1, e_2, \ldots, e_n\} \text{is discourse,} \\
V = \{v_1, v_2, \ldots, v_n\} \text{is vertex set,} \\
W = \{w_{ij}(k), w_{ji}(k)\} \text{is the weight of the kth directed arcs between} \\
\text{the nodes} \; v_i \text{and} \; v_j \text{,} \\
R = \{r_1, r_2, \ldots, r_n\} \text{is the attribute set on} \; U, E = \bigcup e_k < v_i, v_j > \text{is the edge set on} \; V, \text{Among them, considering undirected edges, make} \\
d: e_k \rightarrow d(e_k), \text{is the direction of edge} \; e_k \text{, and} \\
d(e_k) \in \{+, -\} \text{. Edges which given the direction are called arcs, denoted by} a, \text{E} = \bigcup e_k < v_i, v_j > \text{have directions, so} \\
e_k = a^+(v_i, v_j), v_i \text{is the head of} \; a^+, v_j \text{is the tail of} \; a^+, \text{E} = \bigcup e_k < v_i, v_j > \text{.}
\]

\[
\text{Definition 2.2 Given discourse cognitive map} \; U = (V, E, W), U = \{v_1, v_2, \ldots, v_n\}, \\
E = \bigcup e_k < v_i, v_j >, \forall e \in E, \text{make mapping} \; w: e \rightarrow w(e) \text{.call the real number} \; w(e) \text{is the edge weight,} \\
\Omega(e_{w1}^R) = \Omega(e_{w2}^R) = f(w(e)) \text{is the class weight of edge equivalence class} \\
e_{w1}^R, e_{w2}^R \text{.In which,} \; e \in [e_{w1}^R, e_{w2}^R] \text{is the edge equivalence class between node} \; u \text{and node} \; v \text{.}
\]

\[
\text{Definition 2.3 For any attribute set} \; R \in \Omega \text{on} \; E, \text{edges between node} \; u \text{and node} \; v \text{can be divided} \\
\text{into different equivalence classes} \; [e_{w1}^R, e_{w2}^R], \text{if} \; d(e) \subseteq R, \text{call} \; d(e) \text{is the class direction of equivalence classes} \\
[e_{w1}^R, e_{w2}^R] \text{, in which} \; e \in [e_{w1}^R, e_{w2}^R] \text{.The edge equivalence class which given direction is called} \\
equivalence class, \text{denoted} [a_{uv}^R]^+. \text{If} \; d(e) = +, [a_{uv}^R]^+ \text{is with positive direction, denoted} [a_{uv}^R]^+. \text{in which,} \; u \text{is the tail of} \; [a_{uv}^R]^+, \text{v is the head of} \; [a_{uv}^R]^+. \text{If} \; d(e) = -, [a_{uv}^R]^-. \text{is with negative direction, denoted} \\
[a_{uv}^R]^-. \text{in which,} \; u \text{is the head of} \; [a_{uv}^R]^-, \text{v is the tail of} \; [a_{uv}^R].
\]

2.2 RCM Rough centrality

\[
\text{Definition 2.4 In rough cognitive map} \; (\overline{R}(T), \overline{R}(T)), \text{the lower approximation class in-degree of} \\
node \; v \; \text{is the number of different arc equivalence classes which start from node} \; v \text{in} \; \overline{R}(T); \\
\text{the lower approximation class out-degree of node} \; v \; \text{is the number of different arc equivalence classes which end with node} \; v \text{in} \; \overline{R}(T);
\]
Definition 2.5 In rough cognitive map $\overline{(R(T), \overline{R}(T))}$, the upper approximation class in-degree of node $v$ $\overline{\text{Deg}}^+(v)$ is the number of different arc equivalence classes which start from node $v$ in $\overline{R}(T)$; the upper approximation class out-degree of node $v$ $\overline{\text{Deg}}^+(v)$ is the number of different arc equivalence classes which end with node $v$ in $\overline{R}(T)$;

Theorem: For any rough cognitive map $D = (R(T), \overline{R}(T))$, let $\varepsilon_1$ is the total number of different equivalence classes in $R(T)$, $\varepsilon_2$ is the total number of different arc equivalence classes in $\overline{R}(T)$, $\overline{\text{Deg}}^+(v)$ and $\overline{\text{Deg}}^+(v)$ is lower approximation class in-degree and lower approximation class out-degree of node $v$ respectively, $\overline{\text{Deg}}^+(v)$ and $\overline{\text{Deg}}^+(v)$ is upper approximation class in-degree and upper approximation class out-degree of node $v$ respectively, if attribute set $R$ contains vertex attributes, there

$$\sum_{v \in V} \overline{\text{Deg}}^+(v) = \sum_{v \in V} \overline{\text{Deg}}^-(v) = \varepsilon_1; \sum_{v \in V} \overline{\text{Deg}}^+(v) = \sum_{v \in V} \overline{\text{Deg}}^-(v) = \varepsilon_2;$$

In the map, the simplest, most direct measure of the node is centrality. In rough cognitive map, the absolute centrality of the node is distinguished between two aspects: in-centrality and out-centrality.

On the other hand, accordingly, the centrality in $R(T)$ is the lower approximation absolute centrality, contains lower approximation in-centrality and lower approximation out-centrality. The centrality in $\overline{R}(T)$ is the upper approximation absolute centrality, contains upper approximation in-centrality and upper approximation out-centrality.

However, there is a significant limitation in the measure of centrality. The centrality makes sense only between the members in the same map or in the map of same scale. Above other factors, the degree of a point depends on the size of graph. So when the map of different sizes, centrality of different points is not comparable. In view of this, the use of the value of initial degree may be misleading.

To overcome this problem, we propose the relative centrality, when calculate the centrality of directed graph based on the relative number, according to the basic definition, need to calculate in-centrality and out-centrality of a certain node in the same time, and then demand ratio with the total number of connections. The formula is:

$$C^*_o(n_i) = \frac{d_i(n_i) + d_o(n_i)}{2(N-1)};$$

Where $N$ is the size of the map.

Therefore, we need to calculate the lower approximation class in-degree and the lower approximation class out-degree to calculate the relative centrality of the node, and then demand ratio with the total number of equivalence classes in $\overline{R}(T)$,this is called lower approximation relative centrality,

$$C^*_o(v)_{\text{RR}} = \frac{\overline{\text{Deg}}^-(v) + \overline{\text{Deg}}^+(v)}{2(N-1)}.$$  (1)

We need to calculate the upper approximation class in-degree and the upper approximation class out-degree to calculate the relative centrality of the node, and then demand ratio with the total number of equivalence classes in $\overline{R}(T)$,this is called upper approximation relative centrality,

$$C^*_o(v)_{\text{RR}} = \frac{\overline{\text{Deg}}^+(v) + \overline{\text{Deg}}^-(v)}{2(N-1)}.$$  (2)

Definition 2.6 The rough centrality of node $v$ in rough cognitive map can be defined approximately by two accurate relative centralities $C^*_D(v)_{R(T)}$ and $C^*_D(v)_{\overline{R}(T)}$,in which,

$$C^*_o(v)_{\text{RR}} = \frac{\overline{\text{Deg}}^+(v) + \overline{\text{Deg}}^+(v)}{2(N-1)}; C^*_o(v)_{\text{RR}} = \frac{\overline{\text{Deg}}^+(v) + \overline{\text{Deg}}^+(v)}{2(N-1)} \cdot R(T) \text{ and } \overline{R}(T) \text{ is R- lower}$$
approximation and R-upper approximation of map \( T' = (C_D(v)_{R(T)}, C_D(v)_{\overline{R}(T)}) \) is the rough centrality of node \( v \).

In order to compare the relative importance of each node in rough cognitive map more accurately, we introduce the concept of rough centrality \( R \) accuracy, the formula is:
\[
d(v) = \frac{C_D(v)_{R(T)}}{C_D(v)_{\overline{R}(T)}}
\]  

Let \( H \) be threshold of importance degree of node, it can be set according to the needs of users. When \( d(v) \geq H \), node is reserved. Otherwise, delete the node to achieve pruning large-scale rough cognitive maps.

3. Rough central area

3.1 certainty -possible class road

Definition 3.1 The certainty class path \((v_0, v_i)\) of rough cognitive map \( D = (R(T), \overline{R}(T)) \) is a finite non-empty sequence \( W = v_0 \leftarrow v_1 \leftarrow \cdots \leftarrow v_i \) in \( R(T) \), its items are vertices and edge equivalence classes in \( R(T) \) alternately, \( \forall i, j = 0, 1, \ldots, v-1 \), \( e_{v_i,v_j} \in R(T) \) belong to the same edge equivalence class determined by \( R \). \( v_i \) is the length of \( W \). If \( \forall i \neq j, i, j = 0, 1, \ldots, v-1 \), then \( v_i \neq v_j \), the certainty class path is called certainty class road.

Definition 3.2 The possible class path \((v_0, v_i)\) of rough cognitive map \( D = (R(T), \overline{R}(T)) \) is a finite non-empty sequence \( W = v_0 \leftarrow v_1 \leftarrow \cdots \leftarrow v_i \) in \( R(T) \), its items are vertices and edge equivalence classes in \( R(T) \) alternately, \( \forall i, j = 0, 1, \ldots, v-1 \), \( e_{v_i,v_j} \in R(T) \) belong to the same edge equivalence class determined by \( R \). \( v_i \) is the length of \( W \). If \( \forall i \neq j, i, j = 0, 1, \ldots, v-1 \), then \( v_i \neq v_j \), the possible class path is called possible class road.

Obviously, there may be several equivalence classes, so the certainty class road and the possible class road between two nodes may not be only in rough cognitive maps.

Definition 3.3 In rough cognitive map \( D = (R(T), \overline{R}(T)) \), if there is \((u, v)\) certainty class path, then any two nodes in vertex set are certainty class connected.

Definition 3.4 In rough cognitive map \( D = (R(T), \overline{R}(T)) \), if there is \((u, v)\) possible class path, then any two nodes in vertex set are possible class connected.

Definition 3.5 For any rough cognitive maps \( T = (M, X, W) \), assuming its edge set can be divided into equivalence classes \( \{E_1, \ldots, E_n\} \), if there is a certain edge equivalence class \( E_k \) makes all vertices in vertex set are certainty connected with \( E_k \), then rough cognitive \( T \) is certainty class connected graph; If any edge equivalence class \( E_k \) makes all vertices in vertex set are certainty connected with \( E_k \), then rough cognitive \( T \) is certainty strong class connected graph.

Definition 3.6 For any rough cognitive maps \( T = (M, X, W) \), assuming its edge set can be divided into equivalence classes \( \{E_1, \ldots, E_n\} \), if there is a certain edge equivalence class \( E_k \) makes all vertices in vertex set are possible connected with \( E_k \), then rough cognitive \( T \) is possible class connected graph; If any edge equivalence class \( E_k \) makes all vertices in vertex set are possible connected with \( E_k \), then rough cognitive \( T \) is possible strong class connected graph.

Definition 3.7 (central area)

- Certainty central area: Given rough cognitive map \( T = (M, X, W) \) is certainty strong class connected graph, if \( T' = (M', X', W') \), \( M' \subseteq M, X' \subseteq X, \frac{1}{\sum_{u \in X'} |E_{u,x}|} \) then \( T' \) is certainty strong class connected sub graph of \( T \), if \( T' \) contains the node of the highest rough centrality \( R \) accuracy, then \( T \) is maximum certainty central area.

- Possible central area: Given rough cognitive map \( T = (M, X, W) \) is possible strong class connected graph, if \( T' = (M', X', W') \), \( M' \subseteq M, X' \subseteq X, \frac{1}{\sum_{u \in X'} |E_{u,x}|} \) then \( T' \) is possible strong class connected sub graph of \( T \), if \( T' \) contains the node of the highest rough centrality \( R \) accuracy, then \( T \) is maximum possible central area.
3.2 Mining Algorithm

The proposed algorithm is based on the detection of node rough centrality to cut the large scale rough cognitive map structure.

Algorithm steps:
1. **Input:** Rough cognitive map $T = (M, X, W)$, Threshold set by the user
2. **Output:** the maximum certainty central area, the maximum possible central area
3. **Step1:** Using breadth-first search traversal method to traversal the R-lower approximation and R-upper approximation cognitive map of graph $T$ respectively.
4. **Step2:** In $R(T)$, calculate the lower approximation class in-degree, lower approximation class out-degree and the total number of equivalence classes in $R(T)$. Using formula (1) calculate the lower approximation relative centrality of all nodes in $R(T)$.
5. **Step3:** In $\overline{R(T)}$, calculate the upper approximation class in-degree, upper approximation class out-degree and the total number of equivalence classes in $\overline{R(T)}$. Using formula (2) calculate the upper approximation relative centrality of all nodes in $\overline{R(T)}$.
6. **Step4:** By the formula (3), calculate the rough centrality $R$ accuracy of all nodes in $R(T)$ and $\overline{R(T)}$, obtain the maximum rough centrality $R$ accuracy.
7. **Step5:** If rough cognitive map $T = (M, X, W)$ is certainty strong class connected sub graph, let the node that has the maximum rough centrality $R$ accuracy in $R(T)$ be center branching out, determine the scope of expansion by the threshold set by the user, come to the maximum certainty central area.
8. **Step6:** If rough cognitive map $T = (M, X, W)$ is possible strong class connected sub graph, let the node that has the maximum rough centrality $R$ accuracy in $\overline{R(T)}$ be center branching out, determine the scope of expansion by the threshold set by the user, come to the maximum possible central area.

Through the above steps of the algorithm, come to the maximum certainty central area to analysis the causal relations in rough cognitive map; come to the maximum possible central area to forecast the causal relations in rough cognitive map; laid the foundation for the further research of rough cognitive map.

4. Conclusion

This article give another definition of rough cognitive map from the perspective of the arc, introduce the concept of rough centrality $R$ accuracy; define certainty -possible class road, according to the given threshold to determine the rough central area. For the large-scale systems with complex causal connection in practical problems, design an effective rough central area mining algorithm for further analysis and forecast the rough cognitive map, it can analysis and forecast rough cognitive maps accurately.

The next step is to explore the way of study the rough cognitive map model. Research on the rough cognitive maps is still in the exploratory stage, it has not yet formed the corresponding theoretical system and design method. There are still many problems to be research and exploration.

References