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Transverse momentum in semi-inclusive deep inelastic scattering

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Abstract

Within the framework of perturbative quantum chromodynamics we derive the evolution equations for transverse momentum dependent distributions and apply them to the case of semi-inclusive deep inelastic scattering. The evolution equations encode the perturbative component of transverse momentum generated by collinear parton branchings. The current fragmentation is described via transverse momentum dependent parton densities and fragmentation functions. Target fragmentation instead is described via fracture functions. We present, to leading logarithmic accuracy, the corresponding semi-inclusive deep inelastic scattering cross-section, which applies to the entire phase space of the detected hadron. Some phenomenological implications and further developments are briefly outlined.

1. Introduction

The predictivity of perturbative quantum chromodynamics (pQCD) relies upon the factorization of hadronic cross-sections into perturbative process dependent coefficient functions, universal perturbative evolution equations and non-perturbative process independent parton densities [1]. In presence of a hard scale, which justifies the perturbative approach due to asymptotic freedom, the scale dependence of parton densities is predicted through renormalization group equations, i.e., Altarelli–Parisi (AP) evolution equations [2]. Inclusive deep inelastic lepton–hadron scattering (DIS) has been investigated since a long time and in great detail. The evaluations of the splitting functions has been recently performed up to $\mathcal{O}(\alpha_s^3)$ in perturbation theory [3], further constraining non-perturbative dynamics. The evolution of initial state partons in terms of longitudinal and transverse momenta has been also considered within pQCD in Refs. [4–6]. In semi-inclusive deep inelastic scattering (SIDIS) processes, at variance with inclusive DIS, one hadron is detected in the final state, $l + P \to l + h + X$. In this case, on the contrary, an equally accurate theoretical description has not been developed yet. The additional hadronic degrees of freedom require a more detailed description of parton dynamics. Within the usual pQCD-improved parton model approach to SIDIS [7,8], one deals only with current fragmented hadrons. However, another distinct issue enters the perturbative description. Both the struck parton and spectators do evolve according to the hard scale governing the process [9]. As a result also target fragmentation has to be included to describe final state hadrons. Evolution is predictable in terms of new functions dubbed *fracture functions*, whose factorization has been proven in Ref. [10]. An explicit evaluation of the SIDIS cross-section at one loop including fracture functions has been given in Ref. [11].

In the pQCD-improved parton model detected hadrons are expected to have a sizeable transverse momentum $\boldsymbol{P}_{h\perp}$, as a result of perturbative evolution in terms of hard partons emission. We will show that it is possible to reformulate evolution equations in order to include transverse degrees of freedom since such a dependence is fully predictable within pQCD.

The aim of this work is twofold. First, we derive transverse momentum dependent (TMD) evolution equations which enter SIDIS cross-sections in the current fragmentation region. We then extend this treatment to distributions in the target fragmentation region

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via fracture functions. As a result of these generalizations, we finally present the combined SIDIS cross-section which describes the production of one final state hadron on its whole phase space.

2. Transverse evolution equations and kinematics

Evolution equations resum leading logarithmic (LL) contributions due to collinear partons emission. In the time-like case the evolution equations are:

$$Q^2\frac{\partial \mathcal{D}_i^h(z_h,Q^2)}{\partial Q^2}=\frac{\alpha_s(Q^2)}{2\pi}\int_{z_h}^{1}\frac{du}{u}P_{ij}\big(u,\alpha_s\big(Q^2\big)\big)\mathcal{D}_j^h\bigg(\frac{z_h}{u},Q^2\bigg), \tag{1}$$

where the fragmentation function $\mathcal{D}_i^h(z_h,Q^2)$ represents the probability to find, at a scale Q^2, a given hadron h with momentum fraction z_h of its parent parton i. $P(u)$ are the time-like splitting functions which, at least at LL accuracy, can be interpreted as the probabilities to find a parton of type j inside a parton of type i and are expressed as a power series of the strong running coupling $\alpha_s(Q^2)$. Ordinary evolution equations, Eq. (1), contain only longitudinal degrees of freedom of partons inside hadrons although, at each branching, the emitting parton acquires a transverse momentum relative to its initial direction. Transverse momentum dependent (TMD) evolution equations were first derived in Ref. [13] for fragmentation functions in the time-like region:

$$Q^2\frac{\partial \mathcal{D}_i^h(z_h,Q^2,\boldsymbol{p}_\perp)}{\partial Q^2}=\frac{\alpha_s(Q^2)}{2\pi}\int_{z_h}^{1}\frac{du}{u}P_{ij}\big(u,\alpha_s\big(Q^2\big)\big)\frac{d^2\boldsymbol{q}_\perp}{\pi}\delta\big(u(1-u)Q^2-q_\perp^2\big)\mathcal{D}_j^h\bigg(\frac{z_h}{u},Q^2,\boldsymbol{p}_\perp-\frac{z_h}{u}\boldsymbol{q}_\perp\bigg). \tag{2}$$

One-particle distributions $\mathcal{D}_i^h(z_h,Q^2,\boldsymbol{p}_\perp)$ in Eq. (2) give the probability to find, at a given scale Q^2, a hadron h with longitudinal momentum fraction z_h and transverse momentum $\boldsymbol{p}_\perp$ relative to the parent parton i. The $P(u)$ splitting functions are the ordinary splitting functions as in Eq. (1) and flavour indices are understood as in the inclusive case, Eq. (1). Let us discuss the kinematics structure of Eq. (2). Consider an outgoing parton k emerging from a hard collision with virtuality $k^2=Q^2>0$, assigned to have zero transverse momentum ($\boldsymbol{k}_\perp=\boldsymbol{0}_\perp$) and unitary longitudinal momentum fraction, as displayed in Fig. 1. It subsequently branches, with probability $P(u)$, into a couple of partons, q and q', with a fractional momentum u and $1-u$ of k, respectively, and transverse momentum $\boldsymbol{q}_\perp=-\boldsymbol{q}'_\perp$ relative to $\boldsymbol{k}$. The parton q then non-perturbatively hadronizes generating the final hadron h with a fractional momentum z_h and transverse momentum $\boldsymbol{p}_\perp$ and $\tilde{\boldsymbol{p}}_\perp$ relative to $\boldsymbol{k}$ and $\boldsymbol{q}$, respectively. We thus derive the following constraints:

$$\tilde{\boldsymbol{p}}_\perp=\boldsymbol{p}_\perp-\frac{z_h}{u}\boldsymbol{q}_\perp, \tag{3}$$

$$q_\perp^2=u(1-u)Q^2. \tag{4}$$

Eq. (3) takes into account the Lorentz boost of transverse momenta $\tilde{\boldsymbol{p}}_\perp$ from the q-frame to the p-frame. The second equation follows by imposing the virtuality flow of time-like branching. These relations directly enter Eq. (2), respectively, as argument of the distribution $\mathcal{D}_j^h$ and of the invariant-mass conserving δ-function. The unintegrated distributions fulfill the normalization:

$$\int d^2\boldsymbol{p}_\perp\,\mathcal{D}_i^h\big(z_h,Q^2,\boldsymbol{p}_\perp\big)=\mathcal{D}_i^h\big(z_h,Q^2\big), \tag{5}$$

since the boost in Eq. (3) is linear in the transverse variables, i.e., the Jacobian is:

$$\det\bigg(\frac{d^2\boldsymbol{p}_\perp}{d^2\tilde{\boldsymbol{p}}_\perp}\bigg)=1. \tag{6}$$

This property guarantees that we can recover inclusive distributions, Eq. (1), starting from less inclusive ones. The opposite statement is not valid since Eq. (2) contains new physical information.

In order to obtain a complete description of semi-inclusive cross-sections we need the space-like version of Eq. (2). On a general ground we may expect that the infrared structure of space-like evolution equations is the same as the time-like one since it depends

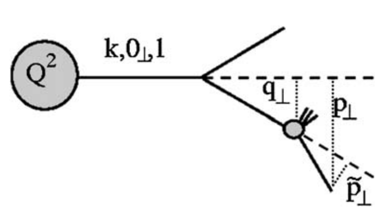

Fig. 1. A parton with momentum k emerges from a hard process with virtual mass $k^2=Q^2>0$ and then evolves into a quasi-real parton with transverse momentum $\boldsymbol{p}_\perp$. The vertex is associated with the time-like splitting functions $P(u)$. The small blob symbolizes resummation of ladder diagrams in LL accuracy.

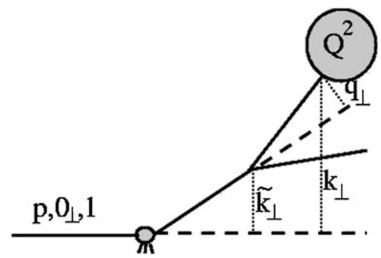

Fig. 2. A quasi-real parton with momentum p perturbatively evolves towards the hard vertex acquiring a virtual mass $p^2 = Q^2 < 0$. The vertex is associated with the space-like splitting functions $P(u)$. The small blob symbolizes resummation of ladder diagrams in LL accuracy.

only upon the dynamics of the underlying gauge theory, the only changes being in the kinematics. In analogy to the time-like case we consider now a initial state parton p in a incoming proton P which undergoes a hard collision, the reference frame being aligned along the incoming proton axis, as in Fig. 2. The boost of transverse momentum and the invariant mass-conserving constraint are in this case:

$$\tilde{\boldsymbol{k}}_\perp = \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}, \tag{7}$$

$$q_\perp^2 = (1-u)Q^2. \tag{8}$$

We thus generalize Eq. (2) to the space-like case:

$$Q^2 \frac{\partial \mathcal{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^{1} \frac{du}{u^3} P_j^i\big(u, \alpha_s\big(Q^2\big)\big) \frac{d^2\boldsymbol{q}_\perp}{\pi} \delta\big((1-u)Q^2 - q_\perp^2\big) \mathcal{F}_P^j\left(\frac{x_B}{u}, Q^2, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}\right). \tag{9}$$

One-particle distributions $\mathcal{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)$ in Eq. (9) give the probability to find, at a given scale Q^2, a parton i with longitudinal momentum fraction x_B and transverse momentum $\boldsymbol{k}_\perp$ relative to the parent hadron P. The *constructive* proof of Eq. (9) will be given elsewhere [27]. The unintegrated distributions fulfill a condition analogous to the one in Eq. (5), i.e.:

$$\int d^2\boldsymbol{k}_\perp \mathcal{F}_P^i\big(x_B, Q^2, \boldsymbol{k}_\perp\big) = \mathcal{F}_P^i\big(x_B, Q^2\big). \tag{10}$$

We note that the inclusion of transverse momentum does not affect longitudinal degrees of freedom since partons always degrade their fractional momenta in the perturbative branching processes. The different Lorentz structure in the transverse arguments of the parton distribution functions $\mathcal{F}_P^i$ arises from the different structure of the Bethe–Salpeter ladder used to derive the evolution equations [27].

3. Transverse momenta in target fragmentation region

Current and target hadron production mechanisms cannot be separated since hadrons produced by current fragmentation may go in to the target remnant direction and vice versa. In these configurations new infrared singularities appear which cannot be reabsorbed through the standard renormalization procedure into parton distribution functions and fragmentation functions. It has been shown [9] that the cross-section can be renormalized by introducing new non-perturbative factorized [10,11] objects, i.e., fracture functions usually indicated by $\mathcal{M}^i_{P,h}(x, z, Q^2)$. These functions express the conditional probability of finding, at a scale Q^2, a parton i with momentum fraction x in a proton P while a hadron h with momentum fraction z is detected. Fracture functions obey non-homogeneous evolution equations [9]:

$$\begin{aligned} Q^2 &\frac{\partial \mathcal{M}^i_{P,h}(x, z, Q^2)}{\partial Q^2} \\ &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^{1} \frac{du}{u} P_j^i(u) \mathcal{M}^j_{P,h}\left(\frac{x}{u}, z, Q^2\right) + \frac{\alpha_s(Q^2)}{2\pi} \int_{x}^{\frac{x}{x+z}} \frac{du}{x(1-u)} \hat{P}_j^{i,l}(u) \mathcal{F}_P^j\left(\frac{x}{u}, Q^2\right) \mathcal{D}_l^h\left(\frac{zu}{x(1-u)}, Q^2\right). \end{aligned} \tag{11}$$

The first term in the above equation (see Fig. 3(a)) describes the evolution of the active parton j while the hadron h is detected. The second term (see Fig. 3(b)) takes into account the production of a hadron h by a time-like cascade initiated by the active parton j. The $\hat{P}_j^{i,l}(u)$ represent the unsubtracted Altarelli–Parisi splitting functions [14]. Since perturbative evolution is at work even in target fragmentation region, we expect that a non-negligible amount of transverse momentum is produced there. We thus generalize these distributions to contain also transverse degrees of freedom. The fracture functions $\mathcal{M}^i_{P,h}(x, \boldsymbol{k}_\perp, z, \boldsymbol{p}_\perp, Q^2)$ give the conditional

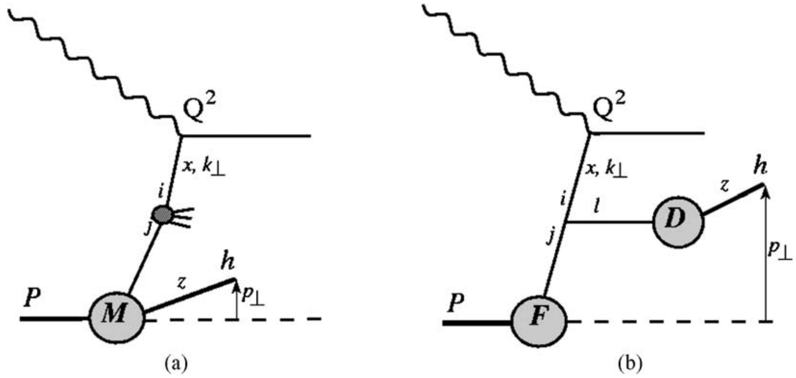

Fig. 3. Evolution of fracture functions M: (a) homogeneous term; (b) inhomogeneous one.

probability to find in a proton P, at a scale Q^2, a parton with momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$ while a hadron h, with momentum fraction z and transverse momentum $\boldsymbol{p}_\perp$, is detected. Under these assumptions the following evolution equations can thus be derived [27]:

$$
\begin{aligned}
&Q^2\frac{\partial\mathcal{M}^i_{P,h}(x,\boldsymbol{k}_\perp,z,\boldsymbol{p}_\perp,Q^2)}{\partial Q^2}\\
&\quad=\frac{\alpha_s(Q^2)}{2\pi}\left\{\int\limits_{\frac{x}{1-z}}^{1}\frac{du}{u^3}P^i_j(u)\int\frac{d^2\boldsymbol{q}_\perp}{\pi}\delta\big((1-u)Q^2-q_\perp^2\big)\mathcal{M}^j_{P,h}\left(Q^2,\frac{x}{u},\frac{\boldsymbol{k}_\perp-\boldsymbol{q}_\perp}{u},z,\boldsymbol{p}_\perp\right)\right.\\
&\qquad\left.+\int\limits_{x}^{\frac{x}{x+z}}\frac{du}{x(1-u)u^2}\hat{P}^{i,l}_j(u)\frac{d^2\boldsymbol{q}_\perp}{\pi}\delta\big((1-u)Q^2-q_\perp^2\big)\mathcal{F}^j_P\left(\frac{x}{u},\frac{\boldsymbol{k}_\perp-\boldsymbol{q}_\perp}{u},Q^2\right)\mathcal{D}^h_l\left(\frac{zu}{x(1-u)},\boldsymbol{p}_\perp-\frac{zu}{x(1-u)}\boldsymbol{q}_\perp,Q^2\right)\right\}.
\end{aligned}
\tag{12}
$$

As in the longitudinal case two terms contribute to the evolution of TMD fracture functions as displayed in Fig. 3. The homogeneous one has a pure non-perturbative nature since involves the fragmentation of the proton remnants into the hadron h. The inhomogeneous one takes into account the production of the hadron h from a time-like cascade of parton j and thus is dubbed *perturbative*. Fracture functions fulfill the normalization condition:

$$
\int d^2\boldsymbol{k}_\perp\int d^2\boldsymbol{p}_\perp\,\mathcal{M}^i_{P,h}\big(x,\boldsymbol{k}_\perp,z,\boldsymbol{p}_\perp,Q^2\big)=\mathcal{M}^i_{P,h}\big(x,z,Q^2\big),
\tag{13}
$$

as direct consequence of the kinematics of both terms in the evolution equations, Eq. (12). The proof of factorization, i.e. that all singularities occurring in the target remnant direction can be properly renormalized by the less inclusive quantity $\mathcal{M}^i_{P,h}(x,\boldsymbol{k}_\perp,z,\boldsymbol{p}_\perp,Q^2)$, is lacking at present. In the following we assume such a factorization to hold.

4. Solutions

Evolution equations (2) and (9) can only be approximately diagonalized by the joint Fourier–Mellin transform

$$
\mathcal{D}_n\big(\boldsymbol{b},Q^2\big)=\int d^2\boldsymbol{p}_\perp\int\limits_0^1 \mathrm{e}^{\frac{-i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}{z}}\,dz\,z^n\mathcal{D}\big(z,Q^2,\boldsymbol{p}_\perp\big),
\tag{14}
$$

where $\boldsymbol{b}$ is the transverse momentum Fourier-conjugated variable [12]. An exact diagonalization is prevented by the kinematics structure of the distribution under integral in Eq. (2) since it combines longitudinal momentum fractions with transverse momenta. Such an exact diagonalization can be, however, obtained in the soft limit, i.e., when the variables x or z approach the edge of phase space. In this case for the non-singlet time-like unintegrated fragmentation functions the solution reads [13,17]:

$$
\mathcal{D}\big(z,Q^2,\boldsymbol{p}_\perp\big)=\mathcal{D}\big(z,Q^2\big)\mathcal{G}\big(Q_0^2,Q^2,z,\boldsymbol{p}_\perp\big),
\tag{15}
$$

where the scale Q_0^2 sets the upper limit of the non-perturbative regime. We recall that in this limit the convergence of the perturbative series can be further improved by taking into account soft gluon radiation enhancements. The form factor $\mathcal{G}$ can therefore be computed to leading logarithmic accuracy [12,15,16] by simply demanding that $\alpha_s(Q^2)\to\alpha_s(\boldsymbol{p}_\perp^2)$. The expression to next-to-leading logarithmic accuracy has been given in Ref. [17]. These issues have been recently specialized to the case of SIDIS processes

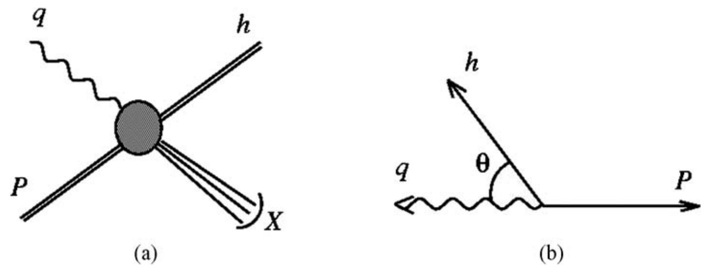

Fig. 4. (a) Semi-inclusive DIS $l + P \to l + h + X$ process; (b) kinematics of the reaction in the photon–proton center of mass frame.

in the current fragmentation region in Ref. [21]. Away from the soft limit, the factorized structure of the solution, Eq. (15), is not automatically preserved. In this case numerical methods have shown to be useful in order to solve the equations in the $x, z \in \mathcal{O}(1)$ range [19,27] and to extend the solutions to be valid in the flavour mixing sector. As in the longitudinal case, distributions at a scale $Q^2 > Q_0^2$ are known if we provide a non-perturbative input density at some arbitrary scale Q_0^2. We assume as initial condition the usual longitudinal fragmentation distribution times a z-independent, flavour-independent factor:

$$\mathcal{D}_i^h\big(z, Q_0^2, \boldsymbol{p}_\perp\big) = \mathcal{D}_i^h\big(z, Q_0^2\big)\frac{\mathrm{e}^{-\boldsymbol{p}_\perp^2/\langle \boldsymbol{p}_\perp^2\rangle}}{\pi\langle \boldsymbol{p}_\perp^2\rangle}. \tag{16}$$

The Gaussian $\boldsymbol{p}_\perp$-distribution is used to model partons intrinsic momenta inside hadrons [22]. This issue has also been considered in Refs. [23,24]. We now discuss the phenomenological implications of these results. According to Ref. [20], single hadron cross-sections are usually parametrized in terms of four independent structure functions $H_{i=1,\dots,4}(x_B, z_h, Q^2, \boldsymbol{P}_{h\perp})$. The hadronic invariants of SIDIS processes are constructed by using external particles four-momenta, as displayed in Fig. 4(a), and therefore the structure functions H_i are supposed to describe both current and target hadron fragmentation mechanisms.

Once evolution equations are solved, theoretical predictions can be compared with semi-inclusive DIS data, as for the longitudinal case, provided that a factorization theorem holds even for transverse momentum dependent distributions. Such a theorem has been shown to hold in the current fragmentation region for the structure function H_2 in Ref. [18]:

$$H_2\big(x_B, z_h, \boldsymbol{P}_{h\perp}, Q^2\big) = \sum_{i=q,\bar{q}} e_q^2 \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp \delta^{(2)}(z_h\boldsymbol{k}_\perp + \boldsymbol{p}_\perp - \boldsymbol{P}_{h\perp})\mathcal{F}_P^i\big(x_B, \mu_F^2, \boldsymbol{k}_\perp\big)\mathcal{D}_i^h\big(z_h, \mu_D^2, \boldsymbol{p}_\perp\big)C\big(Q^2, \mu_F^2, \mu_D^2\big), \tag{17}$$

where the standard SIDIS variables are defined as:

$$z_h = \frac{P\cdot P_h}{P\cdot q}, \qquad x_B = \frac{Q^2}{2P\cdot q}, \tag{18}$$

and μ_F^2 and μ_D^2 are the factorization scales. The above results are accurate up to powers in $(P_{h\perp}^2/Q^2)^n$ for transverse momenta $|P_{h\perp}| \simeq \Lambda_{\mathrm{QCD}}$. Evolution equations for $\mathcal{F}$ and $\mathcal{D}$ are given in Eqs. (2) and (9). The factor C is the process-dependent hard coefficient function computable in perturbative QCD and to LL accuracy we can set $C = 1$. Provided that factorization holds for the TMD fracture functions, we may add, according to Eq. (12), their contributions to H_2:

$$\begin{aligned} H_2\big(x_B, z_h, \boldsymbol{P}_{h\perp}, Q^2\big) = \sum_{i=q,\bar{q}} e_q^2 \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp \big\{&\delta^2(z_h\boldsymbol{k}_\perp + \boldsymbol{p}_\perp - \boldsymbol{P}_{h\perp})\mathcal{F}_P^i\big(x_B, Q^2, \boldsymbol{k}_\perp\big)\mathcal{D}_i^h\big(z_h, Q^2, \boldsymbol{p}_\perp\big)A(0) \\ &+ (1 - x_B)\mathcal{M}_{P,h}^i\big(x_B, \boldsymbol{k}_\perp, z, \boldsymbol{p}_\perp, Q^2\big)\delta^2(\boldsymbol{p}_\perp - \boldsymbol{P}_{h\perp})A(1)\big\}, \end{aligned} \tag{19}$$

where we have identified all the three factorization scales with the hard scale, $Q^2 = \mu_F^2 = \mu_D^2 = \mu_M^2$. Although formally the two contributions are simply added in Eq. (19), at LL accuracy and in photon–proton center of mass frame, the produced hadrons are mainly distributed in two opposite hemispheres. Target fragmented hadrons are produced mainly in the $\theta = \pi$ direction while current fragmented hadrons mainly along $\theta = 0$ direction. Here θ is the angle of the produced hadron h with respect to the photon direction, as shown in Fig. 4(b). In order to keep track of the emission angle of the detected hadron h, we supplement current and target fragmentation terms in Eq. (19) with an angular distribution $A(v)$ [11]. The angular and energy variables v and z are defined as:

$$z = \frac{E_h}{E_p(1 - x_B)}, \qquad v = \frac{1 - \cos\theta}{2}, \qquad z_h = zv. \tag{20}$$

In Eq. (20), E_h and E_p denote, respectively, the energies of the detected hadron and of the incoming proton in the photon–proton center of mass frame. The variables z and v are a useful frame-dependent representation for the hadronic invariant z_h in two respects: z reduces to z_h in the current fragmentation region so that we recover the standard definitions, while for low z_h-values we can distinguish soft hadrons ($z \to 0$) from the ones produced in the target remnant direction ($\theta \to \pi$). Since to LL accuracy all sources of transverse momenta contributing to $\boldsymbol{P}_{h\perp}$ have been taken into account we may represent Eq. (19) as in Fig. 5.

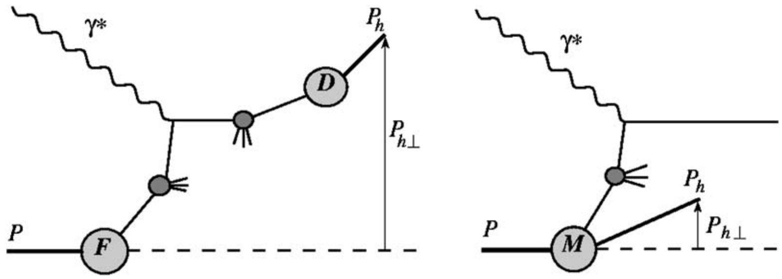

Fig. 5. Sources of transverse momentum in the current (left) and in the target (right) fragmentation region in SIDIS. Dark blobs symbolize hard partons emission. Transverse momentum $\boldsymbol{P}_{h\perp}$ of the detected hadron h is also indicated. F, D and M represent parton distribution, fragmentation and fracture functions, respectively.

5. Conclusions and perspectives

In this work we have extended ordinary distributions to include transverse degrees of freedom both in the current and in the target fragmentation region of semi-inclusive DIS via fracture functions. As long as a factorization theorem holds for transverse momentum dependent fracture functions, the semi-inclusive cross-sections are thus predictable on the whole phase space of the detected hadrons. Although this extension may have its own theoretical relevance, on the phenomenological side it also improves our knowledge of both the perturbative and non-perturbative parton dynamics. Evolution equations (2), (9) and (12) can be straightforwardly extended also to the polarized case. In the target fragmentation region the evolution equations for longitudinal polarized fracture functions have been derived in Ref. [25]. We stress that the accuracy of Eqs. (2), (9) and (12) is set by the accuracy of the Altarelli–Parisi splitting functions. Splitting functions for partons and fragmentation functions are well known. They have been recently calculated in the target region at two loop level in Ref. [26]. Therefore, the evolution equations in the current and in the target fragmentation region can be set to allow the analysis of the semi-inclusive cross-sections to the same level of accuracy. Detailed derivation of the results presented in this work and of the phenomenological implications are postponed to a forthcoming work [27].

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