Remarks on global anomalies in RCFT orientifolds

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Abstract

We check the list of supersymmetric standard model orientifold spectra of Dijkstra, Huiszoon and Schellekens for the presence of global anomalies, using probe branes. Absence of global anomalies is found to impose strong constraints, but in nearly all cases they are automatically satisfied by the solutions to the tadpole cancellation conditions.
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1. Introduction

In previous papers [1,2] coauthored by one of us a large number of supersymmetric open string spectra was found with a chiral spectrum that exactly matches the standard model spectrum. These models were constructed using orientifolds of tensor products of $N = 2$ minimal models. The standard model gauge groups arise due to Chan–Paton multiplicities of boundary states of the underlying rational conformal field theory.

In contrast to the majority of published work on orientifold model building (see, e.g., [3] and references therein), the construction of [2] is algebraic and not geometric. It is based on rational conformal field theory (RCFT) on surfaces with boundaries and crosscaps. The basic RCFT building blocks and the way they are put together are subject to a set of constraints which are the result of many years of work by several groups.

The constraints can be divided into world-sheet and space–time conditions. The boundary states themselves must satisfy the "sewing constraints" [4–8]. There are further constraints on the crosscap states needed to define non-orientable surfaces [9, 10]. These are all worldsheet conditions needed to guarantee the correct factorization of all amplitudes. In addition, some

space–time conditions must be imposed, the tadpole cancellation conditions. They are needed to make sure that tree-level one-point functions of closed string states on the crosscap cancel those of the disk. If these tadpoles are left uncancelled, this will manifest itself in the form of infinities in sum of the Klein bottle, annulus and Möbius diagrams.

If the tadpoles correspond to physical states in the projected closed string spectrum, these infinities merely signal that the corresponding string theory is unstable and might be stabilized by shifting the vacuum expectation value of the corresponding field. However, if the tadpoles do not correspond to physical states their presence implies a fundamental inconsistency in the theory, which may manifest itself in the form of chiral anomalies in local gauge or gravitational symmetries.

There is no proof that the aforementioned set of conditions is sufficient to guarantee consistency of the resulting unoriented, open string theories. It was shown in [11] that for the simple current boundary states derived in [12] and that where used in [2] all sewing constraints are satisfied in the oriented case. To our knowledge, however, there is still no complete proof in the unoriented case, although important progress was made in [13]. Nevertheless, the boundary and crosscap states used in the construction are based on generic simple current modifications of the Cardy boundary states [14] and the Rome crosscap formula [15]. They have been successfully compared with geometric constructions, for example, the circle and its orbifold

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[16] and WZW models. In addition, they can be shown to yield integral partition functions in all cases, a highly non-trivial requirement [17].

With regard to space–time consistency there is a more concrete reason to worry. In other constructions of orientifold models it was observed that in certain cases gauge groups with global anomalies can occur, even though all tadpole conditions are satisfied [18–20]. By "global anomalies" we mean here anomalies in the global definition of field theory path integral, as first described in [21]. The symptom for such an anomaly is an odd number of massless fermions in the vector representation of a symplectic factor of the gauge group (including in particular doublets of $SU(2)$).

Examples of orientifold spectra having such a problem were found in geometric settings, where the problem can be traced back to uncancelled K-theory charges of branes and O-planes. It is known that D-branes are not characterized by (co)homology but by K-theory [22,23]. Tadpole cancellation guarantees, in particular, the cancellation of cohomology charges of branes, which are characterized by long range RR fields coupling to these charges. This cancellation is physically necessary for branes and O-planes that fill all non-compact dimensions, since the field of an uncancelled charge cannot escape to infinity. However, the branes may carry additional Z_2-charges without a corresponding long range field. Tadpole cancellation does not imply the cancellation of these charges. If they remain uncancelled, this may manifest itself in the form of global anomalies.

This implies that also in algebraic constructions one has to be prepared for the possibility of additional constraints. Unfortunately, a complete description of global anomalies in theories of unoriented open strings does not seem to be available at present. Therefore, the best we can do is to examine if the symptoms of the problem are present.

This check was not done systematically for the results presented in [2]. However, since all gauge groups and representations were stored, we have been able to do an a posteriori check. This leads to the following results. The total number of complete spectra at our disposal is 270 058.[1] Of the 270 058 models, only 1015 turn out to have one or more globally anomalous symplectic factors. Interestingly, on average the anomalous models have more than one anomalous symplectic factor: there are 2075 anomalous symplectic factors out of a total of 845 513.

The gauge groups of these models usually (but not always) have "hidden sectors" in addition to the standard model gauge group $SU(3) \times SU(2) \times U(1)$. Since the standard model itself is free of global anomalies, the origin of the anomaly is always related to the hidden sector, but this may happen in two ways. First of all, a symplectic factor within the hidden sector may be anomalous. However, the hidden sector can also cause an $SU(2)$ factor (the weak gauge group or, in a subclass of models, an additional $Sp(2)$ factor) of the standard model to be anomalous.

Since we require any open string stretching between the standard model and hidden branes to be non-chiral, this can only happen if an open string has one end on the standard model $SU(2)$ and the other end on a brane with an $O(N)$ Chan–Paton group, with N odd (if the other end is on a symplectic brane the ground state dimension is automatically even, and when it ends on a complex brane the ground state must be a non-chiral pair, again yielding an even multiplicity). This does indeed occur in a few of the aforementioned anomalous cases.

Even if the massless spectrum does not exhibit this problem, this still does not guarantee that the corresponding string theory is globally consistent. Indeed, if one starts with string theory with a globally anomalous $Sp(2)$ factor, moving the two symplectic branes away from the orientifold plane produces a $U(1)$ theory which presumably is also globally inconsistent, but which does not exhibit the problem in its field theory limit. We assume here and below that continuously moving branes cannot introduce or remove such an inconsistency.

A more powerful constraint was suggested in [18]. In addition to the CP-factors or branes present in a given model, one may introduce "probe-branes". The idea is to add a brane–antibrane pair to a given brane configuration, which we assume to have a field theory limit without global anomalies (and that is tadpole-free, and hence has also no local anomalies). The reason for adding such pair rather than a single brane is that the brane and antibrane cancel each others cohomological brane charges, and hence one does not introduce couplings to long range RR fields. This implies that the result is at least free of local chiral anomalies. If that were not the case, a discussion of global anomalies would not make much sense. The resulting configuration is not free of all tadpoles (dilaton tadpoles will not cancel, for instance), and, in particular, is neither supersymmetric nor stable, but that should not affect the consistency.

The CP gauge group of the new configuration can now be checked for global anomalies. Since the probes are added in pairs, they cannot introduce new global field theory anomalies in the existing configuration, but if the CP groups of the probe-brane pair are symplectic, one may find that the latter gauge groups have a global anomaly (i.e., an odd number of vectors). In that case one should conclude that the original theory was inconsistent as well.

It is not clear that this constraint captures all possible global string anomalies. In the context of our RCFT construction, we have for every choice of $N = 2$ tensor product and modular-invariant partition function a definite number of distinct boundary states at our disposal. For a given orientifold choice, a certain subset of those boundaries will have symplectic CP-factors. Each of them can (and will) be used, with its antibrane, as a probe brane pair. If the algebraic model is viewed from a geometric point of view, perhaps additional branes can be considered that do not have an algebraic description, and that would lead to additional constraints if used as probes. In all cases studied in the literature, the branes not present in the algebraic description simply correspond to rational branes continuously moved to non-rational positions. If this is also true for the more complicated cases considered here, this would not yield anything new. The fact that the set of RCFT boundaries is

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[1] This is larger than the number of spectra mentioned in [2] because the latter were obtained after identifying spectra modulo hidden sector details. In other words, some of the 270 058 stored spectra differ only in the hidden sector.

algebraically complete [7] may imply that they also provide a complete set of probe brane constraints.

The boundary states considered so far all correspond geometrically to space–time filling branes. One may also use probe branes that are not space–time filling, and indeed the corresponding constraints are important to understand the relation between tadpole cancellation and cancellation of local anomalies [18]. For example, in non-chiral theories there may be unphysical tadpoles, but they cannot manifest themselves as chiral anomalies. Instead, they will then appear as local anomalies of gauge theories on lower-dimensional branes. However, we expect lower-dimensional branes to be irrelevant for global anomalies, because there are no global anomalies in 1,2 and 3 dimensions. Nevertheless, clearly a more fundamental discussion of these additional constraints in RCFT constructions is needed, presumably involving the appropriate generalization of K-theory charges to boundary and crosscap states.

In any case, the probe branes described above imply, in general, a very large number of additional constraints. Typically, the models we consider have a few thousand boundary states, and a few hundred of them yield a symplectic gauge group. In principle, any such gauge group imposes a mod-2 condition on the spectrum, and hence each might reduce the number of solutions by a factor of 2. Purely statistically speaking, this could reduce the number of solutions by several orders of magnitude. The aforementioned discussion of manifest global anomalies suggests already that the result will be less dramatic.

The probe-brane constraint cannot be checked as easily as the manifest global anomalies, because the probe brane CP-factors are not listed in our database unless they happen to be part of the CP gauge group themselves. The only way to check it is to generate the models again, and recompute the spectrum in the presence of any probe branes that might occur. Even if a previously recorded spectrum has a global inconsistency, it might still be possible to make a different choice for the hidden sector gauge group and cancel it. However, rather surprisingly this was rarely necessary. For the vast majority of MIPFs we did not encounter any global probe brane anomalies for any solution, even though the number of potential anomalies was often very large. In [2] tadpole solutions were obtained for 333 modular-invariant partition functions. We have encountered global anomalies for only 25 of those (and then only for very few solutions in each case). These MIPFs are listed in the table. We present this list because we hope that the presence of global anomalies in these cases makes sense from another point of view, for example, Calabi–Yau geometry.

It is not possible to state exactly which fraction of the 270 058 stored spectra fails the probe-brane conditions, because we do not have sufficient information in some older cases to regenerate them exactly. Instead, we simply searched the corresponding MIPFs again with the probe brane condition imposed as an additional constraint. In nearly all previous cases a new, global anomaly-free solution turns out to exist. To get an idea of the effect of the probe brane constraint on the original database, consider the MIPF that contributed the largest number of solutions, the one listed in the table for tensor product (1, 6, 46, 46). For this MIPF we had 19 644 full tadpole solutions stored, in-

Table 1
Tensor products and MIPFs for which non-trivial global anomalies affecting previous spectra were found. The first column specifies the tensor product, the second the Hodge numbers of the corresponding Calabi–Yau manifold and the number of singlets it yields in a heterotic string compactification, the third column gives the number of boundaries, and the last a sequence number assigned by the programme kac [25] used to compute the spectra

Tensor	(h_{21}, h_{11}, S)	Boundaries	No.
(1, 6, 46, 46)	(9, 129, 525)	1484	10
(1, 10, 22, 22)	(7, 55, 263)	1148	19
	(20, 32, 237)	1632	27
(2, 4, 14, 46)	(25, 37, 287)	1152	8
	(28, 40, 309)	1440	10
(2, 4, 16, 34)	(26, 62, 339)	1232	17
(2, 4, 22, 22)	(10, 82, 361)	864	42
	(13, 85, 367)	1080	22
	(10, 58, 309)	864	11
	(13, 61, 335)	1080	13
	(21, 69, 344)	1728	16
	(20, 32, 261)	1668	17
(2, 6, 8, 38)	(28, 52, 331)	1200	16
	(22, 34, 265)	720	25
(2, 6, 14, 14)	(9, 57, 273)	768	60
	(10, 58, 271)	768	22
	(9, 33, 233)	768	21
	(10, 34, 251)	768	62
(2, 10, 10, 10)	(9, 45, 243)	832	53
	(13, 49, 251)	1120	18
	(15, 51, 271)	1312	16
	(19, 31, 231)	1664	59
	(19, 31, 235)	1120	24
(4, 4, 6, 22)	(13, 61, 289)	330	12
	(9, 33, 211)	438	8
	(18, 30, 221)	402	34

cluding the precise boundary labels needed to regenerate them (16 243 of these 19 644 solutions are distinct if hidden sector details are ignored). Only 59 of the 19 644 violated the global anomaly conditions, and in 8 of those 59 cases a new solution was found that is free of global anomalies.

This is a very surprising result in view of the large number of constraints implied by the probe brane procedure. In general, for every boundary label b with a symplectic CP group, one obtains a constraint of the form

$$\sum_{i,a} N_a A^i_{ab}(\chi_i)_{0,\mathrm{L}} = 0 \bmod 2, \qquad (1)$$

where A^i_{ab} are the annulus coefficients and χ_i is the Virasoro character of representation i restricted to massless characters of definite (in our case left, L) space–time chirality. This imposes as many mod-2 constraints on the CP-multiplicities N_a as there are symplectic boundaries.

This condition is similar but not identical to the chiral anomaly constraint derived from tadpole cancellation

$$\sum_{i,a} N_a w_i \left(A^i_{ab} + 4M^i_b\right) = 0, \qquad (2)$$

where M^i_b are the Möbius coefficients and w_i is the Witten index, $(\chi_i)_{0,\mathrm{L}} - (\chi_i)_{0,\mathrm{R}}$. Here b can be any boundary, not just those that appear in a given solution with non-vanishing Chan–Paton multiplicity. This implies the absence of local gauge

anomalies associated with probe branes. The index b can represent any probe brane. The antibranes are not represented by any label in this set because they do not satisfy the BPS condition for the given choice of unbroken supersymmetry. However, we do know their anomaly contribution. Consider a $U(N)$ factor in the configuration of interest, and a probe brane pair contributing CP factors $U(M)_1 \times U(M)_2$ (consisting thus of four branes: a brane b, its conjugate, b^c and their antibranes). Strings stretching between $U(M)$ may produce massless chiral states $(N, M, 1)$, but then there is necessarily also a state $(N^*, 1, M^*)$ from the antibrane (the notation $(*, *, *)$ refers to $U(N) \times U(M) \times U(M)_{\text{anti}}$). This cancels the $U(N)$ anomalies. This cancellation is simply a consequence of introducing brane–antibrane pairs. The $U(M)$ anomalies also cancel, but for a different reason. Formula (2) implies that the tadpole multiplicities N_a are such that not only the anomalies within the original configuration cancel, but also for the CP group associated with any brane b that is added to it. This is the local analog of the global anomaly probe brane constraint, and evidently it is automatically satisfied if the tadpoles cancel. Note that this works in a slightly more complicated way if A_{bb}^i and/or M_b^i is non-zero. (Anti)-symmetric tensors contribute anomalies $M \pm 4$. The term proportional to M is cancelled by strings in the representation $(1, M^*, M^*)$ stretching between the brane and the antibrane (which necessarily exist if $A_{bb}^i \neq 0$), whereas the term proportional to 4 cancels against contributions from the probed configuration, as a consequence of (2).

Although Eqs. (1) and (2) look similar, they are not related in any obvious way. Eq. (2) can be rewritten entirely in terms of left-handed fermions, but the set of labels b for the two conditions is disjoint. Therefore, both seem to give a priori independent set of constraints on the Chan–Paton multiplicities N_a. In principle, this gives one mod-2 constraint for every symplectic boundary label b. The total number of constraints is reduced by the following considerations:

- If a is itself symplectic, N_a is even, and hence there is no mod-2 constraint on N_a.
- If a is complex, $N_a = N_{a^c}$, which reduces the number of independent variables.
- There may be linear dependencies among the constraints.
- Since the local anomaly conditions (2) are satisfied, so is their mod-2 reduction. Some of the global anomaly conditions may be already contained in mod-2 reduced local anomaly conditions.
- We may derive additional mod-2 constraints from the tadpole conditions that do not produce local anomaly cancellation conditions. This requires rewriting the remaining tadpole conditions in terms of integers, an operation for which no canonical algorithm is known to us, while in the previous case the anomaly takes care of that. However, all coefficients turned out to be integers automatically in all cases we considered, after reducing the tadpole equations to an independent set.

Even after taking all this into account, often there still are mod-2 conditions left over, and sometimes a substantial number of

them. In some of the simpler cases, for example, the tensor product $(1, 1, 1, 1, 7, 16)$, there are no global anomaly constraints at all, because there are no symplectic factors. The next degree of complication occurs, for example, for the tensor product $(1, 4, 4, 4, 4)$. It has a total of 528 MIPF/orientifold choices, with up to 65 independent probe brane constraints. Nevertheless, in 504 cases these are all already contained in the local anomaly conditions, and in the remaining 22 there is just one mod-2 constraint left over. Roughly speaking, the number of left-over mod-2 conditions increases as the tensor product contains larger tensor factors and has more primaries. At the other extreme we have the aforementioned MIPF of $(1, 6, 46, 46)$, which has 24 tadpole conditions, 10 of which independent from each other. From the local anomaly conditions reduced modulo 2 we get just 2 constraints. Adding the remaining tadpole conditions we get 10 mod-2 constraints. The symplectic factors yield 155 independent mod-2 constraints, which combined with the ones from the tadpoles leads to a total of 157 mod-2 constraints, and hence just 10 of these are automatically satisfied by any solution that was previously found. Therefore, the existing set of solutions has to be checked for 147 mod-2 conditions, which could potentially reduce the number of solutions enormously. It is very surprising that 99.7% of the solutions survive all these constraints, as discussed above.

During the reanalysis we have used a somewhat improved method for solving the tadpole conditions, which has allowed us to push the limits a bit further and solve them in a few more cases that were previously intractable. As a result we now have more solutions than before, namely, 210 782, distinguished in the same way as in [2]. All massless spectra of this set of solutions can be searched and examined via a webpage [24].

The probe branes provide a way to define for each boundary a set of Z_2-charges. A priori there can be as many charges as there are symplectic factors, but usually these are not independent. These charges may be expected to correspond to the K-theory charges of the corresponding D-branes in a geometric setting. We have attempted to make sense of these charges and tried to relate them directly to quantum numbers of the boundary states. Unfortunately, we had little success in this enterprise except for some cases where a clear relation was found between the q quantum numbers of the boundaries[2] and the sum of the K-theory charges of the configuration. Just as an example, for the tensor product $(1, 1, 2, 2, 4, 4)$ with 110 modular invariants, the relation holds for several orientifolds corresponding to 22 of these invariants (between one and four orientifolds for each invariant).

This Letter leaves unanswered the important issue of a derivation, from first principles, of the global anomaly conditions that must be satisfied by orientifold constructions. Just as in other approaches, Uranga's probe brane procedure seems to be the only method at our disposal. This is unsatisfactory and needs to be addressed in the future, but for now the main message is that the set of solutions is barely affected by these seemingly powerful constraints.

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[2] We use the standard notation (l, q, s) for the quantum numbers of the $N = 2$ minimal models.

Acknowledgements

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