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Physics Procedia 15 (2011) 59–63

Physics Procedia

Transition Matrices and Time Travel

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It has been proven by Lee [1] that the grandfather paradox and Deutsch's unproven paradox are precluded for two- and three-state graphical models. We prove that both paradoxes are also precluded for a general n-state model. In addition, we present a new time travel paradox in this paper.

Keywords: Time Travel, Transition Matrices, Graphical Model 04.20.Cv

The idea of traveling back in time to change the past is probably one of the oldest scientific issues that have given rise to much imagination. There are numerous publications presenting the possibilities [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] and impossibilities [16, 17, 18, 19] of building a time machine. There is also a school of thought that hypothesize that time travel is impossible based on reasoning that lead to time travel paradoxes [20]. Two popular time travel paradoxes are the grandfather paradox and Deutsch's unproven theorem paradox [2]. The grandfather paradox states that a time traveler travels back in time to kill his grandfather before his father was conceived. This leads to a paradox because the time traveler would not exist at present and hence could not travel back in time to kill his grandfather. The Deutsch's unproven theorem paradox states that a time traveler travels back in time to present the proof of a mathematical theorem so that it can be recorded in a document in which the time traveler reads in the future. One way to avoid such paradoxes is to forbid time travel. However, time travel could still be possible while avoiding the above mentioned paradoxes if we impose additional conditions that events happening in the time line must be self-consistent. This self-consistency principle has been proposed by several authors [1, 2, 3, 21].

Recently, Lee [1] has shown for the first time that the grandfather paradox and Deutsch's unproven theorem paradox are related to the basic axioms of probabilities. Imposing the basic axioms of probabilities, such as normalization and semi-positive definiteness, precludes both the grandfather paradox and Deutsch's unproven paradox. The mathematical derivation to preclude these paradoxes was done on two- and three-state discrete time graphical models [22, 23, 24, 25, 26]. This model resembles a Markov Chain with a single backward loop in time. Fig. 1 illustrates such a model where σ_j, $j = 1, \cdots n$ represents the state sampled at time j.

$$\sigma_1 \to \sigma_2 \to \sigma_3 \to \sigma_4 \cdots \to \sigma_i \cdots \to \sigma_k \cdots \to \sigma_n$$

Figure 1: A simple cyclic graph to model traveling back in time from $t = k$ to $t = i$.

We proof in this paper that the grandfather paradox and Deutsch's unproven paradox are precluded for a general n-state model (with $n < \infty$). Following the formulation of Lee [1], suppose a signal is send back at time $t = k$ to the past at time $t = i$. For all other times, information propagates forward in a Markovian manner (as shown in Fig. 1). Let the transition matrices for the forward propagating signal at time $t = j$ be $T_j(\sigma_j|\sigma_{j-1})$. Time $t = i$ is special as signals are propagating from $t = i - 1$ and $t = k > i$. Let the transition matrix be $\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k)$. Finally, let π_m be a

doi:10.1016/j.phpro.2011.05.060

sequence of states starting from $t = 1$ to $t = m$. The basic equations governing the transition matrices are given in [1],

$$0 \leq T_j(\sigma_j|\sigma_{j-1}) \leq 1 \forall j \tag{1}$$

$$0 \leq \hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) \leq 1 \tag{2}$$

$$\sum_{\pi_m} P(\pi_m) = 1 \tag{3}$$

$$\sum_{\sigma_j} T_j(\sigma_j|\sigma_{j-1}) = 1 \tag{4}$$

$$\sum_{\sigma_i} \hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) = 1 \tag{5}$$

$$\sum_{\sigma_i, \sigma_k} \hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) V(\sigma_k|\sigma_i) = 1 \tag{6}$$

Where $P(\pi_m)$ is the probability of sampling the sequence π_m and $V(\sigma_k|\sigma_i)$ is the probability of sampling the state σ_k at time $t = k$ given that the state is σ_i at time $t = i$. Refer to [1] for a thorough explanation.

It is sufficient to describe both grandfather paradox and Deutsch's unproven theorem paradoxes using two states, i.e. dead or alive and proven or unproven. In this paper, we want to study how these paradoxes can be embedded in a higher dimensional space in which the world consists of more states than just dead/alive or proven/unproven. Using the grandfather paradox as an illustration, let Ω be the set of n discrete states describing the world. Among these states, there is a non-empty subset of states $\Omega_d \subset \Omega$ in which the agent is dead and a subset of states $\Omega_a \subset \Omega$ in which the agent is alive. We presume dead and alive events are mutually exclusive, that is, $\Omega_d \cap \Omega_a = \emptyset$ and since the agent can only be dead or alive, $\Omega_d \cup \Omega_a = \Omega$. We use a simple specific example to explain further. Suppose that the agent can be male or female and dead or alive. If for simplicity, we ignore all other states in the world except for male/female and dead/alive. Then Ω consists of four states, (1) male and dead, (2) male and alive (3) female and dead (4) female and alive. Ω_a consists of two states, male and alive and female and alive. Ω_d consists of the other two states, male and dead, female and dead. If we consider dead/alive to be in one subspace and male/female to be in another subspace, then Ω is just the direct product of these subspaces. Once, this simple example is understood, it is not difficult to generalize to arbitrary number of states. The above argument holds for the Deutsch's unproven theorem paradox.

1. Probabilistic View of Causality

Two alternative views, deterministic and probabilistic view of causality existed for a long time. For example, in the deterministic view, if you study hard, you will pass the exams. However, one might get unlucky and fail the exams even if he studies hard. There are other arguments to resolve this apparent contradiction, like one might study hard, but not hard enough or there are other "hidden variables" not factored into the cause and effect equation when making the statement.

Probabilistic view of causality states that you will have a higher chance of passing the exams if you study hard. The observation of studying hard and not passing the exams in this case is attributed to just being unlucky since there is still non-zero chance of failing although this non-zero chance gets smaller with more hard work.

We take the probabilistic view of causality in this paper. A sequence of events happening in time happens with a probability distribution. A realization of sequence of events also happens with a distribution. In addition, we assume a Markov model, except for a single time point where a signal is send from the future (See Fig. 1). At this point, we would like to elaborate on the effect of signal from the future. This signal, does not cause an event to happen at time $t = i$, but simply change the probability distribution of events at time $t = i$. This change in probability propagates forward in time to affect the probability distribution at time $t = k$ (the time at which the signal was sent to i). The change in probability distribution at various times should occur self-consistently.

2. Generalized Grandfather Paradox

The grandfather paradox can be formulated in a more abstract form. Let Ω be the set of n discrete states. Let $\Omega_d \subset \Omega, \Omega_d \neq \emptyset$ be a non-empty subset of states that are analogous to the absorbing states in an absorbing Markov Chain. The transition matrices $T_j(\sigma_i|\sigma_{i-1}) = 0$ and $\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) = 0$ for $\sigma_{i-1} \in \Omega_d$ and $\sigma_i \notin \Omega_d$. We can't go from a state belonging to Ω_d to a state not belonging Ω_d. For example, the agent cannot transit from a dead state to an alive state. Let $\Omega_a = \Omega\backslash\Omega_d \neq \emptyset$. Clearly,

$$\Omega_d \cup \Omega_a = \Omega \text{ and } \Omega_d \cap \Omega_a = \emptyset \tag{7}$$

Now let,

$$\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) = \begin{cases} N(\sigma_i|\sigma_{i-1}) & \text{for} \quad \sigma_k \in \Omega_d \\ S(\sigma_i|\sigma_{i-1}) & \text{for} \quad \sigma_k \in \Omega_a \end{cases} \tag{8}$$

with S being a generalization of the killing matrix described in Lee [1]. That is, S brings all states $\sigma_{i-1} \in \Omega$ into Ω_d, $S(\sigma_i|\sigma_{i-1}) = 0$ for all $\sigma_i \in \Omega_a$. Next define,

$$A(\sigma_i) = \sum_{\sigma_k \in \Omega_a} V(\sigma_k|\sigma_i) \tag{9}$$

we have $A(\sigma_i) = 0$ for $\sigma_i \in \Omega_d$ and $0 \leq A(\sigma_i) \leq 1$ for $\sigma_i \in \Omega_a$. Using Eq. (6) we obtain,

$$\sum_{\sigma_i} (N - S)(\sigma_i|\sigma_{i-1}) A(\sigma_i) = 0 \tag{10}$$

simplifying some more using the fact that $S(\sigma_i|\sigma_{i-1}) = 0$ for any $\sigma_i \in \Omega_a$ and $A(\sigma_i) = 0$ for $\sigma_i \in \Omega_d$,

$$\sum_{\sigma_i \in \Omega_a} N(\sigma_i|\sigma_{i-1}) A(\sigma_i) = 0 \tag{11}$$

since $A(\sigma_i) \geq 0$ and $N(\sigma_i|\sigma_{i-1}) \geq 0$, for Eq. (11) to hold, we have either $A(\sigma_i) = 0$ or $N(\sigma_i|\sigma_{i-1}) = 0$ or both equal zero for any σ_i. Let $N(\sigma_i|\sigma_{i-1}) = 0$ for $\sigma_i \in \Omega_a^{(1)}$ and $N(\sigma_i|\sigma_{i-1}) > 0$ for $\sigma_i \in \Omega_a^{(2)}$ with $\Omega_a^{(1)} \cup \Omega_a^{(2)} = \Omega_a$. $N(\sigma_i|\sigma_{i-1}) = 0$ implies that transition into a state $\sigma_i \in \Omega_a^{(1)}$ happens with zero probability. Hence all transitions must be into a state $\sigma_i \in \Omega_a^{(2)} \cup \Omega_d$. If $\sigma_i \in \Omega_a^{(1)}$ then $A(\sigma_i) = 0$ implies that $\sigma_k \in \Omega_d$ (see Eq. (9)). If $\sigma_i \in \Omega_d$, then clearly $\sigma_k \in \Omega_d$. Hence for any possible σ_{i-1} we have $\sigma_k \in \Omega_d$. All possible sequences of states leads to $\sigma_k \in \Omega_d$. The generalized grandfather paradox is precluded because a contradiction cannot be generated by the matrix S since $\sigma_k \in \Omega_d$ with probability one. This conclusion is consistent with the grandfather paradox for two- and three-state systems presented in Lee [1].

3. Generalized Deutsch's Unproven Theorem Paradox

Let Ω be the set of states. Let $\Omega_p \subset \Omega, \Omega_p \neq \emptyset$ and let $\Omega_u = \Omega\backslash\Omega_p$. Ω_p represents the states in which the theorem has been proven and Ω_u represents the states in which the theorem is unproven. Let $\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) = N(\sigma_i|\sigma_{i-1})$ for $\sigma_k \in \Omega_u$ and $\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) = S(\sigma_i|\sigma_{i-1})$ for $\sigma_k \in \Omega_p$. This paradox assumes that the theorem is not proven in the past, but is simply documented using signals from the future, once the proof is documented, it cannot get lost. V and N must be of the form,

$$\begin{aligned} V(\sigma_k|\sigma_i) = 0 \qquad & \text{for} \qquad \sigma_i \in \Omega_p \text{ and } \sigma_k \in \Omega_u \\ V(\sigma_k|\sigma_i) = 0 \qquad & \text{for} \qquad \sigma_i \in \Omega_u \text{ and } \sigma_k \in \Omega_p \\ \sum_{\sigma_k \in \Omega_p} V(\sigma_k|\sigma_i) = 1 \qquad & \text{for} \qquad \sigma_i \in \Omega_p \\ \sum_{\sigma_k \in \Omega_u} V(\sigma_k|\sigma_i) = 1 \qquad & \text{for} \qquad \sigma_i \in \Omega_u \\ \sum_{\sigma_i \in \Omega_u} N(\sigma_i|\sigma_{i-1}) = 1 \qquad & \text{for} \qquad \sigma_{i-1} \end{aligned} \tag{12}$$

We also have $\sigma_{i-1} \in \Omega_u$ because the signal travels from the future at $t = i$ and does not affect the state at time $t = i - 1$. Using Eq. (6),

$$\begin{aligned}
&\textstyle\sum_{\sigma_i\in\Omega_u,\sigma_k\in\Omega_u} N(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i)+ \\
&\textstyle\sum_{\sigma_i\in\Omega_p,\sigma_k\in\Omega_u} N(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i)+ \\
&\textstyle\sum_{\sigma_i\in\Omega_u,\sigma_k\in\Omega_p} S(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i)+ \\
&\textstyle\sum_{\sigma_i\in\Omega_p,\sigma_k\in\Omega_p} S(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i) = \quad 1
\end{aligned} \tag{13}$$

and using Eq. (12), we obtain

$$\Rightarrow \sum_{\sigma_i\in\Omega_p} S(\sigma_i|\sigma_{i-1}) = 0 \tag{14}$$

Since $S(\sigma_i|\sigma_{i-1}) \geq 0$, the only way for its sum to be zero is that each term is zero $S(\sigma_i|\sigma_{i-1}) = 0$ for $\sigma_i \in \Omega_p$. The Deutsch's unproven paradox is precluded because the transition matrix S which is suppose to bring the proof from the future is zero $S(\sigma_i|\sigma_{i-1}) = 0$ for and $\sigma_i \in \Omega_p$. That is to say, it is impossible to "document" the proof from the future even if such signal travels from the future.

4. A Complimentary Set Paradox

We can construct another paradox by "combining" the grandfather paradox and the Deutsch's unproven theorem paradox. We call it the complimentary set paradox for reasons that will be obvious later. Let Ω be the set of n discrete states. Let Ω_b be a non-empty subset of Ω and $\bar{\Omega}_b = \Omega\backslash\Omega_b$ be the complimentary subset. Assume that the transition matrix $V(\sigma_k|\sigma_i)$ is non-ergodic, that is $V(\sigma_k|\sigma_i) = 0$ if σ_i and σ_k does not belong to the same subset Ω_b and $\bar{\Omega}_b$. Note that this is also the case for the Deutsch's unproven theorem paradox. Let σ_k be the state at the future time $t = k$ and a signal is send to the time $i < k$ such that if $\sigma_k \in \Omega_b$ then the transition matrix brings σ_i into $\sigma_i \in \bar{\Omega}_b$ and vice versa. In this construction, the subset σ_i belongs to (Ω_b or $\bar{\Omega}_b$) is always complimentary to the subset σ_k belongs to. This explain why we call this a complimentary set paradox. The transition matrix $\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k)$ therefore can be written in the form,

$$\hat{T}_i(\sigma_i|\sigma_{i-1}, \sigma_k) = \begin{cases} N(\sigma_i|\sigma_{i-1}) & \text{for} \quad \sigma_k \in \Omega_b \\ S(\sigma_i|\sigma_{i-1}) & \text{for} \quad \sigma_k \in \bar{\Omega}_b \end{cases} \tag{15}$$

With $N(\sigma_i|\sigma_{i-1}) = 0$ for $\sigma_i \in \Omega_b$ and $S(\sigma_i|\sigma_{i-1}) = 0$ for $\sigma_i \in \bar{\Omega}_b$. This paradox is fundamentally different from the grandfather paradox and Deutsch's unproven paradox. The grandfather paradox is precluded because the agent *must* be dead at time $t = k$ hence cannot travel back in time to kill himself. The normalization condition of Eq. (3) can still be satisfied. The summation sums over all sequences in which the agent is died before time $t = k$. The Deutsch's unproven theorem paradox is precluded because the probability of recording the proof is zero. The normalization condition of Eq. (3) is still satisfied where the summation sums over only sequences in which the proof is non-existence. This paradox is radically different in which the normalization condition of Eq. (3) cannot hold at all! To proof this, we show that Eq. (6) cannot be satisfied.

$$\begin{aligned}
\sum_{\sigma_i,\sigma_k} T(\sigma_i|\sigma_{i-1}, \sigma_k)V(\sigma_k|\sigma_i) &= \sum_{\sigma_k\in\Omega_b,\sigma_i} N(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i) + \sum_{\sigma_k\in\bar{\Omega}_b,\sigma_i} S(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i) \\
&= \sum_{\substack{\sigma_k\,\in\,\Omega_b \\ \sigma_i\,\in\,\Omega_b}} N(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i) + \sum_{\substack{\sigma_k\,\in\,\bar{\Omega}_b \\ \sigma_i\,\in\,\bar{\Omega}_b}} S(\sigma_i|\sigma_{i-1})V(\sigma_k|\sigma_i) \\
&= 0 + 0 \neq 1
\end{aligned} \tag{16}$$

In the second line, we make use of the fact that $V(\sigma_k|\sigma_i) = 0$ for σ_i, σ_k not belonging to the same subset. For the last line, we use $N(\sigma_i|\sigma_{i-1}) = 0$ for $\sigma_i \in \Omega_b$ and $S(\sigma_i|\sigma_{i-1}) = 0$ for $\sigma_i \in \bar{\Omega}_b$. In fact, any sequence π_m happens with zero probability. This combination of transition matrices N, S, V cannot exist under the current framework. The paradox can be resolved if we relax the condition of V to make it ergodic, that is, $V(\sigma_k|\sigma_i) \neq 0$ when σ_i, σ_k does not belong to the same subset.

5. Conclusion

It seems the apparent time travel paradoxes come from the fact that $V(\sigma_k|\sigma_i)$ is non-ergodic. This is the case for the time travel paradoxes described in this paper. It is also suggestive to infer that there will be no paradox if $V(\sigma_k|\sigma_i)$ is ergodic and k is far enough into the future from i. This statement remains to be proven rigorously, however a hand-waving argument is as follows. Suppose a signal is send from time $t = k$ to time $t = i$ to change the distribution of states at time $t = i$. Since $V(\sigma_k|\sigma_i)$ is ergodic and k is far away in the future, one can find a self-consistent set of sequences of states linking σ_i and σ_k.

Sections (2) and (3) avoided the time travel paradoxes by precluding them. These paradoxes can also be avoided by allowing $V(\sigma_k|\sigma_i)$ to be ergodic. For the grandfather paradox can be avoided if we allow a small chance of resurrection. The agent travels back in time from $t = k$ to $t = i$ to kill himself, between the time $t = i$ and $t = k$, the "dead" agent resurrected so that he can then travel back in time again at $t = k$ to kill himself. Another way to avoid the paradox is to assume that the agent travels back in time but did not succeed in killing himself.

The Deutsch's unproven theorem paradox can be avoided if we allow the document to get lost and then the theorem re-derived again between the times $t = i$ and $t = k$. In this way, the re-derived prove can be sent to the past to be recorded and lost before it was re-derived again.

We have shown in this paper that the grandfather paradox and the Deutsch's unproven theorem paradox are precluded for the case in which the agent lives in a world with n discrete states. For future work, we would like to find out if similar paradoxes can arise when the transition matrix $V(\sigma_k|\sigma_i)$ is ergodic.

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