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FULL LENGTH ARTICLE

A polynomial regression model for stabilized turbulent confined jet diffusion flames using bluff body burners

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Abstract Thermal structure of stabilized confined jet diffusion flames in the presence of different geometries of bluff body burners has been mathematically modeled. Two stabilizer disc burners tapered at 30° and 60° and another frusted cone of 60°/30° inclination angle were employed all having the same diameter of 80 (mm) acting as flame holders. The measured radial mean temperature profiles of the developing stabilizing flames at different normalized axial distances were considered as the model example of the physical process.

A polynomial mathematical model of fourth degree has been investigated to study this phenomenon to find the best correlation representing the experimental data. Least Squares regression analysis has been employed to estimate the coefficients of the polynomial and investigate its adequacy. High values for $R^2 > 0.9$ obtained for most of the investigated bluff burners at the various locations of x/d_j prove the adequacy of the suggested polynomial for representing the experimental results. Very small values of significance $F < (\alpha = 0.05)$ for all investigated cases indicate that there is a real relationship between the independent variable r and the dependant variable T. The low values of $p < (\alpha = 0.05)$ obtained reveal that all the recorded parameters for all the investigated cases are significant.

1. Introduction

Turbulent diffusion flames are usually used in industrial applications such as gas burners of industrial furnaces, gas turbine combustion chamber and flaring of petroleum industry. To improve the efficiency of practical burners, the design has been widely studied and received renewed attention in recent years. The co-axial jet diffusion flames have been found to be a viable method for enhancing flame stability. In such an arrangement, a flame holder such as bluff bodies is necessary to generate a recirculation zone in which the fuel and oxidizer mix thoroughly. Bluff body wakes play a very important role in stabilizing the flame [1]. It can be noted that the aerodynamic wake provides sufficient residence time for the fuel to ensure a stable flame creating a pilot flame which serves as a continuous

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ignition source to stabilize the main flame even at a higher velocity [2,3].

Several studies on bluff body flame stabilization have been reported revealing the complex flow pattern, chemistry and pressure gradient interaction which present in the reactive recirculatory flow field [4–6].

Also the effect of bluff body shape in flame stabilization was investigated experimentally [7–9]. Bluff bodies with different geometry and aerodynamic characteristics had a more obvious effect on flow structure and mixing mechanism. The flow features influenced by the different shapes of bluff bodies creating a large scale motion of the re-circulated vortices, prolong stagnation of reactants which is a key factor to flame stabilization regime.

Moreover, the effect of bluff body geometry such as lip thickness, for the LPG jet diffusion flames, on several physical parameters like flame length, gas temperature and flame stability were experimentally studied [10–12]. Results indicated that with increase in lip thickness, the flame length gets reduced, increasing the flame temperature and enhancing flame stability. This can be attributed to the enhanced reactivity and residence time of the mixture gas with increasing lip thickness of the bluff body. Also the recirculation zone formed in the wake of this bluff body allows better mixing in this region shifting the reaction zone toward the bluff body realizing an improvement in the combustion domain.

The present study analyses through mathematical modeling the previously reported experimental data of thermal structure of turbulent stabilized confined jet diffusion flames in the presence of different geometries of bluff body burners [13].

2. Experimental

The experimental setup comprised a vertical combustor of 150 (mm) diameter, 5 (mm) thickness and 1 (m) height. The combustion chamber was fitted with an arrangement of supplying the fuel and combustion air. The burner section consisted of an outer cylinder of the same diameter as the combustion chamber and a central pipe of 25 (mm) diameter. The latter holds the bluff body and the fuel supply line is connected to the fuel jet nozzle of (d_j) 2.5 (mm) inner diameter and 10 (mm) outer diameter at the centre of the bluff body at the base of the combustor. In this experimental example three bluff-bodies were used. The first stabilizer disc was tapered at 30°; the second was tapered at 60° of the same diameter of 80 (mm) and 10 (mm) high. The third bluff body was stabilizer frusted cone having inclination angles of 60°/30° and 50 (mm) high with the same surface diameter of 80 (mm) facing the jet flame. Commercial LPG fuel was used in all experiments. The developing jet flames operated at the same fuel mass flow rate ($\dot{m}_f$) of 2.6 kg/h, combustion air flow rate ($\dot{m}_a$) of 40 kg/h, air/fuel ratio (A/F) = 15.34 at the stoichiometric condition and overall flames equivalence ratio (Φ) = 1 in the presence of each bluff body geometry [13].

The mean radial temperature distribution was measured at different normalized axial distances along these developing flames over the different bluff-body burners.

3. Regression analysis and mathematical model

Regression analysis is a statistical tool for the investigation of relationships between two or more variables of which at least one is subject to random variation, and to test whether such a relation, either assumed or calculated, is statistically significant. Usually, the investigator seeks to ascertain the causal effect of one variable upon another. To explore such issues, the investigator assembles data on the underlying variables of interest and employs regression to estimate the quantitative effect of the causal variables upon the variable that they influence. The investigator also typically assesses the "statistical significance" of the estimated relationships, that is, the degree of confidence that the true relationship is close to the estimated relationship. The statistical tests, which normally accompany regression analysis, serve in model identification, model verification, and efficient design of the physical process. Regression analysis produces an equation that will predict a dependent variable using one or more independent variables. Numerous references dealt with the concept of Regression analysis [14–17].

When running regression, we are trying to discover whether the coefficients of the independent variables are really different from 0 (so the independent variables are having a genuine effect on the dependent variable) or if alternatively any apparent differences from 0 are just due to random chance. The null (default) hypothesis always states that each independent variable is having absolutely no effect (has a coefficient of 0) and you are looking for a reason to reject this hypothesis.

3.1. Polynomial regression

From the experimental result we assume that the behavior of the dependent variable can be explained by a polynomial, additive relationship between the dependent variable and a set of power in the independent variable. Polynomial regression models contain squared and higher order terms of the predictor variables making the response surface curvilinear.

In statistics, polynomial regression is a form of linear regression in which the relationship between the independent variable x and the dependent variable y is modeled as an nth order polynomial. Polynomial regression fits a nonlinear relationship between the value of x and the corresponding y, and has been used to describe nonlinear phenomena. Although polynomial regression fits a nonlinear model to the data, as a statistical estimation problem it is linear, in the sense that the regression function $E\ (y|x)$ is linear in the unknown parameters that are estimated from the data. In addition, it is assumed that the Ordinary Least Squares (OLS) assumptions hold that minimizes the variance of the unbiased estimators of the coefficients. We then proceed to develop a complete fourth degree polynomial model. We eliminate non-significant terms based on statistical parameter tests (if the recorded $p > (\alpha = 0.05)$ and the coefficient confidence interval spans zero) then rerun the model without these non-significant parameters. The final model should contain only significant parameters.

3.2. Goodness of fit

The *OLS* technique ensures that we find the values of coefficients which 'fit the sample data best', in the specific sense of minimizing the sum of squared residuals. To guarantee that the 'best fitting' equation fits the data well we assess the adequacy of the 'fitted' equation through the following indicators [16,17].

3.2.1. Coefficient of multiple determinations, R^2

This is the most important number of the output ranging from 0 to 1, and it is a measure of how well the regression line approximates the real data. *R* square tells you how much of the output variable's variance is explained by the input variables' variance. An *R* square of 1.0 is a perfect fit, with every point falling right on the line, and zero means there's absolutely no pattern or fit whatsoever. Ideally we would like to see this at least 0.6 (60%).

An analysis of variance is used to test the hypothesis that the polynomial fit is a better fit than the mean. The total variance, the variance of the predictor fitted to just the mean, is partitioned into variance explained by the polynomial regression model and residual variance (the difference from the fitted line to the observations). An *F*-test then compares the partitioned variances to determine if they are significantly different.

Regression Significance F is the probability that the output results by chance rather than from a real correlation between independent and dependent variables and Eq. (1) do not explain the variation in *y*. The test is used to check if a linear statistical relationship exists between the response variable and at least one of the predictor variables which means that at least one of the coefficients does not equal 0. The smaller the value, the greater the probability that the results have not arisen by chance. Thus you should reject the claim that there is no significant relationship between the independent and dependent variables if this value < ($\alpha = 0.05$).

The p-value for each regression coefficient tells you how likely it is that the coefficient for that independent variable emerged by chance and *does not* describe a real relationship. Thus you should reject the claim that there is no significant relationship between your independent variable (in the corresponding row) and dependent variable if $p < \alpha$.

Confidence Limits are the 95% probability that the true value of the coefficient lies between the Lower 95% and Upper 95% values. The narrower this ranges the better. If the lower value is negative and the upper value positive, try correlating the data with this variable left out. A significant *p*-value, or

Table 1 Regression statistics and analysis of variance.

	Normalized axial distances x/d_j							
	32	56	68	82	132	178	200	240
Disc tapered at 30°								
R square	0.807	0.927	0.975	0.985	0.993	0.966	0.976	0.983
F ratio	48.11	145.70	443.02	741.42	763.48	327.663	476.909	302.04
Significance *F*	6.1E−09	8.68E−14	2.32E−18	1.3E−21	2.08E−22	1.25E−17	1.89E−19	3.14E−18
Disc tapered at 60°								
R square	0.593	0.741	0.859	0.952	0.985	0.973	0.982	0.994
F ratio	16.75	32.93	70.34	229.79	739.31	412.24	623.56	929.35
Significance *F*	3.24E−05	1.78E−07	1.58E−10	6.29E−16	1.35E−21	9.69E−19	9.24E−21	2.68E−23
Frusted cone at 60°/30°								
R square	0.922	0.957	0.989	0.992	0.995	0.997	0.994	0.960
F ratio	135.785	253.203	1072.02	1405.46	1051.11	1737.445	802.49	277.56
Significance *F*	1.84E−13	2.17E−16	1.98E−23	9.07E−25	7.42E−24	3.88E−26	1.24E−22	7.88E−17

Table 2 Estimated regression parameters for stabilized disc tapered at 30°.

Regression parameters		Normalized axial distances x/d_j							
		32	56	68	82	132	178	200	240
b_0	Coeff.	814.61	961.52	1054.39	1096.80	1225.95	1411.40	1348.14	1278.56
	±C.L.	59.29	40.32	23.98	17.46	13.843	35.00	25.62	18.59
	P-value	2.03E−19	7.75E−25	6.50E−31	1.80E−34	3.52E−35	4.69E−30	1.05E−32	7.09E−33
b_1	Coeff.					−0.8661			−1.1151
	±C.L.					0.457			0.615
	P-value					7.60E−04			1.12E−03
b_2	Coeff.	−0.2232	−0.2555	−0.2497	−0.2.196	−0.236	−0.2427	−0.2694	−0.2312
	±C.L.	0.0710	0.0482	0.0287	0.0209	0.0167	0.0419	0.0307	0.0223
	P-value	1.23E−06	1.34E−10	4.80E−15	7.85E−17	1.28E−18	2.27E−11	3.87E−15	7.92E−16
b_3	Coeff.								2.511E−04
	±C.L.								1.60E−04
	P-value								3.69E−03
b_4	Coeff.	2.59E−05	2.87E−05	2.64E−05	2.14E−05	2.1742E−05	1.80E−05	2.76E−05	2.53E−05
	±C.L.	1.33E−05	9.02E−06	5.37E−06	3.91E−06	3.097E−6	7.83E−06	5.73E−06	4.16E−06
	P-value	5.07E−04	1.02E−06	5.57E−10	6.61E−11	1.81E−12	8.56E−05	8.10E−10	2.66E−11

a coefficient confidence interval that doesn't span zero, implies the term has a significant contribution to the response.

A mathematical model based on experimental data has been investigated to study the thermal structure of stabilized turbulent confined jet diffusion flames in the presence of different geometries of bluff body burners. It is used to determine the relationship between the independent (radius r in (mm)) and the observed dependent (mean temperature T in (°C)) variable. The complete estimated polynomial model may be written as

$$Y = T = b_0 + b_1 r + b_2 r^2 + b_3 r^3 + b_4 r^4 \quad (1)$$

where T = mean temperature (°C), r = radius (mm).

In a similar study, for modeling and optimization of catalytic combustion of turbulent confined lifted diffusion flames, the temperature profiles at different axial locations along the flames over the discs have been correlated with the radial distance (r) employing a linearized form of an exponential function [18].

The regression has been performed employing Microsoft Excel 2007 which determines the coefficients of the equation along with the statistical parameters which validate the results. Among these statistical parameters are R squared, F-ratio, significance F, confidence interval and the P-value for the parameters.

4. Results and discussions

Table 1 presents the values of R^2, Fisher ratio (F) together with the significance F. As for the significance F very small values < (α = 0.05) have been obtained for all the relations. This indicates that there is a real relation between the independent variable r and the dependant variable T for all the investigated

Table 3 Estimated regression parameters for stabilized disc tapered at 60°.

Regression parameters		Normalized axial distances x/d_j							
		32	56	68	82	132	178	200	240
b_0	Coeff.	639.78	763.68	876.63	979.69	1057.47	1096.29	1295.77	1185.57
	±C.L.	69.98	66.14	52.67	33.43	20.03	27.03	26.93	12.50
	P-value	1.63E−15	9.77E−18	2.72E−21	7.01E−27	9.71E−33	4.12E−30	8.15E−32	8.31E−36
b_1	Coeff.								−1.506
	±C.L.								4.13E−01
	P-value								1.95E−07
b_2	Coeff.	−0.169	−0.214	−0.241	−0.268	−0.270	−0.251	−0.294	−0.245
	±C.L.	8.37E−02	7.92E−02	6.30E−02	4.00E−02	2.40E−02	3.23E−02	3.22E−02	1.50E−02
	P-value	3.60E−04	1.07E−05	5.36E−08	1.20E−12	1.67E−17	5.57E−14	1.72E−15	7.32E−20
b_3	Coeff.								2.25E−04
	±C.L.								1.07E−04
	P-value								2.78E−04
b_4	Coeff.	2.12E−05	2.58E−05	2.81E−05	3.03E−05	2.87E−05	2.42E−05	2.67E−05	2.39E−05
	±C.L.	1.56E−05	1.48E−05	1.18E−05	7.48E−06	4.48E−06	6.05E−06	6.03E−06	2.80E−06
	P-value	9.99E−03	1.47E−03	5.61E−05	1.88E−08	3.1E−12	2.39E−08	3.66E−09	3.73E−14

Table 4 Estimated regression parameters for frusted cone at 60°/30°.

Regression parameters		Normalized axial distances x/d_j							
		32	56	68	82	132	178	200	240
b_0	Coeff.	989.83	1139.01	1241.22	1305.91	1571.22	1482.66	1398.33	1334.72
	±C.L.	47.27	39.31	20.06	17.34	17.15	10.06	10.53	19.17
	P-value	1.50E−23	9.06E−27	2.54E−34	2.78E−36	1.72E−35	8.03E−40	7.07E−39	1.69E−35
b_1	Coeff.					−0.962	−0.913	−0.6552	
	±C.L.					5.67E−01	3.33E−01	3.48E−01	
	P-value					2.00E−03	1.16E−05	7.97E−04	3.05E−12
b_2	Coeff.	−0.316	−0.332	−0.308	−0.267	−0.301	−0.227	−0.1772	0.1468
	±C.L.	5.66E−02	4.70E−02	2.40E−02	2.07E−02	2.05E−02	1.20E−02	1.26E−02	2.29E−02
	P-value	4.73E−11	4.05E−13	9.27E−19	9.03E−19	6.96E−19	4.00E−21	1.67E−18	
b_3	Coeff.							0.0002	
	±C.L.					1.47E−04	8.64E−05	9.04E−05	
	P-value					8.00E−03	5.02E−05	8.78E−04	
b_4	Coeff.	3.86E−05	3.77E−05	3.07E−05	2.22E−05	2.26E−05	1.70E−05	1.55E−05	1.43E−05
	±C.L.	1.05E−05	8.79E−06	4.48E−06	3.88E−06	3.84E−06	2.25E−06	2.35E−06	4.29E−06

cases of burners at the different discrete points of the normalized axial distances x/d_j. As regards R squared, that indicates the goodness of fit between the experimental values and the corresponding predicted ones employing Eq. (1), high values for $R^2 > 0.9$ have been obtained for most of the relations and this proves the adequacy of the polynomial (1) for representing the experimental results.

Tables 2–4 depict the values of only the significant coefficients of the polynomial Eq. (1) along with the corresponding $p < (\alpha = 0.05)$ values and coefficients limits. The empty cells

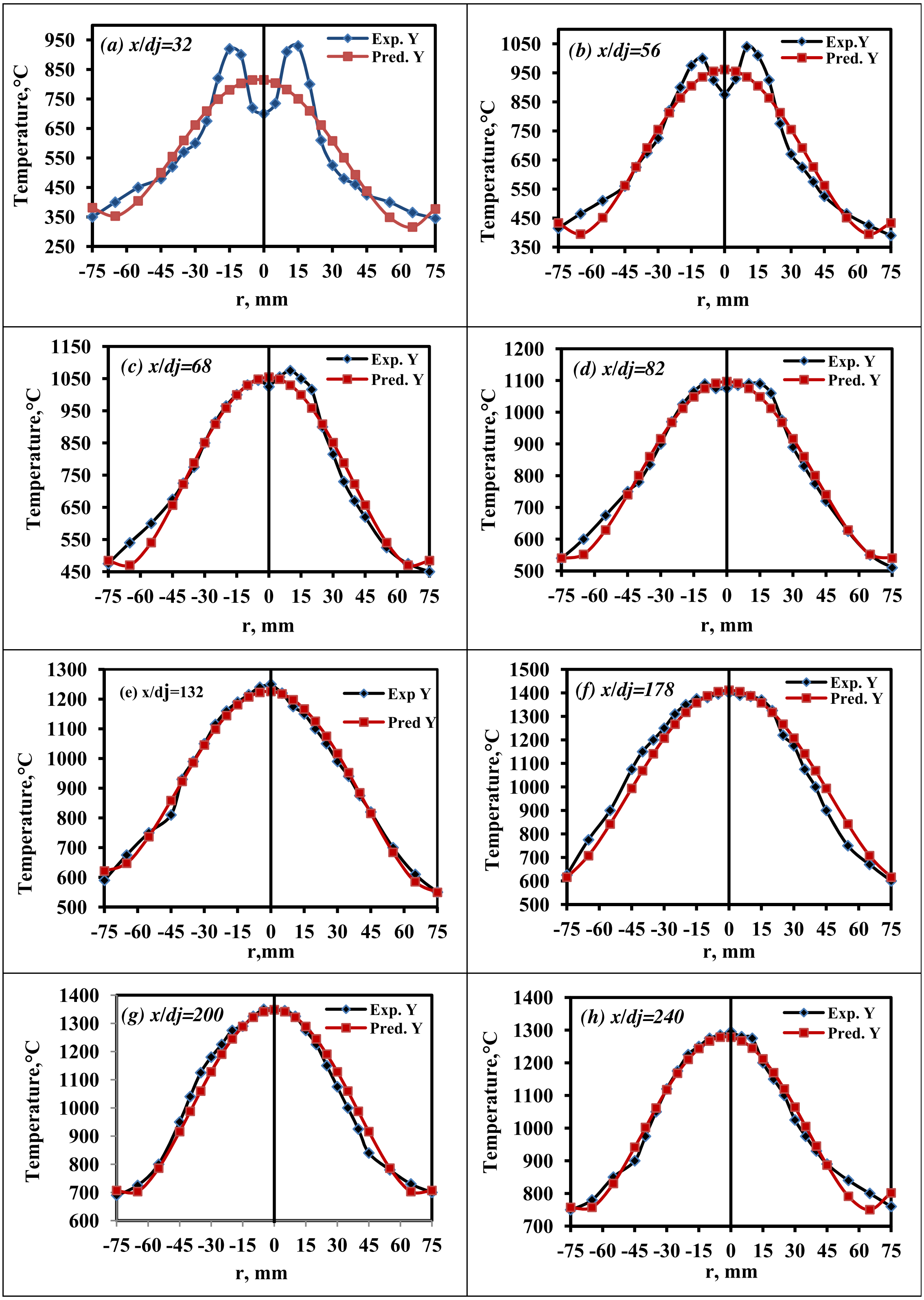

Figure 1 Experimental and predicted values of temperature for stabilizer disc burner tapered at 30° at different axial distances x/d_j.

in Tables 2–4 belong to the eliminated non significant coefficients. The low values of p indicate that all the recorded coefficients are significant. This is also manifested in the small values of coefficient limits in comparison with their corresponding ones which mean that they do not span the zero as a value for the parameter.

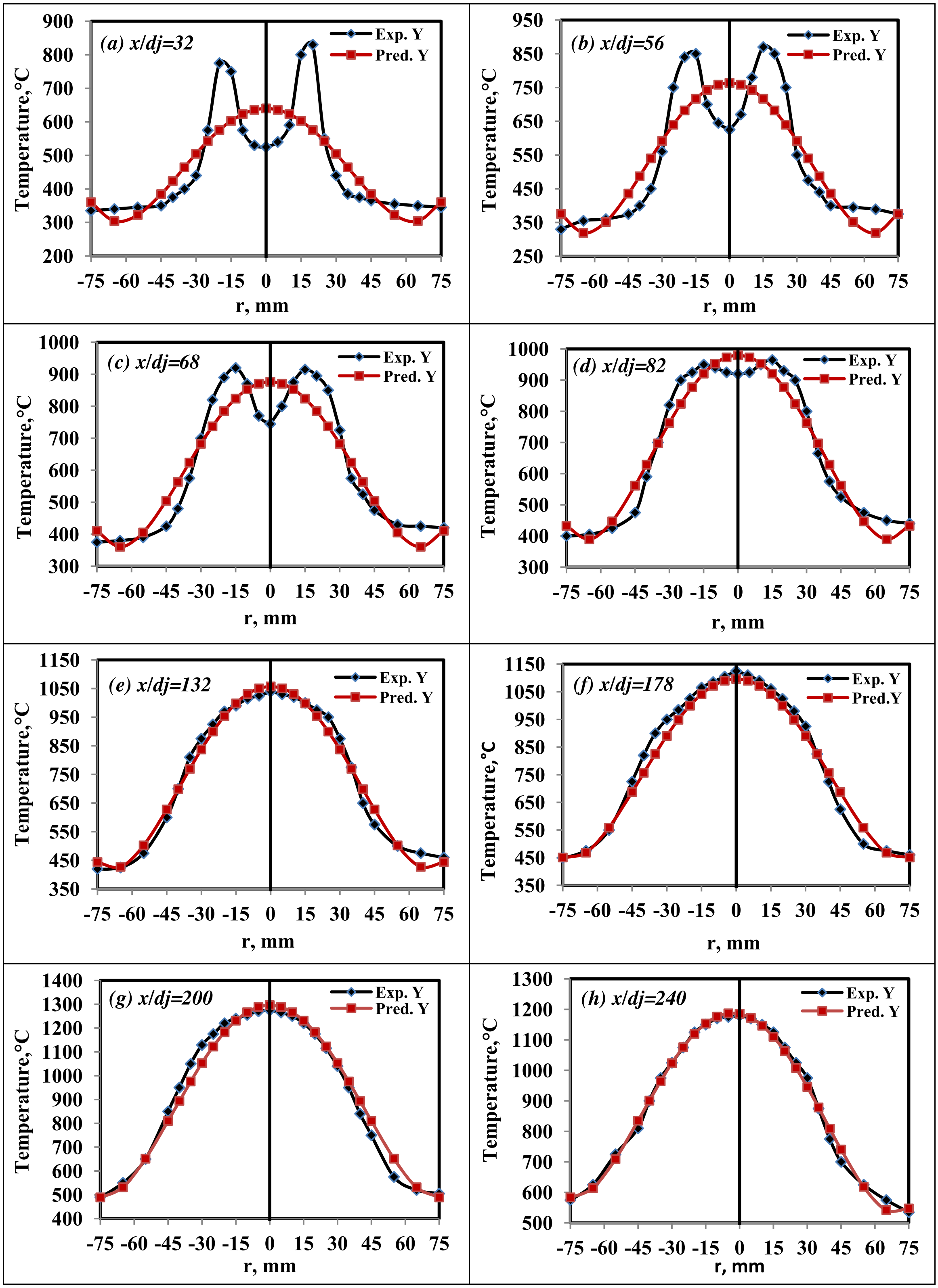

Figure 2 Experimental and predicted values of temperature for stabilizer disc burner tapered at 60° at different axial distances x/d_j.

For the various geometries of bluff body burners (Figs. 1–3) compare the experimental data with the corresponding predicted values obtained from Eq. (1) employing the predicted parameters obtained through the regression technique. The percentage residual = [(experimental value – corresponding predicted value)/experimental value] was calculated and is presented in Table 5 for x/d_j = 32, 56, 68 and 82.

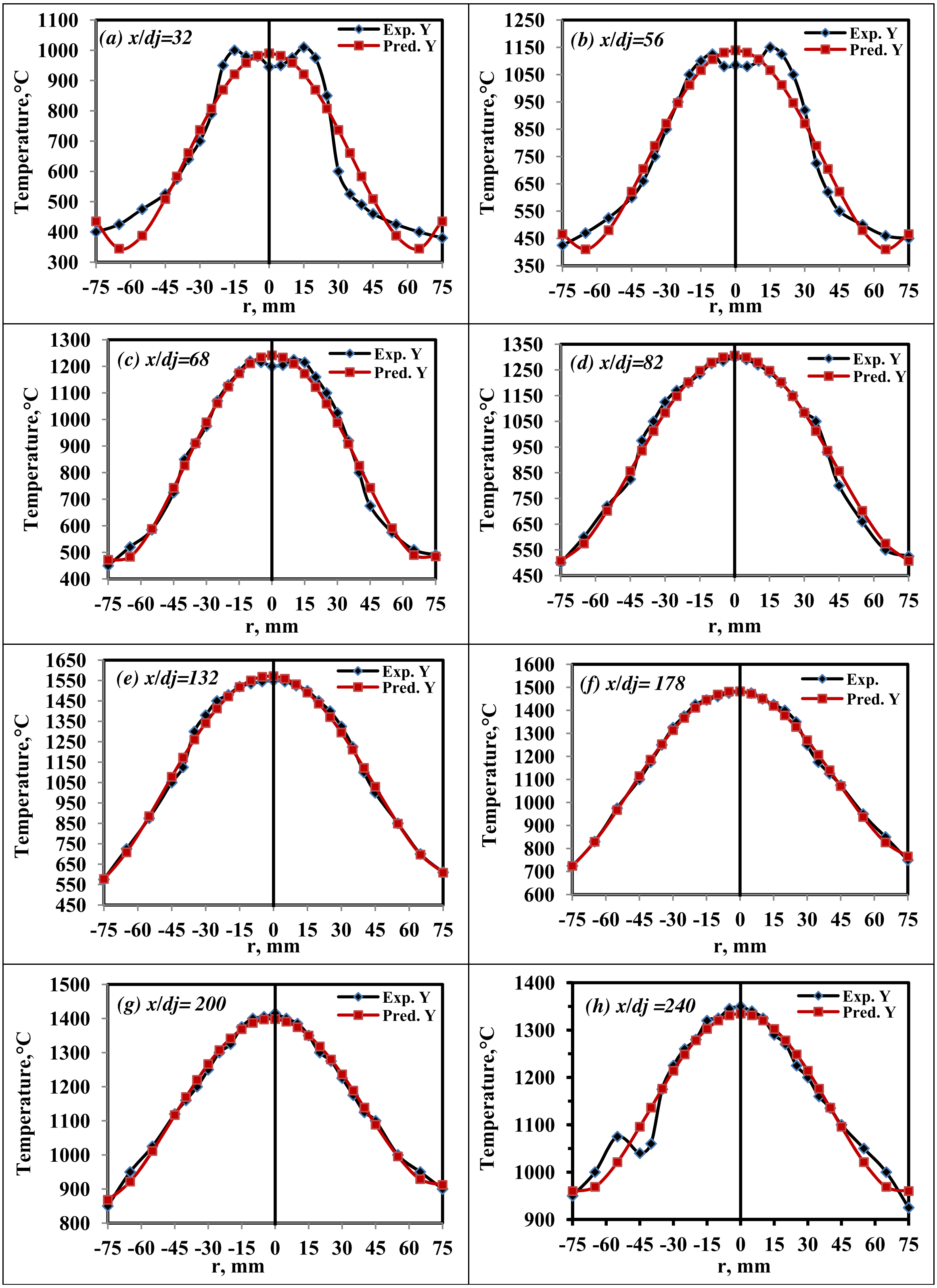

Figure 3 Experimental and predicted values of temperature for frusted cone stabilizer disc burner of 60°/30° inclination angle at different axial distances x/d_j.

Table 5 The percentage of residual.

Burner type	x/d_j	$-r_{mm}$	% residual	% residual at $r = 0$	$+r_{mm}$	% residual
Disc burner of 30° tapered angle	32	25	16.7	−16.37	20	17.87
	56	15	6.37	−9.89	15	10.35
	68	10	0.65	−2.87	15	5.64
	82	15	1.55	−2.03	20	4.49
Disc burner of 60° tapered angle	32	20	19.63	−21.86	20	30.66
	56	15	18.8	−22.19	15	19.75
	68	15	11.8	−17.67	20	13.27
	82	20	5.15	−6.49	15	5.66
Frusted cone of 60°/30° inclination angle	32	15	2.18	−4.74	20	8.43
	56	15	1.67	−4.98	20	7.28
	68	15	0.67	−3.43	20	3.2
	82	15	−1.61	−0.45	20	0.62

For the stabilized disc tapered at 30° (Table 2) presents the estimated regression parameters along with the confidence limits and the corresponding p values < ($\alpha = 0.05$) indicating that all the recorded coefficients are significant. Fig. 1 shows the predicted temperature profile employing Eq. (1) and the corresponding predicted constants of Table 2 compared with the measured radial mean temperature distribution of the developing stabilized flame at different discrete points of the normalized axial distances ranging from $x/d_j = 32$ up to $x/d_j = 240$.

A close inspection of Fig. 1(a–d) and the percentage residual of Table 5 indicates some deviation between the measured radial mean temperature profiles and the corresponding predicted values within the creative recirculation zone very near upstream distances along the stabilized flames at $x/d_j = 32$, 56, 68 and 82 with maximum value of 18% at $x/d_j = 32$. This is also manifested in the relatively low value of $R^2 = 0.81$ at this distance which could be considered a reasonable relation. This can be explained by the presence of the stabilizer disc burner creating a recirculation zone which enhances the mixing process of fuel jet and entrained combustion air where the combustion occurs. In this intense zone of reversed flow around the stabilizer disc burners, a complex aerodynamics wake can be generated producing a region of low velocity, providing low residence time into the incoming fresh mixture. Also, it is accompanied by negative pressure gradient interaction in this reactive re-circulatory flow field.

As for the other normalized axial distances a value of $R^2 > 0.9$ has been obtained. This is also clear from Fig. 1(e–h) which indicates that the predicted values of temperatures through the correlated polynomial function coincide with the experimental temperature profiles. This occurs as the combustion process proceeds to completion gradually downstream locations at all regions surrounding the main reaction zone of the developing stabilized flame operating using stabilizer disc burner of 30° tapered angle.

For the stabilized disc tapered at 60° (Table 3) demonstrates the significance of the recorded estimated regression parameters indicated with the low confidence limits and the corresponding p values < ($\alpha = 0.05$). Fig. 2(a–h) displays the predicted temperature profile employing Eq. (1) and the corresponding predicted constants of Table 2 compared with the measured radial mean temperature distribution of the developing stabilized flame at different discrete points of the normalized axial distances ranging from $x/d_j = 32$ up to $x/d_j = 240$.

Fig. 2(a–d) indicates a remarkable deviation of the experimental data compared to the corresponding predicted ones employing Eq. (1) and the tabulated values of constants in Table 3 specially at $x/d_j = 32$, 56, 68 and 82. This is obvious from the high values of residual recorded in Table 5 with a value = 31% at $x/d_j = 32$. This is also confirmed by the low values of R^2 of 0.59, 0.74 and 0.86 that have been obtained at $x/d_j = 32$, 56 and 68, respectively. However these relations could be accepted as these values of R^2 are $\geqslant 0.6$. This is due to the effect of the geometry of the 60° tapered angle stabilizer disc burner creating strong recirculation with strong turbulence intensity resulting from the disturbances of mixing, so reducing the spreading of the fuel jet and leading to retardation of combustion process.

As for the frusted cone stabilizer burner of 60°/30° inclination angle (Table 4) depicts the estimated regression parameters along with the confidence limits and the corresponding p values < ($\alpha = 0.05$) indicating that all the recorded coefficients are significant. Fig. 3(a–h) displays the good agreement between the measured radial mean temperature profiles and the corresponding predicted values. This also has been verified by the high recorded values of $R^2 > 0.9$ in Table 1 and low values of % residual listed in Table 5 at all listed locations of x/d_j.

The presence of the frusted cone stabilizer burner in the combustion domain promotes the combustion process as a result of the successful mixing of fuel jet and the entrained combustion air at the central upstream region at the base of the developed flame. This flow aerodynamic around the frusted cone has a great role for enhancing the occurrence of intense chemical reactions which are accompanied by rapid combustion very near within the creative recirculation zone and proceed to completion at all regions surrounding the main reaction zone recording higher temperatures very near downstream distance.

5. Conclusions

The results of the present investigation verify that:

(1) The thermal structure of stabilized confined jet diffusion flames in the presence of two stabilizer disc burners tapered at 30° and 60° and another frusted cone of 60°/30° inclination angle at different normalized axial distances has been successfully analyzed through mathematical modeling.
(2) A polynomial mathematical model of fourth degree has been investigated to study this phenomenon and was found to be the best correlation representing the experimental data. It results in the best values of statistical parameters in comparison to other attempted correlations.
(3) The high values for $R^2 > 0.9$ obtained for most of the investigated bluff body burners at the various locations of x/d_j prove the adequacy of the employed equation for representing the experimental results. The low values obtained at some upstream locations along the developed flame of the stabilized disc tapered at 60° could be attributed to the aerodynamic problems associated with the geometry of this type of burner.
(4) The very small values of significance $F < (\alpha = 0.05)$ indicate that there is a real relationship between the independent variable r and the dependant variable T for all the investigated cases of burners at the different discrete points of the normalized axial distances x/d_j.
(5) All the recorded parameters for all the investigated cases are significant. This is revealed in the low values of $p < (\alpha = 0.05)$ obtained.
(6) There is a good agreement between the experimental measured radial mean temperature profiles and the corresponding predicted values employing the suggested polynomial and the estimated parameters.

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