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Short Combo Strategy Using Barrier Options and its Application in Hedging

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Abstract

This paper deals with a Short Combo option strategy and its application in hedging against an underlying price increase assuming the given underlying asset will be bought in the future. The key difference between the previous studies is that in this paper we are concentrated on single barrier options. Barrier options were formed to provide risk managers with cheaper means to hedge their exposures without paying for the price changes they believed unlikely to occur. The methodology is based on the profit functions in analytical form. We propose various hedging possibilities and show its practical application. In our analysis we used vanilla and barrier European options on SPDR Gold Shares. The results show that the Short Combo strategy formation using barrier options gives the end-users greater flexibility to express a precise view in the specific future price situations.

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Introduction

There are many types of market risk, i.e. risk of unfavourable price changes that can bring losses for financial and non-financial institutions. At present, in the context of globalization process, the market risk becomes more important than ever. The price fluctuations affect the activity of companies and banks. There are a wide range of instruments, methods, techniques to identify measure and hedge the market risk, from the simplest to the most

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complicated. The analysis of available hedging strategies is regular theme of scientific papers. For example, Campello et al. (2011) investigates the implications of hedging for corporate financing and investment. Loss (2012) studies firm's optimal hedging strategies. Adam and Fernando (2006) and Brown et al. (2006) analyze the corporate risk management policies of gold mining firms. Tichý (2009) focuses on currency hedging of non-financial institutions. Judge (2007) analyzes why it is important to hedge. According to Zmeškal (2004), the main idea of hedging is to add new asset or assets (usually derivatives) to risky asset in order to create new portfolio, so-called hedging portfolio, hedged against a market risk.

In this paper we will discuss the most sophisticated instrument to hedge the market risk − options. Options and option strategies (the simultaneous combination of one or more option position) can offer advantages to protection from changes in price of various underlying asset (e.g. stocks, bonds, commodities, currencies, indices). Bull, bear, butterfly, condor, straddles, strangles, ladders, combo are some of the options strategies described in popular books including Cohen (2005), Carol (2008), Hull (2008), Chorafas (2008), Smith (2008), Mullaney (2009). We want to demonstrate that options, respectively option strategies can by a very important risk management tool. At present, there are only literature concerning on trading in options strategies using vanilla option, for example (Mugwagwa et al., 2008), (Santa-Clara and Saretto, 2009), (Dewobroto, 2010), (Fahlenbrach et al., 2010), (Chang et al., 2010), (Lazar and Lazar, 2011). To the best of our knowledge, no study has yet utilized barrier options to investigate option strategies and hedging using option strategies as well excepting our up to date written papers. In the context of a constant development of derivative products, new kinds of options are formed beside vanilla or else classical options. The whole group of these options are called exotic option. Barrier options are one of the most famous exotic options. They are options with a second strike price, called barrier. Crossing of the barrier level during the life of an option implies activation (knock-in barrier level) or deactivation (knock-out barrier level) of particular barrier option. The activation, respectively deactivation of a barrier option can be determined by a higher/lower barrier than an underlying spot price at time of contract conclusion (up barrier level) or vice versa (down barrier level). For example Briys et al. (1998), Zhang (1998), Weert (2008) explain barrier options more detail.

The aim of this paper is to analyse the Short Combo strategy using barrier options and proposed its theoretical application in hedging against a price increase of the underlying. Our theoretical analysis will be useful for financial and non-financial institutions. The proposed hedging possibilities can be used as a model cases in practical investment. The practical application in hedging of the real underlying asset SPDR Gold Shares is also designed to demonstrate the benefit of our findings.

Methodology of the theoretical analysis

In our analysis we use an interesting method based on finding of the income function. This approach was used by authors in the analysis of hedging using classical options. For example, there are studies (Amaitiek et al., 2010), (M. Šoltés, 2010a), (V. Šoltés and Amaitiek, 2010). Recently, the authors also published the papers dealing with hedging against a price decrease using barrier option.

Following the mentioned studies we analyse Short Combo strategy using barrier options and proposed its application in hedging. Firstly, we derive the income functions for barrier option positions. These functions simplify the application in hedging. Furthermore, we select the suitable positions for hedging. We use these positions in deriving of the income functions from secured position. Followed, these functions are used in the practical application to SPDR Gold Shares.

Short Combo formation using barrier options

The Short Combo strategy is formed by selling n put options with a lower strike price X_1, premium $p^0_{1SELL(S)}$ per option and at the same time by buying n call options with a higher strike price X_2, premium $c^0_{2BUY(B)}$ per option. Put and call options are on the same underlying and they have the same expiration time T.

As we have mentioned earlier, the barrier option can be type knock-in or knock-out, down or up. Up and knock-In (UI) call/put option is activated if an underlying price during a life of an option increases above upper barrier U or only touches it. Down and knock-In (DI) call/put option is activated if an underlying price during the life of an option decreases below lower barrier D or only touches it. Up and knock-Out (UO) call/put option is deactivated if

an underlying price during a life of an option increases above upper barrier. Down and knock-Out (DO) call/put option is deactivated if an underlying price during the life of an option decreases below lower barrier.
It is evident that there are 16 possibilities of Long Combo strategy formation using barrier options. In detail, we present a construction of Short Combo strategy by selling of *n* down and knock-in put options with a lower strike price X_1, premium p^0_{1SDI} per option, barrier level *D* and at the same time by buying *n* up and knock-in call options with a higher strike price X_2, premium c^0_{2BUI} per option, barrier level *U*.
Selling of a down and knock-in put option is an obligation to buy a particular underlying asset for a strike price X_1 at expiration time *T* if an option is activated, i.e. the underlying price during the option live *t* does not exceed the barrier *D*. The following formula represents the fulfilment of this condition:

$$\min_{0\le t\le T}(S_t) \le D. \tag{1}$$

Conversely, a knock-out option is deactivated if this condition is fulfilled. Once the option is activated or deactivated it becomes a classical option. Down and knock-in/out (up and knock-in/out) option has barrier level below (above) the underlying spot price S_0 at time of contract conclusion. Following the study (Ye, 2009) we assume $D < X$, because otherwise DI/DO put option is equivalent to a correspondent vanilla put. The same assumption is valid for DI/DO call option. For UI/UO call/put option we suppose $X < U$. The seller (writer) of the barrier option receives from the buyer the option premium.
The profit function from selling *n* down and knock-in put option has the following form:

$$P(S_T) = \begin{cases} n(S_T - X_1 + p_{1SDI}) & if \ \min_{0\le t\le T}(S_t) \le D \wedge S_T < X_1, \\ np_{1SDI} & if \ \min_{0\le t\le T}(S_t) > D \wedge S_T < X_1, \\ np_{1SDI} & if \ S_T \ge X_1, \end{cases} \tag{2}$$

where is the option premium at expiration time.
Buying of an up and knock-in call option is the right to buy the particular underlying asset for the strike price at the expiration time *T* if an option is activated, i.e. the following condition is fulfilled:

$$\max_{0\le t\le T}(S_t) \ge U. \tag{3}$$

The profit function from buying *n* up and knock-in call options is:

$$P(S_T) = \begin{cases} -nc_{2BUI} & if \ S_T < X_2, \\ n(S_T - X_2 - c_{2BUI}) & if \ \max_{0\le t\le T}(S_t) \ge U \wedge S_T \ge X_2, \\ -nc_{2BUI} & if \ \max_{0\le t\le T}(S_t) < U \wedge S_T \ge X_2. \end{cases} \tag{4}$$

The profit function from the Short Combo strategy is the sum of the individual profit functions (2) and (4). The Short Combo strategy profit function is expressed by the equation:

$$P(S_T) = \begin{cases} n(S_T - X_1 + p_{1SDI} - c_{2BUI}) & if \ \min_{0\le t\le T}(S_t) \le D \wedge S_T < X_1, \\ n(p_{1SDI} - c_{2BUI}) & if \ \min_{0\le t\le T}(S_t) > D \wedge S_T < X_1, \\ n(p_{1SDI} - c_{2BUI}) & if \ X_1 \le S_T < X_2, \\ n(S_T - X_2 + p_{1SDI} - c_{2BUI}) & if \ \max_{0\le t\le T}(S_t) \ge U \wedge S_T \ge X_2, \\ n(p_{1SDI} - c_{2BUI}) & if \ \max_{0\le t\le T}(S_t) < U \wedge S_T \ge X_2. \end{cases} \tag{5}$$

If the following condition is fulfilled:

$$p^0_{1SDI} - c^0_{2BUI} \ge 0, \tag{6}$$

the Short Combo strategy is zero cost strategy.

The profit functions from other possibilities of Short Combo strategy construction have different barrier conditions. In generally, the profit function has the form:

$$P(S_T)=\begin{cases} n(S_T-X_1+p_{1S}-c_{2B}) & if\ it\ it\ fulfilled\ condition\ 1\ \wedge\ S_T<X_1, \\ n(p_{1S}-c_{2B}) & if\ it\ it\ fulfilled\ condition\ 2\wedge S_T<X_1, \\ n(p_{1S}-c_{2B}) & if\ X_1\le S_T<X_2, \\ n(S_T-X_2+p_{1B}-c_{2B}) & if\ it\ it\ fulfilled\ condition\ 3\wedge S_T\ge X_2, \\ n(p_{1S}-c_{2B}) & if\ it\ it\ fulfilled\ condition\ 4\wedge S_T\ge X_2. \end{cases} \quad (7)$$

Corresponding barrier conditions for the Short Combo strategy possibilities using barrier options are in Table 1. Substituting the barrier in the profit function (7) we get the profit function of the particular possibility of this strategy construction.

Table 1: Barrier conditions for the individual possibilities of Short Combo strategy formation

	The possibility of the strategy formation	Barrier condition 1	Barrier condition 2	Barrier condition 3	Barrier condition 4
1.	selling DI put option and buying UI call option	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$
2.	selling DO put option and buying UI call option	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$
3.	selling UI put option and buying UI call option	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$
4.	selling UO put option and buying UI call option	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$
5.	selling DI put option and buying UO call option	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$
6.	selling DO put option and buying UO call option	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$
7.	selling UI put option and buying UO call option	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$
8.	selling UO put option and buying UO call option	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$
9.	selling DI put option and buying DI call option	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$
10.	selling DO put option and buying DI call option	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$
11.	selling UI put option and buying DI call option	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$
12.	selling UO put option and buying DI call option	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$
13.	selling DI put option and buying DO call option	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$
14.	selling DO put option and buying DO call option	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$
15.	selling UI put option and buying DO call option	$\max_{0\le t\le T}(S_t)\ge U$	$\max_{0\le t\le T}(S_t)<U$	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$
16.	selling UO put option and buying DO call option	$\max_{0\le t\le T}(S_t)<U$	$\max_{0\le t\le T}(S_t)\ge U$	$\min_{0\le t\le T}(S_t)>D$	$\min_{0\le t\le T}(S_t)\le D$

In the next section we analyze the possibilities of Short Combo strategy formation using barrier options suitable for hedging against a price increase.

Hedging analysis

Let us suppose that at time *T* in the future we will buy a portfolio consisting of *n* pieces of the underlying asset, but we are afraid of its price increase. Profit function from unsecured position (UP) in the portfolio at time *T* is

$$UP(S_T)=-\,n\,S_T, \quad (8)$$

Let us suppose that we have decided to hedge the maximum acceptable buying price of some underlying asset at time *T* using the Short Combo strategy formed by European type options with expiration at time of hedging. Hedging process does not eliminate the amount of loss completely, but it ensures the maximum acceptable loss.

The secured position profit by the Short Combo strategy using classical options is known. It has the following form:

$$SP(S_T) = \begin{cases} -n(X_1 - p_{1S} + c_{2B}) & if\ S_T < X_1, \\ -n(S_T - p_{1S} + c_{2B}) & if\ X_1 \leq S_T < X_2, \\ -n(X_2 - p_{1S} + c_{2B}) & if\ S_T \geq X_2, \end{cases} \tag{9}$$

Based on the analysis of all possibilities of this strategy formation using barrier options we can conclude that only four possibilities secure a maximum purchasing price for every possible future price scenarios.

I. Let us hedge using Short Combo strategy formed by selling n of down and knock-in put options with a lower strike price X_1, premium p_{1SDI}^0 per option, barrier level D and at the same time by buying n of up and knock-in call options with a higher strike price X_2, premium c_{2BUI}^0 per option, barrier level U $(D < X_1 < X_2 < U)$. Options are on the same underlying asset and their expiration time is equal to the time of hedging.
We get the income function from secured position as a sum of the profit function from Short Combo strategy (5) and the income function from unsecured position in the portfolio (8). The income function is:

$$SP_I(S_T) = \begin{cases} -n(X_1 - p_{1SDI} + c_{2BUI}) & if\ \min\limits_{0\leq t\leq T}(S_t) \leq D \wedge S_T < X_1, \\ -n(S_T - p_{1SDI} + c_{2BUI}) & if\ \min\limits_{0\leq t\leq T}(S_t) > D \wedge S_T < X_1, \\ -n(S_T - p_{1SDI} + c_{2BUI}) & if\ X_1 \leq S_T < X_2, \\ -n(X_2 - p_{1SDI} + c_{2BUI}) & if\ \max\limits_{0\leq t\leq T}(S_t) \geq U \wedge S_T \geq X_2, \\ -n(S_T - p_{1SDI} + c_{2BUI}) & if\ \max\limits_{0\leq t\leq T}(S_t) < U \wedge S_T \geq X_2. \end{cases} \tag{10}$$

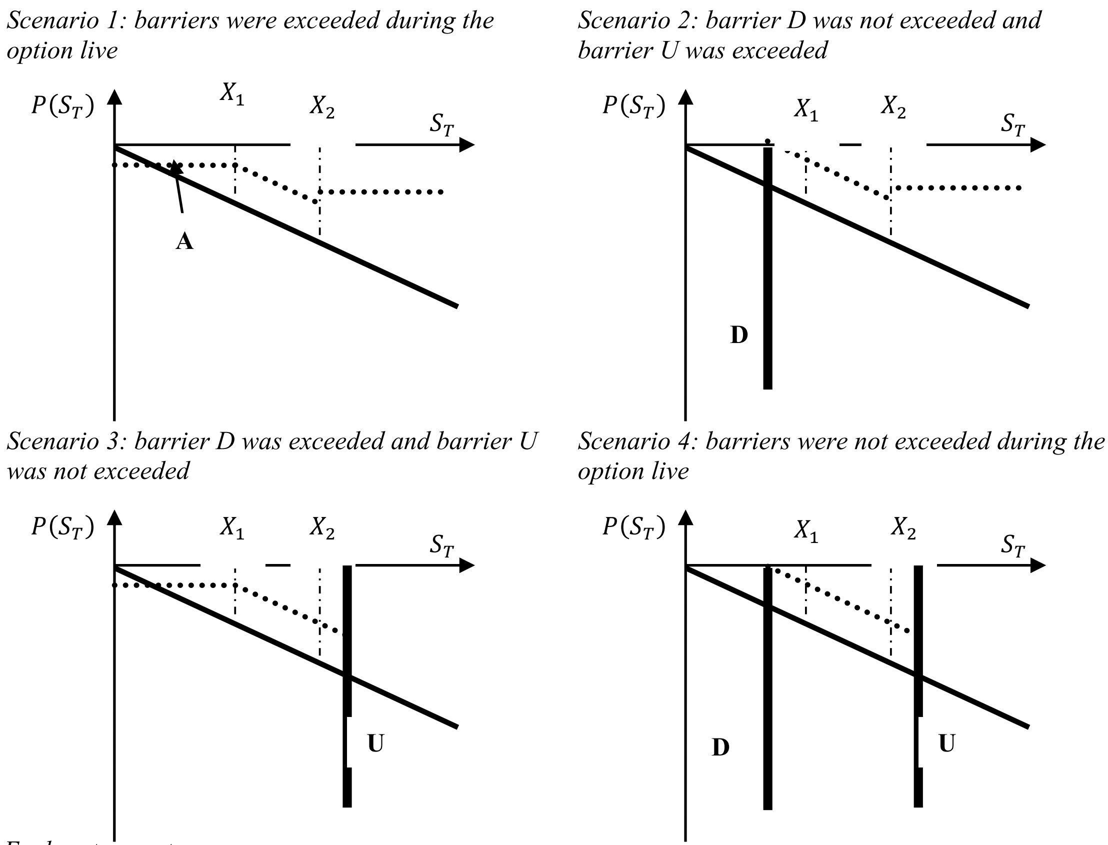
It is true that call/put vanilla option premium is the sum of DI/UI call/put barrier option premium and DO/UO call/put barrier option premium.

By comparing the secured positions (9) and (10) we have formulated the following statements:

- If the price of the underlying falls below *D* and grows above *U* during the option life, then profit of hedging is similar to profit of hedging using classical options with same strike prices, expiration time and underlying asset. We have hedged the price from the interval $\langle X_1, X_2 \rangle$. We have hedged the maximum purchasing price. On the other hand, we cannot participate in the price decrease under X_1.
- If the price of the underlying does not decrease below *D* and at the expiration time is below X_1, then we participate in the price decrease. The minimum price is bounded by the barrier *D*. The option premium receives from the selling down and knock-in put option is lower than the option premium from the selling corresponding classical put option with the same parameters.
- If the price of the underlying does not increase above *U* and at the expiration time is above X_2, then we have hedged the maximum price in the amount of upper barrier. The reason is lower price paid for buying of call barrier option in the comparison to the classical option price with the same parameters.
- In the case of this hedging possibilities we have hedged the price from the interval $\langle D, U \rangle$.

The Figure 1 shows the income function of unsecured position (8) and the income function from secured position using the Short Combo strategy (10) meeting the condition $p_{1SDI}^0 \geq c_{2BUI}^0$ at the expiration time for the possible future price scenarios during expiration time.

We see that this hedging strategy is inappropriate for the end-users who expect exceeding the barrier level *D* during the expiration time and the price at expiration time less than the price *A*.



Explanatory notes:

▬ *Unsecured position*

••••• *Secured position by Short Combo strategy using barrier options*

$A = X_1 - p_{1SDI} + c_{2BUI}$

Fig 1: Graphs of the income functions from unsecured position and secured position by Short Combo strategy using barrier options

II. Let us create this option strategy by selling n down and knock-out put options with a lower strike price X_1, premium p^0_{1SDO} per option, barrier D and at the same time by buying n up and knock-in call options with a higher strike price X_2, premium c^0_{2BUI} per option, barrier $U (D < X_1 < X_2 < U)$.

The income function from secured position is:

$$ZP_{II}(S_T) = \begin{cases} -n(X_1 - p_{1SDO} + c_{2BUI}) & if \ \min\limits_{0 \le t \le T}(S_t) > D \wedge S_T < X_1, \\ -n(S_T - p_{1SDO} + c_{2BUI}) & if \ \min\limits_{0 \le t \le T}(S_t) \le D \wedge S_T < X_1, \\ -n(S_T - p_{1SDO} + c_{2BUI}) & if \ X_1 \le S_T < X_2, \\ -n(X_2 - p_{1SDO} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) \ge U \wedge S_T \ge X_2, \\ -n(S_T - p_{1SDO} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) < U \wedge S_T \ge X_2. \end{cases} \quad (11)$$

By analyzing the function (11) we have concluded the following:

- The maximum buying price of the underlying is hedged by the strike price X_2 or the barrier *U*.

- The hedger can participate in the price decrease underX_1 if the price exceeds the barrier level *D* during the option life.
- On the other hand, if the price does not exceed the barrier level *D* during the option life, than the hedger hedges the strike price X_1.
- In the case of this hedging possibilities the hedger have hedged the price from the interval $\langle 0,\ U \rangle$.

III. Let us form the Short Combo strategy by selling n up and knock-in put options and buying n up and knock-in call options with the same barrier level U $(X_1 < X_2 < U)$.
The income function from secured position in this case is expressed by function:

$$ZP_{III}(S_T) = \begin{cases} -n(X_1 - p_{1SUI} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) \ge U \wedge S_T < X_1, \\ -n(S_T - p_{1SUI} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) < U \wedge S_T < X_1, \\ -n(S_T - p_{1SUI} + c_{2BUI}) & if \ X_1 \le S_T < X_2, \\ -n(X_2 - p_{1SUI} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) \ge U \wedge S_T \ge X_2, \\ -n(S_T - p_{1SUI} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) < U \wedge S_T \ge X_2. \end{cases} \quad (12)$$

Analysis results:

- If the barrier level is exceeded, the hedger hedges the minimum and maximum price.
- The maximum possible price is in the amount of the barrier *U*.
- If the barrier level is exceeded, the hedger can participate in the price decrease.
- The hedger have hedged the price from the interval $\langle 0,\ U \rangle$.

IV. Finally, we hedge by selling n up and knock-out put options and buying n up and knock-in call options with the same barrier level U $(X_1 < X_2 < U)$.
The income function from secured position is expressed by the following equation:

$$ZP_{IV}(S_T) = \begin{cases} -n(X_1 - p_{1SUO} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) < U \wedge S_T < X_1, \\ -n(S_T - p_{1SUO} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) \ge U \wedge S_T < X_1, \\ -n(S_T - p_{1SUO} + c_{2BUI}) & if \ X_1 \le S_T < X_2, \\ -n(X_2 - p_{1SUO} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) \ge U \wedge S_T \ge X_2, \\ -n(S_T - p_{1SUO} + c_{2BUI}) & if \ \max\limits_{0 \le t \le T}(S_t) < U \wedge S_T \ge X_2. \end{cases} \quad (13)$$

By analogy, we have formulated the statements:

- If the barrier level is exceeded, the hedger hedges the maximum price and can participate in the price decrease.
- If the barrier level is not exceeded, the hedger hedges the minimum price X_1 and the maximum price *U*.
- The hedger has hedged the price from the interval$\langle 0,\ U \rangle$.

The aim of the hedging transactions against an underlying price increase is hedged the purchasing maximum price. Other hedging possibilities formed by barrier options have also unprotected scenarios, i.e. scenarios without hedging the maximum price. Therefore we recommend the hedging possibilities described above, the remaining possibilities are the combination of hedging and speculation.

Methodology of the practical analysis and data

We use the obtained theoretical results in the application to SPDR Gold Shares. SPDR Gold Shares offer investors an innovative, relatively cost efficient and secure way to access the gold market without being necessary to take care of delivery and safekeeping. GLD are an appropriate tool for those who want "to play" in the gold market, not for those who want to buy real gold. It is possible to use them for hedging, forming of option strategies etc. For these reasons they are very popular and SPDR Gold Trust is currently one of the largest holders of the gold in the world.

SPDR Gold shares reached a value of USD 155-175 in year 2012. The share value was approximately USD 160 in January 2013. The value of these shares has dropped by almost 26% since January 2013. Now, at December 17, 2013 the share price is USD 118.98. We expect exceeding the price of USD 150 till January 2015.

The objective of this section is to hedge the portfolio of SPDR Gold shares against a price growth to the January 17, 2015. We are going to show which parameters the hedger should pay attention to, when deciding to use a given hedging strategy. We propose hedging variants and evaluate their profitability with respect to the income of unsecured portfolio for particular intervals of underlying spot price at the time of expiration. In the end, we realize the comparative analysis of the proposed variants.

We look at vanilla and standard barrier European options on the SPDR Gold Shares with various strike prices and barrier levels. In the case of vanilla options we use real data (source: www.finance.yahoo.com and www.morningstar.com).
Due to the lack of the real-traded barrier option data the barrier option premiums are calculated. We use the most popular method for option pricing − the Black-Scholes model (Black and Scholes, 1973). The classic version of this model is not designed for barrier options. By its modification Merton (1973) derived the first analytical formula for a down and out call European type option. Later Rubinstein and Reiner (1991) provided the formulas for eight types of barrier options. Haug (1998) gave the formulas for all types of European single barrier options. Barrier options can also be priced via lattice tree (the binomial model was first proposed by Cox et al. (1979)), Monte Carlo simulation for example (Ross and Ghamami, 2010) and others.
We will consider analytical closed formulas under the Black-Scholes-Merton framework provided by Haug. To simplify the calculations of particular barrier option premiums we use the statistical program R.
The mentioned model for shares without paying dividend is based on the following parameters: type of option (DI/DO/UI/UO CALL/PUT), actual underlying spot price, strike price (selected according to strike prices of vanilla options), expiration time (according to European standard 30E/360), barrier level, risk-free interest rate (US Government bond yield (source: www.bloomberg.com), cost of carry rate, Black-Scholes implied volatility.
The dataset consists of 30 vanilla call and put option premiums, 130 DI and DO put barrier option premiums, 130 UI and UO put barrier option premiums and 110 UI and UO call barrier option premiums. Strike prices are in the range of 90-150, barrier levels of DI/DO options are in the range of 60-110 and barrier levels of UI/UO options are in the range of 120-170 (all in the multiples of 5). All data used in our analysis can be provided upon request.

Application to hedging of SPDR Gold Shares

Suppose that we will buy 100 SPDR Gold shares at January 2015 but we are afraid of the price growth. The actual spot price of these shares at December 17, 2013 is USD 118.98. The hedging instrument will be Short Combo strategy formed by options with expiration in January 2015.

Assume following requirements and expectations. We want to hedge against more than USD 150 growth. At the same time, we consider a drop below the value of 90 improbable. We will propose the zero-cost hedging variants, which meet the above stated requirements.
First hedging variant is formed by selling $n=100$ DI put options with the strike price $X_1=110$, the barrier $D=90$ and the premium $p_{1SDI}=5.13$ per option and at the same time, by buying $n=100$ UI call options with the strike price $X_2=145$, the barrier $U=150$ and the premium $c_{2BUI}=2.93$ per option.

The income function from secured position has the form:

$$SP_1(S_T)=\begin{cases}-10780 & if \min\limits_{0\le t\le T}(S_t)\le 90 \wedge S_T<110,\\ -100(S_T-2.2) & if \min\limits_{0\le t\le T}(S_t)>90 \wedge S_T<110,\\ -100(S_T-2.2) & if\ 110\le S_T<135,\\ -14280 & if \max\limits_{0\le t\le T}(S_t)\ge 150 \wedge S_T\ge 145,\\ -100(S_T-2.2) & if \max\limits_{0\le t\le T}(S_t)<150 \wedge S_T\ge 145.\end{cases} \tag{14}$$

This hedging variant ensured a maximum expense from buying 100 SPDR Gold shares at the amount of USD 14 280 in the case of the upper barrier exceeding. If the upper barrier is not exceeded, the maximum expense can be USD 14 780.

The comparison of the hedging variant 1 and other proposed hedging variants constructed by selling DI put options with the barrier *D=90* and buying UI call options with the barrier *U=150* both with modified strike prices at various SPDR Gold price scenarios is shown in Fig 2.

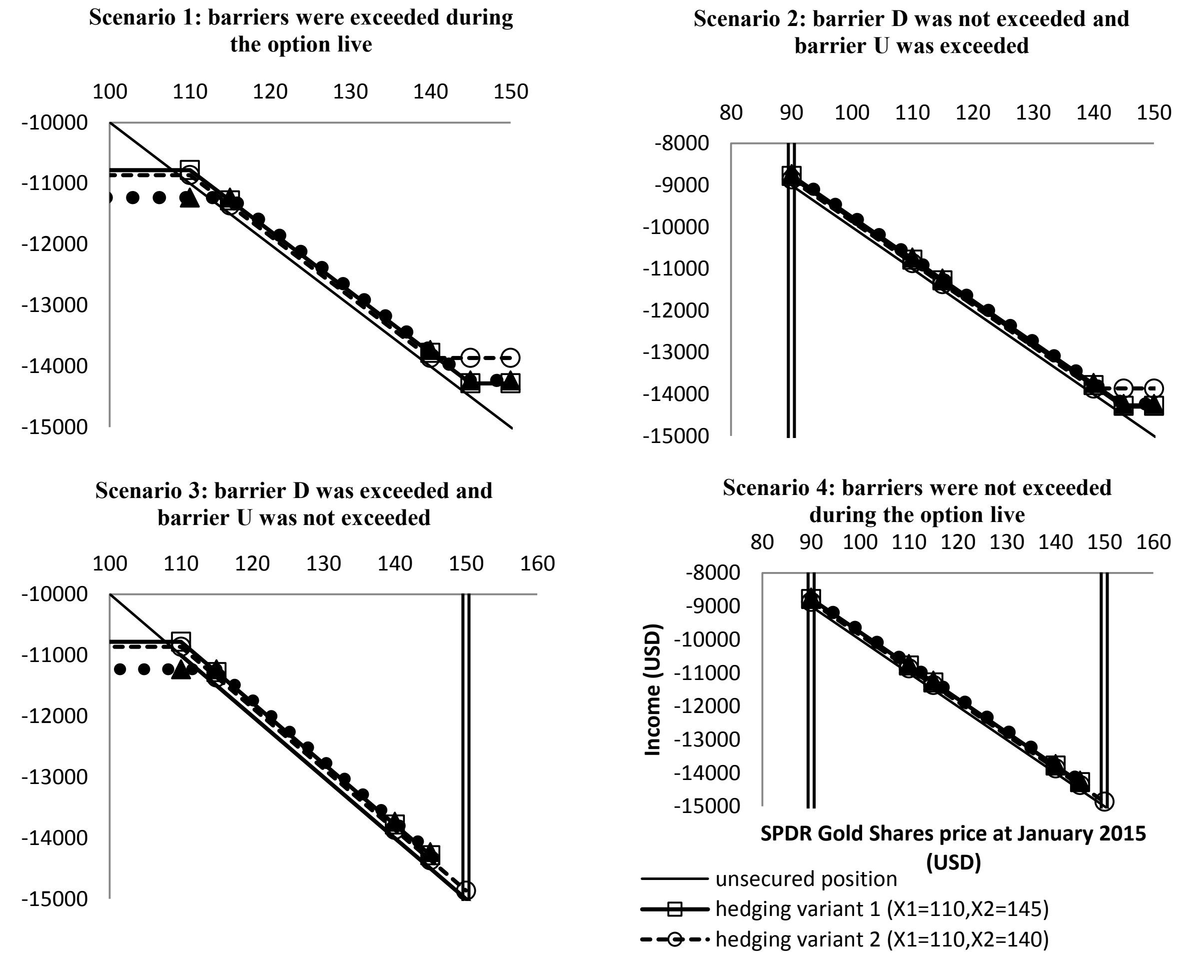


Fig 2: Graphs of 1, 2 and 3 hedging variant income functions

It can be seen, but it also can be calculated using data that the hedging variant 1 ensures the higher income if the spot price of shares is lower than USD 114.52 at expiration time and at the same time the barrier level 90 was exceeded during the maturity. If the spot price is higher than USD 141.31 and at the same time the barrier level 150 was exceeded during the maturity, then the better results is obtained by the hedging variant 2. Otherwise, the hedging variant 3 is the best. It should be noted, the lower strike price X_2 of call option, the higher income in the case of significant higher price at expiration time. The higher strike price X_1 of put options, the higher income in the case of lower price of these shares.

Let us suppose that the significant price increase at the maturity date is most expected. At expected price development and for mentioned assumptions the hedging variant 2 ensures the highest income. Therefore, we will analyse this particular variant in the next section. We will compare this variant with different potential hedging variants.

Fourth hedging variant is formed by selling 100 UO put options with the strike price 135, the barrier 150 and the premium 19.94 per option and at the same time, by buying 100 UI call options with the strike price 140, the barrier 150 and the premium 3.76 per option.

Using the function (13) we obtain the income function of this hedging variant:

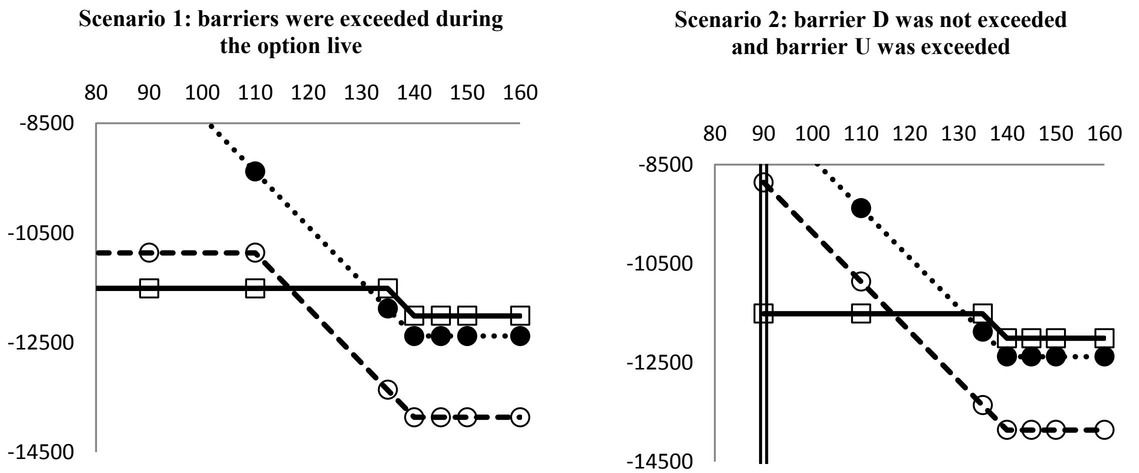
$$SP_4(S_T) = \begin{cases} -11882 & if \ \max\limits_{0\le t\le T}(S_t) < 150 \wedge S_T < 135, \\ -100(S_T - 16.18) & if \ \max\limits_{0\le t\le T}(S_t) \ge 150 \wedge S_T < 135, \\ -100(S_T - 16.18) & if \ 135 \le S_T < 140, \\ -12382 & if \ \max\limits_{0\le t\le T}(S_t) \ge 150 \wedge S_T \ge 140, \\ -100(S_T - 16.18) & if \ \max\limits_{0\le t\le T}(S_t) < 150 \wedge S_T \ge 140. \end{cases} \tag{15}$$

By analogy, we can easily derived the income function of the hedging variant 5 formed by selling 100 vanilla put options with the strike price 135 and by buying 100 vanilla call options with the strike price 140.

Using the function (9) we derive the income function of the hedging variant 5:

$$SP_5(S_T) = \begin{cases} -11512 & if \ S_T < 135, \\ -100(S_T - 19.88) & if \ 135 \le S_T < 140, \\ -12012 & if \ S_T \ge 140. \end{cases} \tag{16}$$

The comparison of hedging variants 2, 4 and 5 is shown in Fig 3.



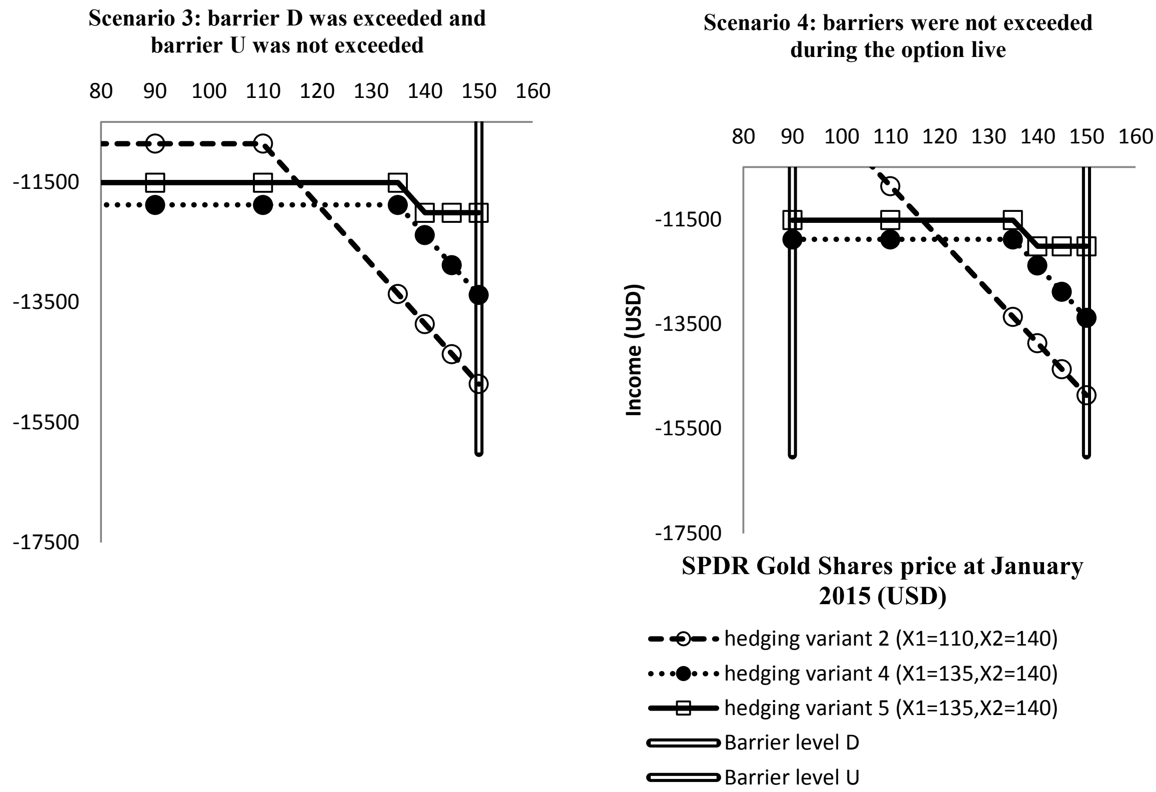


Fig 3: Graphs of 2, 4 and 5 hedging variant income functions

It can be concluded that the variants 4 and 5 are the best hedging variants occurring the significant price increase, i.e. higher than 150.

The variant 5 ensured the highest income if the spot price of shares at expiration time is higher than USD 131.3. Assuming the assumptions mentioned earlier, we recommend this variant to use in hedging.

Now we will analyze hedging variants 1-5 and unsecured variant (UV) providing a comparison of all possible scenarios.

Further, we will select the best variant in terms of expense for particular intervals at time T and barrier conditions during time T. We will also calculate a minimum and maximum expense for the best variants.

Results of the comparative analysis are in Table 2.

The comparative analysis had not shown the best results. The selection of appropriate variant must be made depending on the investor expectations. At the same time, it confirmed that the Short Combo strategy using barrier options gives investors a greater flexibility to express a precise view.

The Short Combo strategy formation using barrier options was better than this strategy formation using classical options in specific situation but not in every practical situation.

The results showed that the Short Combo strategy formation using classical options is cheaper anticipating significant price growth in all scenarios. Therefore we recommended using the hedging variant formed by classical option in the case of low probability of other scenarios occurring.

Table 2: Comparative analysis of hedging variants 1-5 and unsecured variant

Spot price intervals at time T	Best hedging variant	Minimum expense	Maximum expense
Scenario 1 $\min_{0\le t\le T}(S_t) \le 90 \wedge \max_{0\le t\le T}(S_t) \ge 150$			
$0 \le S \le 131.3$	4	0	-11512
$131.3 \le S \le 135$	5	-11512	-11512
$135 \le S \le 140$	5	-11512	-12012
$S \ge 140$	5	-12012	-12012
Scenario 2 $\min_{0\le t\le T}(S_t) > 90 \wedge \max_{0\le t\le T}(S_t) \ge 150$			
Spot price intervals at time T	Best hedging variant	Minimum expense	Maximum expense
$90 \le S \le 131.3$	4	-7382	-11512
$131.3 \le S \le 135$	5	-11512	-11512
$135 \le S \le 140$	5	-11512	-12012
$S \ge 140$	5	-12012	-12012
Scenario 3 $\min_{0\le t\le T}(S_t) \le 90 \wedge \max_{0\le t\le T}(S_t) < 150$			
Spot price intervals at time T	Best hedging variant	Minimum expense	Maximum expense
$0 \le S \le 107.8$	UV	0	-10780
$107.8 \le S \le 110$	1	-10780	-10780
$110 \le S \le 114.52$	1	-10780	-11232
$114.52 \le S \le 115$	3	-11232	-11232
$115 \le S \le 117.8$	3	-12232	-11512
$117.8 \le S \le 135$	5	-11512	-11512
$135 \le S \le 140$	5	-11512	-12012
$S \ge 140$	5	-12012	-12012
Scenario 4 $\min_{0\le t\le T}(S_t) > 90 \wedge \max_{0\le t\le T}(S_t) < 150$			
Spot price intervals at time T	Best hedging variant	Minimum expense	Maximum expense
$90 \le S \le 117.8$	3	-8732	-11512
$117.8 \le S \le 135$	5	-11512	-11512
$135 \le S \le 140$	5	-11512	-12012
$S \ge 140$	5	-12012	-12012

We could see that the unsecured variant ensures the lower expense from buying the SPDR Gold shares compared to the proposed zero-cost hedging variants in one practical situation, i.e. in the case of significant price decrease.

Conclusions

This paper investigated hedging of a portfolio consisting of a risky underlying asset using Short Combo strategy to study the difference between hedging using barrier and vanilla options. The Short Combo strategy is useful for hedging against a price growth assuming the underlying asset will be bought in the future. To the best of our knowledge, no study has yet provided hedging analysis using option strategies formed by barrier options. This work therefore contributed to the literature by filling this gap including. In the analysis we used the unknown approach based on finding income functions which simplify the comparative analysis of hedging variants.
We focused on the Short Combo strategy formation using barrier options and the application of this strategy in hedging including practical application in hedging of SPDR Gold shares. We used SPDR Gold Shares prices and vanilla and barrier option prices on these shares. Barrier options data was calculated using the analytical model of Haugh in the statistical program R.
It is not possible to explicitly conclude that one of the described hedging variant is the best in every practical situation. It depends on the real spot price of the underlying asset at the particular future time and the price development during this time. The selection of appropriate hedging variant must be made by the investor depending on his preferences and expectations.
Our results indicated that hedging using barrier options expands hedging opportunities. Thereby it offers more alternatives for price hedging. It allows securing more unfavourable future price movement scenarios, i.e. it allows adaptation to hedger's specific individual requirements, which reduces costs of hedging. On the other hand, there were price scenarios for which using hedging variant formed by classical options was more preferably. The findings also indicated that the selection of strike prices, lower and upper barriers is extremely significant for the profit profile.

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