PARALLEL ALGORITHMS FOR SOLVING LINEAR EQUATIONS USING GIVENS TRANSFORMATIONS

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Abstract—The use of the Givens method to solve linear equations on a parallel computer is reviewed, and a new alogrithm which requires fewer time steps in the infinite processor case is presented.

1. INTRODUCTORY REVIEW

This paper is a discussion of the use of Givens transformations to solve systems of linear equations on massively parallel computers. After a brief review of Givens transformations in this section, a new parallel algorithm is presented in Section 2, and compared with the existing algorithm of Sameh and Kuck [**1**].

A serial algorithm to solve an augmented matrix using a series of Givens transformations might typically proceed as follows:

(1) row 1 is used to successively rotate to zero all elements in column 1 below the diagonal;
(2) row 2 is used to successively zero all elements below the diagonal in column 2;
(3) this process is continued until the matrix is triangularized and can then be solved by back-substitution.

This algorithm is intrinsically serial because new rotations usually require the modified row generated by the previous rotation, so that only one can be done at a time. On serial computers, the Givens method is generally only competitive with conventional equation solvers when the coefficient matrix is sparse.

2. THE GIVENS METHOD IN THE PARALLEL SETTING

The Givens method is of interest in parallel computing because pivoting which can dominate parallel Gauss-Jordan and Gaussian elimination algorithms [**4**] is not required.

Theorem 3.1. Let λ_l^h be as described in Section 1, and the closed square $S_i \subset \mathcal{D}$ be as described after Lemma 2.2. Assume that

$$\min_{P \in P_h} \{p_h(P) - \lambda_l^h\} > 0 \tag{14}$$

and

$$p_h(P) > \frac{4}{h^2}\left(1 - \cos\frac{h\pi}{l}\right) \tag{15}$$

$\forall P$ at the nodes on S_l and all h small enough, $q > 0$. Then the unique positive solutions $w(x,y)$ in $\mathcal{D}$ and $v_h(P)$ in $\mathcal{D}_h$ satisfy

$$\|w_h(P) - v_h(P)\|_2 \to 0 \qquad as \qquad h \to 0^+, \quad P \in \mathcal{D}_h \cup \partial\mathcal{D}_h,$$

$$\hat{\mathbf{\Phi}}(x_n, \hat{\mathbf{y}}_n, h_n) = \sum_{i=1}^{s} \hat{b}_i \mathbf{g}_i,$$

$$\mathbf{\Phi}(x_n, \hat{\mathbf{y}}_n, h_n) = \sum_{i=1}^{s} b_i \mathbf{g}_i$$

and

$$\mathbf{g}_i = \mathbf{f}(x_n + c_i h_n, \hat{\mathbf{y}}_n + h_n \sum_{j=1}^{i-1} a_{ij} \mathbf{q}_j), \qquad i - 1, 2, \ldots, s \tag{2}$$

But standard typesetting conventions usually align equations only when they are not separated by text.)

In order to compare the rayleigh quotients for problems (18) and (19), we need to estimate $I_h(z,z)/A(M_h a, M_h z)$. We have

$$\begin{aligned} A(M_h z, M_h z) &= \int_D [(\nabla M_h a)^2 + Q(M_h z)^2]\, dx\, dy \\ &\geq \int_D (\nabla M_h z)^2\, dx\, dy + \min_D Q \int (M_h z)^2\, dx\, dy \\ &\geq \int_D (\nabla M_h z)^2\, dx\, dy + \min_D q \Big[\sum z(rh, sh)^2 h^2 \\ &\quad - \tfrac{1}{4} h^2 \sum \{ [z((r+1)h, sh) - z(rh, sh)]^2 \\ &\quad + [z(rh, (s+1)h) - z(rh, sh)]^2 \} \Big] \\ &= \int_D (\nabla M_h z)^2\, dx\, dy + \min_D Q \Big[\sum z^2 h^2 - \tfrac{1}{4} h^2 \int_D (\nabla M_h z)^2\, dx\, dy \Big] \\ &\geq \min_D Q \sum z(rh, sh)^2 h^2, \end{aligned} \tag{28}$$

for h sufficiently small. Here we have used equation (25), the triangle inequality and the identity (24) of Polya [7] below. From equation (28), we have the estimate

$$\left| \frac{I_h(z,z)}{A(M_h z, M_h z)} \right| \leq \frac{I_h(z,z)}{\left(\min_D Q\right) I_h(z,z)} = \frac{1}{\min_D Q} \tag{29}$$

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