A CFO Matrix Method for Interleaved Uplink OFDMA Systems

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Abstract

In this paper, a CFO matrix method is presented for carrier frequency offset (CFO) estimation in interleaved uplink orthogonal frequency division multiplexing access (OFDMA) system. The proposed method utilizes the information of eigenvalues and eigenvectors of the defined CFO matrix to estimate the CFOs of all involved users. Compared with the previous works, the main advantages of the proposed approach are that it can obtain the CFOs of all users simultaneously within only one OFDMA block without pilot symbols, provides better performance such as smaller estimation error and superior noise-resistant capability. Simulation results verify the high accuracy and effectiveness of the proposed method.

1. Introduction

Orthogonal frequency-division multiple access (OFDMA) has received considerable attention and has been proposed for the uplink of wireless communication system, such as 802.16 [1] and 802.22 [2]. Such systems, however, rely crucially on the carrier frequency synchronization. Since multiple user signals are mixed together in both time and frequency domains in the uplink, carrier frequency offsets (CFOs) between the user transmitters and the uplink receiver may destroy the orthogonality among subcarriers, which will introduce intercarrier interference (ICI) from the user itself and multiple access interference (MAI) from other users [3]. It is crucial to accurately estimate and compensate the CFO for reliable OFDMA transmission. In the uplink of OFDMA, the received signal at the uplink receiver is the superposition of the unsynchronized signals from all involved users. That is, different users have different CFOs. CFO estimation of such system, in this case, is a challenging multi-parameter estimation problem.
The CFO estimation algorithm for uplink OFDMA is closely related to the carrier assignment scheme (CAS). There are three types of CAS for OFDMA subcarrier allocation: subband-based, generalized, and interleaved [4]. CFO estimation algorithms for subband-based CAS have been proposed in [5]-[7]. In these techniques, a frequency-guard band between subbands is added to separate signals from different users by filter banks. Subband-based OFDMA systems, however, are vulnerable to frequency selective fading. CFO estimation with generalized CAS can be found in [8]-[10]. These algorithms estimate timing and frequency offset based on the training block or pilot symbols, which leads to decrease the spectrum efficiency. A typical blind CFO estimation algorithm for interleaved subcarrier allocation is developed in [11]. In this contribution, the CFO estimation problem is translated into the direction of arrival (DOA) problem, which exploits the periodic feature of the interleaved subcarriers. Due to the grid-search involved, its computational complexity is quite high. Meanwhile, in the case of low signal-to-noise (SNR), the peak values searched by MUSIC [12] technique often become ambiguous, which results severe performance degradation in the CFO estimation.

In this paper, we propose a new CFO estimator for the interleaved OFDMA uplink systems. The estimation of signal parameters can be derived by the CFO matrix which defines on the basis of the special matrix composed of the received signals. The proposed method based on the CFO matrix can obtain the CFOs of all users simultaneously within only one OFDMA block without pilot symbols, which increases the spectral efficiency and has better performance.

The rest of this paper is organized as follows. The signal model of the interleaved OFDMA uplink systems is presented in Section II. In Section III, the proposed CFO matrix method is introduced and the computational complexity is analyzed. Section IV describes the simulation results, and Section V presents the conclusion.

2.System Model for the OFDMA uplink

Consider a $K$-user OFDMA uplink system with $N$ sub-carriers. Suppose that the $N$ subcarriers are interleaved into $Q(Q > K)$ subchannels, each of which has $P = N/Q$ uniformly spaced subcarriers. The $q$ th subchannel consists of subcarriers with the index set \{q, Q + q, \ldots, (P-1)Q + q\}.$ For convenience, it is assumed that each user has only one subchannel, and the $P$ subcarriers of the $q$ th subchannel are assigned to the $k$ th user. After the cyclic prefix (CP) is removed, the received $N$ signal samples of the OFDMA block from the $k$ th user at the uplink receiver can be described as [11]

$$r^{(k)}(n) = \sum_{p=0}^{P-1} H^{(k)}_p X^{(k)}_p e^{j2\pi(pQ + q + \xi^{(k)})n} = e^{j2\pi\theta^{(k)}n} \sum_{p=0}^{P-1} H^{(k)}_p X^{(k)}_p e^{j2\pi pQn}$$

where \(n = 0, 1, \ldots, N-1\). $X^{(k)}_p$ is the data symbol of the $k$ th user, and $H^{(k)}_p$ denotes the channel frequency response on the $(q + pQ)$ th subcarrier during one OFDMA block of the $k$ th user. $\xi^{(k)} = \Delta f^{(k)}/\Delta f$ is the normalized CFO of the $k$ th user. $\Delta f$ denotes the subcarrier spacing and $\Delta f^{(k)}$ is the CFO between the $k$ th user and the uplink receiver. For practical purpose, the absolute value of $\Delta f^{(k)}$ is assumed to be less than half of OFDMA subcarrier spacing, i.e. $\xi^{(k)} \in (-0.5, 0.5)$.

It is necessary to notice the fact that

$$r^{(k)}(n + \mu P) = e^{j2\pi\mu\theta^{(k)}n} r^{(k)}(n)$$

where $\mu$ is an integer, $\theta^{(k)} = (q + \xi^{(k)})/Q$ is the effective CFO of the $k$ th user. (2) implies that the received signal set has a special periodic structure with every $P$ samples. Thus the $N$ received signal samples $\{r^{(k)}(n)\}_{n=0}^{N-1}$ can be arranged into the following $Q \times P$ matrix:
Furthermore, (3) can be expressed as

$$A^{(k)} = \nu^{(k)}(u^{(k)} \odot (b^{(k)} W))$$

where $\odot$ represents the Schur product of the matrix, $W$ is a $P \times P$ IFFT matrix, and $\nu^{(k)}$, $u^{(k)}$ and $b^{(k)}$ are defined as

$$\nu^{(k)} = [1 e^{j2\pi\delta^{(k)}} \ldots e^{j2\pi((Q-1)\delta^{(k)})}]^T, \quad u^{(k)} = [1 e^{j2\pi\delta^{(k)}/P} \ldots e^{j2\pi((P-1)\delta^{(k)})/P}],$$

$$b^{(k)} = [H_0^{(k)} X_0^{(k)} H_1^{(k)} X_1^{(k)} \ldots H_{P-1}^{(k)} X_{P-1}^{(k)}].$$

Here the superscript $(\cdot)^T$ denotes transpose.

We know that, at the uplink receiver, the signal of one OFDMA block is the superposition of signals from all $K$ involved users, i.e.

$$r(n) = \sum_{k=1}^{K} \left( \sum_{p=0}^{P-1} H_p^{(k)} X_p^{(k)} e^{j\frac{2\pi}{N}(p+q+\xi^{(k)})} \right)$$

The $N$ received samples $\{r(n)\}_{0}^{N-1}$ can be arranged into a $Q \times P$ matrix in the following manner:

$$A = \sum_{k=1}^{K} A^{(k)} = VS = V\{U \odot (BW)\}$$

where $S = U \odot (BW)$ and $W$ is the same as in (4), $V = [\nu^{(1)} \nu^{(2)} \ldots \nu^{(K)}]$ is a $Q \times K$ Vandermonde matrix, where

$$\nu^{(k)} = [1 e^{j2\pi\delta^{(k)}} \ldots e^{j2\pi((Q-1)\delta^{(k)})}]^T, \quad (k = 1, 2, \ldots, K).$$

$B$ and $U$ are $K \times P$ and $K \times P$ matrices, respectively, which are defined as

$$B = [\{b^{(1)}\}^T \{b^{(2)}\}^T \ldots \{b^{(K)}\}^T]^T, \quad U = [\{u^{(1)}\}^T \{u^{(2)}\}^T \ldots \{u^{(K)}\}^T]^T.$$ 

3. Proposed Algorithm

In this section, we estimate the CFOs of all involved users from the received signal samples of one OFDMA block based on (6). First, Let $Y$ denotes a received OFDMA signal matrix plus a noise matrix as follows:

$$Y = A + Z = VS + Z$$

where $Z$ is a $Q \times P$ additive white complex Gaussian noise (AWGN) matrix. Each element of $Z$ is a Gaussian random variable with zero mean and variance $\sigma^2$.

From (7), we construct two submatrices $Y_1$ and $Y_2$, which are composed of the first and the last $Q-1$ rows of matrix $Y$, respectively, i.e.

$$Y_1 = J_1 Y = J_1 VS + J_1 Z$$
$$Y_2 = J_2 Y = J_2 VS + J_2 Z$$

where $J_1 = [I_{Q-1}, 0]$, $J_2 = [0 I_{Q-1}]$, $\Phi = \text{diag}\{\varphi_1, \varphi_2, \ldots, \varphi_K\}$ with $\varphi_k = e^{j2\pi\delta^{(k)}}$ and $I_{Q-1}$ is the $(Q-1) \times (Q-1)$ identity matrix.

Let

$$R_1 = E\{Y_1 Y_1^H\} = V_1 R_1 V_1^H + \sigma_n^2 I = R_{11} + \sigma_n^2 I$$

$$R_2 = E\{Y_2 Y_2^H\} = V_2 R_2 V_2^H + \sigma_n^2 I = R_{22} + \sigma_n^2 I$$

where

$$\begin{bmatrix}
    r^{(k)}(0) & r^{(k)}(1) & \ldots & r^{(k)}(P-1) \\
    r^{(k)}(P) & r^{(k)}(P+1) & \ldots & r^{(k)}(2P-1) \\
    \vdots & \vdots & & \vdots \\
    r^{(k)}(N-P) & r^{(k)}(N-P+1) & \ldots & r^{(k)}(N-1)
\end{bmatrix}$$

(3)
and
\[ R_i = E\{Y_2^H Y_1\} = V_i \Phi R_2 V_1^H + \sigma_s^2 I_1 = R_{22} + \sigma_s^2 I_1 \]  
(9)
where \( E\{\cdot\} \) is the statistical average, \( R_2 = E\{S S^H\} \), \( V_i = J_i V \), \( R_i = V_i R_2 V_1^H \), \( R_{22} = V_i \Phi R_2 V_1^H \), and
\[ I_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \cdots & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}. \]

From the definition of \( J_i \), it is easy to know that \( V_i \) is composed of the first \((Q-1)\) rows of matrix \( V \). Defining the CFO matrix as
\[ R \triangleq R_{22} R_{i1}^{-1} \]  
(10)
where the superscript(-1) denotes matrix pseudo-inverse. Following [14], we have
\[ R V_i = V_i \Phi \]  
(11)

The equation (11) indicates that the diagonal elements of \( \Phi \) are the nonzero eigenvalues of \( R \) and the columns of \( V_i \) are the eigenvectors of \( R \). We note that both \( V_i \) and \( \Phi \) include the information of all the effective CFOs. If we find \( R \), the effective CFOs of all the \( K \) users can be estimated by either of the eigenvalues and the eigenvectors of \( R \). To improve the performance of the estimator, we choose the average value of the two estimations derived by the \( k \)th eigenvalue and the second element of the corresponding eigenvector of \( R \) as the effective CFO of the \( k \)th user, i.e.
\[ \theta(k) = \frac{1}{2} \left[ \frac{\text{angle}(\hat{\phi}_k)}{2\pi} + \frac{\text{angle}(\hat{v}_k)}{2\pi} \right], \]
where \( \hat{\phi}_k \) is the \( k \)th eigenvalue and \( \hat{v}_k \) is the second element of the corresponding eigenvector of \( R \).

Under the assumption \( \xi(k) \in (-0.5,0.5) \), we know that if a user is assigned the subchannel \( q \), the range of its effective CFO is \( ((q-0.5)/Q,(q+0.5)/Q) \). Because each user lies in its own range, one-to-one mapping is possible between \( \theta(k) \) and \( \xi(k) \). Then the normalized CFOs can be calculated by \( \xi(k) = Q\theta(k) - q \).

In summary, the CFOs of all the \( K \) users are estimated using the CFOs matrix method as shown in the following.

**Step 1.** Formulate the received signal samples into matrix form \( Y \), and get data matrices \( \hat{Y}_1 \) and \( \hat{Y}_2 \).

**Step 2.** Calculate \( \hat{R}_1 \) and \( \hat{R}_2 \) by \( \hat{Y}_1 \) and \( \hat{Y}_2 \).

**Step 3.** Compute the eigendecomposition of \( \hat{R}_1 \) by \( \hat{R}_1 \hat{E} = \hat{E} \Lambda \), where \( \Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{Q-1}\} \) with \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{K-1} = \cdots = \hat{\lambda}_{Q-1} = \sigma_n^2 \).

**Step 4.** Estimate \( \hat{R}_{11} \) by \( \hat{R}_1 = \hat{R}_1 - \sigma_n^2 I \) and estimate \( \hat{R}_{22} \) by \( \hat{R}_{22} = \hat{R}_2 - \sigma_n^2 I \).

**Step 5.** Calculate the pseudo-inverse of \( \hat{R}_{11} \), denoting \( \hat{R}_{11}^{-1} \), and the CFO matrix \( \hat{R} = \hat{R}_{22} \hat{R}_{11}^{-1} \).

**Step 6.** Solve for the eigensystem \( \hat{R} \hat{E} = \hat{E} \Gamma \), where \( \Gamma = \text{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_K\} \) with \( \gamma_i \) is the nonzero eigenvalue of \( \hat{R} \) and the columns of \( \hat{E} \) are the eigenvectors of \( \hat{R} \).

**Step 7.** Obtain \( \hat{\theta}(k) \) by \( \hat{\theta}(k) = \frac{1}{2} \left[ \frac{\text{angle}(\hat{\phi}_k)}{2\pi} + \frac{\text{angle}(\hat{v}_k)}{2\pi} \right], \) where \( \hat{\theta}(k) \) is the estimation of the effective CFO \( \theta(k) \) of the \( k \)th user, \( \hat{\phi}_k \) is the estimation of the \( k \)th eigenvalue and \( \hat{v}_k \) is the estimation of the second element of the corresponding eigenvector of \( \hat{R} \).
Step 8. Estimate the normalized CFO of each user by \( \tilde{\xi}^{(k)} = Q\tilde{\theta}^{(k)} - q, (k = 1, 2, \cdots, K) \).

4. Simulation Results

In this section, computer simulations are performed to evaluate the performances of the proposed CFO matrix method. Assume that the total number of subcarriers in the interleaved uplink OFDMA system is 512, which are divided into 16 subchannels, and each subchannel has 32 subcarriers. The length of the cycle prefix (CP) is 128. The normalized root mean square error (NRMSE) criterion is used to verify the estimation performance and is defined as

\[
\text{NRMSE} = \sqrt{\frac{1}{K \Pi} \sum_{\rho=1}^{\Pi} \sum_{k=1}^{K} (\tilde{\xi}^{(k)} - \xi_{\rho}^{(k)})^2}
\]

where \( K \) is the number of users, \( \Pi \) is the total number of Monte Carlo tests, and \( \tilde{\xi}^{(k)} \) is the estimation of \( \xi_{\rho}^{(k)} \). The subscript \( (\cdot)_{\rho} \) denotes the index of the Monte Carlo test. The normalized RMSE is computed after averaging all participating users for 500 independent Monte Carlo tests. The SNR of the \( k \)th user is written as

\[
\text{SNR}^{(k)} = E\{|r^{(k)}(n)|^2\}/\sigma^2, k = 1, 2, \cdots, K.
\]

Fig. 1 illustrates the performance comparison among the proposed CFO matrix method, the method [13] by ESPRIT method under multipath fading channel. The normalized RMSEs of the three methods for \( K = 4 \) are calculated. The CFO matrix method can provide better performance such as smaller estimation error and superior noise-resistant capability for CFO estimation.

Second, the normalized RMSEs of the proposed CFO matrix method are calculated for different number of users. As is shown in Fig. 2, the CFO method can work well for different number of users and the performance increases at the fixed SNR when the user number in one OFDMA block decreases.

Fig. 3 shows the performance improvement of the proposed algorithm using multiple OFDMA blocks when \( Q = 16 \) and \( K = 3 \). If a normalized RMSE is given, such as 0.02, the use of two OFDMA blocks results in a 2 dB gain over the case with one block, and the use of four blocks yields a 2 dB gain over the two block case.

![Fig. 1. Comparison of the NRMSE performance between the propose method and ESPRIT.](image-url)
5. Conclusion

In this paper, the challenging CFO problem in the interleaved uplink OFDMA system is investigated. The proposed algorithm is performed based on the CFO matrix, which includes the information of CFOs of all involved users. All CFOs can be estimated by eigenvalues or eigenvectors of the CFO matrix. In addition, to improve the precision, the proposed method selects the average value of the two estimations calculated by eigenvalues and the second element of corresponding eigenvectors of the CFO matrix as the effective CFO estimation. Compared with the previous works, the proposed method can obtain the CFOs of all users simultaneously within only one OFDMA block without pilot symbols and has superior performance such as smaller estimation error and superior noise-resistant capability. Simulation results verified the effectiveness of the proposed method. The proposed method shows its great potential for the interleaved uplink OFDMA system in the multipath fading channels.

Fig. 2. Comparison of the NRMSE performance of the proposed method when K = 2, 4, 6

Fig. 3. The NRMSE performance of multiple blocks

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