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Passivity and Passification of Fuzzy Systems with Time Delays

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Abstract—Takagi-Sugeno (T-S) fuzzy model provides an effective representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning applied to a set of linear input/output submodels. Recently, a number of authors studied the T-S fuzzy systems with time delays. In this paper, the passivity and feedback passification of T-S fuzzy systems with time delays are considered. Both delay-independent and delay-dependent results are presented, and the theoretical results are given in terms of linear matrix inequalities (LMIs). Numerical examples are given which illustrate the effectiveness of the theoretical results. © 2006 Elsevier Ltd. All rights reserved.

Keywords—Fuzzy systems, Time delay, Passivity, Passification, Linear matrix inequality (LMI).

1. INTRODUCTION

During the last decade, fuzzy systems based on the Takagi-Sugeno (T-S) model [1] have attracted great interests from scientific and engineering communities. The T-S fuzzy model provides an efficient method to represent complex nonlinear systems via fuzzy sets and fuzzy reasoning. It is well known that delays appear in many dynamical systems. Generally speaking, the dynamic behaviors of systems with delay are more complicated than that of systems without any delays.

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Recently, stability and stabilization of T-S fuzzy systems with time delays have been studied by a number of authors [2–9]. In [2,3], using the Lyapunov-Krasovskii approach and the Lyapunov-Razumikhin method, the authors studied the stability of delayed fuzzy systems. In [4] output feedback robust H_∞ control of uncertain fuzzy dynamic system with time-varying delays has been studied. In [5], the authors studied the global exponential stability of fuzzy systems with bounded uncertain delays by using the method of functional differential inequalities analysis. In [6], delay-dependent results of guaranteed cost control for fuzzy delayed systems were presented. In [7], robust stability of uncertain stochastic fuzzy systems with time delays was studied. In [8], delay-dependent result for uncertain delayed fuzzy systems was presented. In [9], we also studied the stability of fuzzy large-scale systems with time delays.

On the other hand, the passivity theory intimately related to the circuit analysis methods [10] has received a lot of attention from the control community since 1970s (see, for example [11–14]). The passivity theory provide a nice tool for analyzing the stability of systems, and has found applications in diverse areas such as stability, complexity, signal processing, and chaos control and synchronization. In recent years, the passivity and passification of linear delayed systems were also studied in literatures [15–17]. In [18], we studied the passivity of delayed neural networks. Although there exists a lot of results in linear and nonlinear systems, as well as time delay systems, the passivity of fuzzy systems has not been fully investigated. In [19,20], the author studied Mamdani type fuzzy control system by using passivity approach. In [21], we studied the passivity and state feedback passification of T-S fuzzy systems with parameter uncertainties.

In this paper, we study the passivity and feedback passification of T-S fuzzy systems with time delays. Both delay-independent and delay-dependent results are presented, and the results are given in terms of linear matrix inequalities (LMIs) [22]. The rest of this paper is organized as follows. In Section 2, the problem to be studied is stated and some preliminaries are presented. The delay-independent passivity condition and state feedback passification of delayed fuzzy systems are presented in Section 3. In Section 4, a delay-dependent passivity condition is presented. In Section 5, numerical examples are given to demonstrate the effectiveness of the theoretical results. And finally, conclusions are drawn in Section 6.

Notations. Throughout this paper, $W^\top$ and W^{-1} denote, respectively, the transpose of, and inverse of a square matrix W. The notation $M > (<)\, 0$ is used to define a symmetric positive definite (negative definite) matrix. R^m denotes the m-dimensional Euclidean space; and $R^{n\times m}$ denotes the set of all $n \times m$ real matrices. In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. PROBLEM FORMATION AND PRELIMINARIES

Takagi and Sugeno [1] proposed an effective way to represent a fuzzy model of a nonlinear dynamic system. It uses a linear input/output (I/O) relation as its consequence of individual plant rules. We consider a continuous time T-S fuzzy system with time delays in this paper. The i^{th} rule of this system is of the following form.

Plant Rule i

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip} THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + A_{1i} x(t-\tau) + B_i w(t), \\ y(t) &= C_i x(t) + C_{1i} x(t-\tau) + D_i w(t), \qquad i = 1, 2, \ldots, r, \\ x(t) &= \psi(t),\ t \in [0, \tau], \end{aligned} \tag{1}$$

where $z_1(t), z_2(t), \cdots, z_p(t)$ are the premise variables, each $M_{ij}(j = 1, 2, \cdots, p)$ is a fuzzy set, $x(t) \in R^n$ is the state vector, $w(t) \in R^q$ is the square-integrable exogenous input and $y(t) \in R^q$ is the output vector. r is the number of IF-THEN rules. $A_i, A_{1i}, B_i, C_i, C_{1i}, D_i$ are some constant matrices with appropriate dimensions. $0 \le \tau \le h$ denotes the delay, and $\psi(t) \in C_{n,\tau}$ is a vector-valued initial continuous function.

Let $\mu_i(t)$ be the normalized membership function of the inferred fuzzy set $\omega_i(t)$, i.e.,

$$\mu_i(t) = \frac{\omega_i(t)}{\sum_{i=1}^{r} \omega_i(t)}, \tag{2}$$

where

$$\omega_i(t) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \tag{3}$$

where $M_{ij}(z_j(t))$ is the grade of membership function of $z_j(t)$ in $M_{ij}(t)$. It is assumed that $\omega_i(t) \ge 0, i = 1, 2, \ldots, r$, and $\sum_{i=1}^{r} \omega_i(t) > 0$ for all t. Therefore, $\mu_i(t) \ge 0$ and $\sum_{i=1}^{r} \mu_i(t) = 1$ for all t. So, the fuzzy system model (1) can be expressed as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(t)[A_i x(t) + A_{1i} x(t-\tau) + B_i w(t)] \\ &= A(t)x(t) + A_1(t)x(t-\tau) + B(t)w(t), \\ y(t) &= \sum_{i=1}^{r} \mu_i(t)[C_i x(t) + C_{1i} x(t-\tau) + D_i w(t)] \\ &= C(t)x(t) + C_1 x(t-\tau) + D(t)w(t), \end{aligned} \tag{4}$$

with

$$\begin{aligned} A(t) &= \sum_{i=1}^{r} \mu_i(t) A_i, & A_1(t) &= \sum_{i=1}^{r} \mu_i(t) A_{1i}, \\ B(t) &= \sum_{i=1}^{r} \mu_i(t) B_i, & C(t) &= \sum_{i=1}^{r} \mu_i(t) C_i, \\ C_1(t) &= \sum_{i=1}^{r} \mu_i(t) C_{1i}, & D(t) &= \sum_{i=1}^{r} \mu_i(t) D_i. \end{aligned} \tag{5}$$

There are some different definitions of passivity. A less restrictive definition of passivity is given by [23].

DEFINITION 1. *The fuzzy system (1) is called passive if there exists a scalar $\gamma \ge 0$ such that*

$$2\int_0^{t_p} w^\top(s) y(s)\, ds \ge -\gamma \int_0^{t_p} w^\top(s) w(s)\, ds, \tag{6}$$

for all $t_p \ge 0$ and for all solution of (1) with $x_0 = 0$.

LEMMA 1. SCHUR COMPLEMENT. *(See [22].) The LMI*

$$\begin{bmatrix} Q(x) & S(x) \\ S^\top(x) & R(x) \end{bmatrix} > 0,$$

with $Q(x) = Q^\top(x)$, $R(x) = R^\top(x)$, and $S(x)$ depend affinely on x, is equivalent to

$$R(x) > 0, \qquad Q(x) - S(x)R(x)^{-1}S^\top(x) > 0.$$

In obtaining the main result of delay-dependent passivity of the fuzzy systems with time delays in this paper, the following lemma about the upper bound for the inner product of two vectors plays an important role.

LEMMA 2. *(See [24].) Assume that* $a(.) \in R^{n_a}$, $b(.) \in R^{n_b}$, *and* $M(.) \in R^{n_a \times n_b}$ *are defined on the interval* Ω. *Then, for any matrices* $X \in R^{n_a \times n_a}$, $Y \in R^{n_a \times n_b}$, *and* $Z \in R^{n_b \times n_b}$, *the following holds*

$$-2\int_\Omega a^\top(\alpha) M b(\alpha) \mathrm{d}\alpha \le \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^\top \begin{bmatrix} X & Y - M \\ Y^\top - M^\top & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} \mathrm{d}\alpha, \tag{7}$$

where

$$\begin{bmatrix} X & Y \\ Y^\top & Z \end{bmatrix} \ge 0.$$

LEMMA 3. *(See [6].) For any real matrices* X_i, Y_i, $1 \le i \le n$, *and* $S > 0$ *with appropriate dimensions, we have*

$$2\sum_{i=1}^{n}\sum_{j=1}^{n} h_i h_j X_i^\top S Y_j \le \sum_{i=1}^{n} h_i \left(X_i^\top S X_i + Y_i^\top S Y_i\right)$$

where h_i *are defined as* $h_i \ge 0$, $\sum_{i=1}^{n} h_i = 1$.

3. DELAY-INDEPENDENT PASSIVITY

In this section, we will firstly derive a delay-independent passivity criterion for the delayed fuzzy system (1), and then we consider the passification problem, that is, designing a state feedback controller to make the closed-loop fuzzy system passive. The main result on the delay-independent passivity of this system is summarized in the following theorem.

THEOREM 1. *If there exist common symmetric positive matrices* $P > 0$, $S > 0$ *and* $\gamma \ge 0$ *such that the following LMIs hold*

$$\begin{bmatrix} A_i^\top P + PA_i + S & PA_{1i} & PB_i - C_i^\top \\ A_{1i}^\top P & -S & -C_{1i}^\top \\ B_i^\top P - C_i & -C_{1i} & -(D_i^\top + D_i + \gamma I) \end{bmatrix} < 0, \tag{8}$$

for $i = 1, 2, \ldots, r$. *Then the delayed fuzzy system (1) is passive in the sense of Definition 1.*

PROOF. Choose a Lyapunov-Krasovskii functional as

$$V(x(t)) = x^\top(t)Px(t) + \int_{t-\tau}^{t} x^\top(s)Sx(s)\,ds, \tag{9}$$

where $P > 0$, $S > 0$. Clearly, $V(x(t))$ is positive definite and radially unbounded. We have

$$\begin{aligned} &\dot V(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) \\ &= \dot x^\top(t)Px(t) + x^\top(t)P\dot x(t) + x^\top(t)Sx(t) - x^\top(t-\tau)Sx(t-\tau) \\ &\quad - [x^\top(t)C^\top(t)w(t) + w^\top(t)C(t)x(t) + x^\top(t-\tau)C_1^\top(t)w(t) + w^\top(t)C_1(t)x(t-\tau) \\ &\quad + w^\top(t)D^\top(t)w(t) + w^\top(t)D(t)w(t)] - \gamma w^\top(t)w(t) \\ &= x^\top(t)[A^\top(t)P + PA(t) + S]x(t) + x^\top(t)PA_1(t)x(t-\tau) \\ &\quad + x^\top(t-\tau)A_1^\top(t)Px(t) - x^\top(t-\tau)Sx(t-\tau) \\ &\quad + w^\top(t)[B^\top(t)P - C(t)]x(t) + x^\top(t)[PB(t) - C^\top(t)]w(t) \\ &\quad - x^\top(t-\tau)C_1^\top(t)w(t) - w^\top(t)C_1(t)x(t-\tau) \\ &\quad - w^\top(t)[D^\top(t) + D(t) + \gamma I]w(t) \\ &= \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}^\top \begin{bmatrix} A^\top(t)P + PA(t) + S & PA_1(t) & PB(t) - C^\top(t) \\ A_1^\top(t)P & -S & -C_1^\top(t) \\ B^\top(t)P - C(t) & -C_1(t) & -(D^\top(t) + D(t) + \gamma I) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}. \end{aligned} \tag{10}$$

From the LMIs (8), we have

$$\begin{aligned}&\sum_{i=1}^{r}\mu_i(t)\begin{bmatrix} A_i^\top P+PA_i+S & PA_{1i} & PB_i-C_i^\top \\ A_{1i}^\top P & -S & -C_{1i}^\top \\ B_i^\top P-C_i & -C_{1i} & -(D_i^\top+D_i+\gamma I)\end{bmatrix}\\ &=\begin{bmatrix} A^\top(t)P+PA(t)+S & PA_1(t) & PB(t)-C^\top(t) \\ A_1^\top(t)P & -S & -C_1^\top(t) \\ B^\top(t)P-C(t) & -C_1(t) & -(D^\top(t)+D(t)+\gamma I)\end{bmatrix}<0.\end{aligned} \tag{11}$$

So we have, from (10) and (11), that

$$\dot V(x(t))-2y^\top(t)w(t)-\gamma w^\top(t)w(t)<0. \tag{12}$$

It follows by integrating (12) with respect to t over the time period $0\sim t_p$ that (6) holds, and hence the delayed fuzzy system (1) is passive in the sense of Definition 1.

From the proof of Theorem 1, it is easy to know that if there is no time delays in the fuzzy system (1), i.e., $A_{1i}=0, C_{1i}=0$ for all i, we have the following corollary by selecting the Lyapunov function as $V(x(t))=x^\top(t)Px(t)$.

COROLLARY 1. *If there exist a common symmetric positive matrix $P>0$, and scalar $\gamma\geq 0$ such that the following LMIs hold*

$$\begin{bmatrix} A_i^\top P+PA_i & PB_i-C_i^\top \\ B_i^\top P-C_i & -(D_i^\top+D_i+\gamma I)\end{bmatrix}<0, \tag{13}$$

for $i=1,2,\dots,r$. Then, the fuzzy system (1) without delay is passive in the sense of Definition 1.

By using the above results, the passivity condition for the delayed fuzzy systems can be easily verified by solving the LMIs numerically using the interior-point algorithm [22].

Next, we consider the passification problem, that is, designing a state feedback controller to make the closed-loop fuzzy system passive. Extending on system (1), we consider a delayed fuzzy system with control input of the following form.

Plant Rule i

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip} THEN

$$\begin{aligned}\dot x(t)&=A_ix(t)+A_{1i}x(t-\tau)+B_iw(t)+B_{1i}u(t),\\ y(t)&=C_ix(t)+C_{1i}x(t-\tau)+D_iw(t),\end{aligned}\qquad i=1,2,\dots,r, \tag{14}$$

where $u(t)\in R^m$ is the control input, B_{1i} is a constant matrix of appropriate dimension.

Based on the concept of parallel distributed compensation (PDC) [25], we consider the following fuzzy control law for the fuzzy model (14).

Controller Rule i

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip} THEN

$$u(t)=K_ix(t),\qquad i=1,2,\dots,r. \tag{15}$$

The overall state feedback controller is presented by

$$u(t)=\sum_{i=1}^{r}\mu_i(t)K_ix(t), \tag{16}$$

where $\mu_i(t)$ is defined as before. The closed-loop fuzzy system can be represented as

$$\begin{aligned}\dot x(t)&=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(t)\mu_j(t)[(A_i+B_{1i}K_j)x(t)+A_{1i}x(t-\tau)+B_iw(t)],\\ y(t)&=\sum_{i=1}^{r}\mu_i(t)[C_ix(t)+C_{1i}x(t-\tau)+D_iw(t)].\end{aligned} \tag{17}$$

The following theorem establishes the main result of the state feedback passification.

Theorem 2. *If there exist common symmetric positive matrices $X > 0$, $Q > 0$, matrix Y_j and scalar $\gamma \geq 0$ such that the following LMIs hold*

$$\begin{bmatrix} XA_i^\top + A_iX + Y_j^\top B_{1i}^\top + B_{1i}Y_j + Q & A_{1i}X & B_i - XC_i^\top \\ XA_{1i}^\top & -Q & -XC_{1i}^\top \\ B_i^\top - C_iX & -C_{1i}X & -(D_i^\top + D_i + \gamma I) \end{bmatrix} < 0, \tag{18}$$

for $i, j = 1, 2, \ldots, r$. Then, the closed-loop delayed fuzzy system (14) is passive in the sense of Definition 1. And, the state feedback gain can be constructed as

$$K_i = Y_i X^{-1}, \qquad i = 1, 2, \ldots, r.$$

Proof. Select a Lyapunov-Krasovskii functional as

$$V(x(t)) = x^\top(t)Px(t) + \int_{t-\tau}^{t} x^\top(s)Sx(s)\,ds, \tag{19}$$

where $P = X^{-1} > 0$ and $S = PQP > 0$. We have

$$\begin{aligned} &\dot V(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) \\ &\quad = \dot x^\top(t)Px(t) + x^\top(t)P\dot x(t) + x^\top(t)Sx(t) - x^\top(t-\tau)Sx(t-\tau) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) \\ &\quad = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(t)\mu_j(t)\{x^\top(t)[(A_i + B_{1i}K_j)^\top P + P(A_i + B_{1i}K_j) + S]x(t) \\ &\qquad + x^\top(t)PA_{1i}x(t-\tau) + x^\top(t-\tau)A_{1i}^\top Px(t) + w^\top(t)B_i^\top Px(t) + x^\top(t)PB_iw(t) \\ &\qquad - x^\top(t-\tau)Sx(t-\tau) - x^\top(t)C_i^\top w(t) - w^\top(t)C_ix(t) - x^\top(t-\tau)C_{1i}^\top w(t) - w^\top(t)C_{1i}x(t-\tau) \\ &\qquad - w^\top(t)(D_i + D_i^\top + \gamma I)w(t)\} \\ &\quad = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(t)\mu_j(t) \\ &\qquad \cdot \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}^\top \begin{bmatrix} A_i^\top P + PA_i + K_j^\top B_{1i}^\top P + PB_{1i}K_j + S & PA_{1i} & PB_i - C_i^\top \\ A_{1i}^\top P & -S & -C_{1i}^\top \\ B_i^\top P - C_i & -C_{1i} & -(D_i^\top + D_i + \gamma I) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}. \end{aligned} \tag{20}$$

From the Proof of Theorem 1, we know that we need $\dot V(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) < 0$ to prove the passivity of system (14). From (20), we know that if the following inequalities holds for all i, then we will have $\dot V(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) < 0$.

$$\begin{bmatrix} A_i^\top P + PA_i + K_j^\top B_{1i}^\top P + PB_{1i}K_j + S & PA_{1i} & PB_i - C_i^\top \\ A_{1i}^\top P & -S & -C_{1i}^\top \\ B_i^\top P - C_i & -C_{1i} & -(D_i^\top + D_i + \gamma I) \end{bmatrix} < 0. \tag{21}$$

The matrix inequalities (21) are not LMIs but quadratic matrix inequalities (QMIs). In order to use the convex optimization technique to find solutions for these matrix inequalities, the QMIs must be converted to LMIs via some transformation. For this purpose, define the following transformation matrix as

$$\begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & P^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{22}$$

Multiplying both sides of the matrices in (21) by the matrix (22), and denoting $X = P^{-1}$, $Q = P^{-1}SP^{-1}$, and $Y_j = K_jX$, yields the linear matrix inequalities (18) in Theorem 2. This completes the proof of Theorem 2.

Similar to Corollary 1, for the closed-loop fuzzy system (14) without time delay, i.e., $A_{1i} = 0$, $C_{1i} = 0$, we have the following corollary.

COROLLARY 2. *If there exist a common symmetric positive matrix $X > 0$, matrix Y_j and scalar $\gamma \geq 0$ such that the following LMIs hold*

$$\begin{bmatrix} XA_i^\top + A_iX + Y_j^\top B_{1i}^\top + B_{1i}Y_j & B_i - XC_i^\top \\ B_i^\top - C_iX & -(D_i^\top + D_i + \gamma I) \end{bmatrix} < 0, \tag{23}$$

for $i, j = 1, 2, \ldots, r$, then the closed-loop fuzzy system (14) without time delay is passive in the sense of Definition 1. And the state feedback gain can be constructed as

$$K_j = Y_j X^{-1}, \qquad i = 1, 2, \ldots, r.$$

4. DELAY-DEPENDENT PASSIVITY

There is another type of passivity result for delayed system, that is the delay-dependent passivity. The delay-dependent passivity is concerned with the size of the delay and usually provides an upper bound of the delay such that the system is passive for any delay less than the upper bound. In general, the delay-dependent passivity is considered less conservative than the delay-independent one. In the following, we will provide a delay-dependent criterion for the passivity of the delayed fuzzy system (1).

The first equation of equation (4) can be recast as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(t) \left[A_i x(t) + A_{1i} x(t-\tau) + B_i w(t)\right] \\ &= \sum_{i=1}^{r} \mu_i(t) \left[(A_i + A_{1i}) x(t) + B_i w(t) - A_{1i} \int_{t-\tau}^{t} \dot{x}(s)\, ds\right]. \end{aligned} \tag{24}$$

We now summarize our main result of this section in the following theorem.

THEOREM 3. *If there exist common matrices $P > 0, Q > 0, X, Y, Z$, and scalar $\gamma \geq 0$, such that*

$$\begin{bmatrix} A_i^\top P + PA_i + hZ + Y^\top + Y + Q & PA_{1i} - Y & PB_i - C_i^\top & hA_i^\top Z \\ A_{1i}^\top P - Y^\top & -Q & -C_{1i}^\top & hA_{1i}^\top Z \\ B_i^\top P - C_i & -C_{1i} & -(D_i^\top + D_i + \gamma I) & hB_i^\top Z \\ hZA_i & hZA_{1i} & hZB_i & -hZ \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{bmatrix} X & Y \\ Y^\top & Z \end{bmatrix} \geq 0, \tag{26}$$

for $i = 1, 2, \ldots, r$. Then, the fuzzy system (1) is passive in the sense of Definition 1 for any time delay $0 \leq \tau \leq h$.

PROOF. Choose a Lyapunov-Krasovskii functional as

$$V(x(t)) = V_1 + V_2 + V_3, \tag{27}$$

where

$$\begin{aligned} V_1 &= x^\top(t) P x(t), \\ V_2 &= \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{x}^\top(\alpha) Z \dot{x}(\alpha)\, \mathrm{d}\alpha\, \mathrm{d}\beta, \\ V_3 &= \int_{t-\tau}^{t} x^\top(\alpha) Q x(\alpha)\, \mathrm{d}\alpha. \end{aligned}$$

We have

$$\dot{V}_1 = \sum_{i=1}^{r} \mu_i(t) \Big\{ x^\top(t)[(A_i^\top + A_{1i}^\top)P + P(A_i + A_{1i})]x(t) + w^\top(t)B_i^\top Px(t) \\ + x^\top(t)PB_iw(t) - 2x^\top(t)PA_{1i} \int_{t-\tau}^{t} \dot{x}(s)\,ds \Big\} \tag{28}$$

Define $a(.)$, $b(.)$, and M in (7) as $a(s) = x(t)$, $b(s) = \dot{x}(s)$, and $M = PA_{1i}$ for all $s \in [t-h, t]$ and applying Lemma 2 will supply (26) and

$$\begin{aligned} \dot{V}_1 \le & \sum_{i=1}^{r} \mu_i(t) \Big\{ x^\top(t) \left[\left(A_i^\top + A_{1i}^\top\right) P + P(A_i + A_{1i}) \right] x(t) \\ & + w^\top(t)B_i^\top Px(t) + x^\top(t)PB_iw(t) \\ & + \tau x^\top(t)Xx(t) + 2x^\top(t)(Y - PA_{1i}) \int_{t-\tau}^{t} \dot{x}(s)\,ds + \int_{t-\tau}^{t} \dot{x}^\top(s)Z\dot{x}(s)\,ds \Big\} \\ \le & \sum_{i=1}^{r} \mu_i(t) \left[x^\top(t)(A_i^\top P + PA_i + hX + Y^\top + Y)x(t) \right. \\ & + w^\top(t)B_i^\top Px(t) + x^\top(t)PB_iw(t) \\ & \left. - 2x^\top(t)(Y - PA_{1i})x(t-\tau) \right] + \int_{t-\tau}^{t} \dot{x}^\top(s)Z\dot{x}(s)\,ds, \end{aligned} \tag{29}$$

and

$$\begin{aligned} \dot{V}_2 &= \tau \dot{x}^\top(t)Z\dot{x}(t) - \int_{t-\tau}^{t} \dot{x}^\top(s)Z\dot{x}(s)\,ds \\ &= \tau \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(t)\mu_j(t) \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}^\top [\,A_i \quad A_{1i} \quad B_i\,]^\top Z\,[\,A_j \quad A_{1j} \quad B_j\,] \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix} \\ & \quad - \int_{t-\tau}^{t} \dot{x}^\top(s)Z\dot{x}(s)\,ds \end{aligned} \tag{30}$$

According to Lemma 3, we have

$$\begin{aligned} \dot{V}_2 \le & \, h\sum_{i=1}^{r} \mu_i(t) \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}^\top [\,A_i \quad A_{1i} \quad B_i\,]^\top Z\,[\,A_i \quad A_{1i} \quad B_i\,] \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix} \\ & - \int_{t-\tau}^{t} \dot{x}^\top(s)Z\dot{x}(s)\,ds \\ \le & \sum_{i=1}^{r} \mu_i(t) \left[hx^\top(t)A_i^\top ZA_ix(t) + 2hx^\top(t)A_i^\top ZA_{1i}x(t-\tau) + 2hx^\top(t)A_i^\top ZB_iw(t) \right. \\ & \left. + 2hw^\top(t)B_i^\top ZA_{1i}x(t-\tau) + hx^\top(t-\tau)A_{1i}^\top ZA_{1i}x(t-\tau) + hw^\top(t)B_i^\top ZB_iw(t) \right] \\ & - \int_{t-\tau}^{t} \dot{x}^\top(s)Z\dot{x}(s)\,ds. \end{aligned} \tag{31}$$

Moreover,

$$\dot{V}_3 = x^\top(t)Qx(t) - x^\top(t-\tau)Qx(t-\tau). \tag{32}$$

So, we have

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) \\
&= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 - 2\sum_{i=1}^{r}\mu_i(t)[x^\top(t)C_i^\top w(t) + x^\top(t-\tau)C_{1i}^\top w(t) + w^\top(t)D_i^\top w(t)] \\
&\quad - \gamma w^\top(t)w(t) \\
&= \sum_{i=1}^{r}\mu_i(t)\begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t)\end{bmatrix}^\top M_i \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t)\end{bmatrix},
\end{aligned}
\tag{33}
$$

with

$$
M_i = \begin{bmatrix} (1,1) & PA_{1i} - Y + hA_i^\top ZA_{1i} & PB_i - C_i^\top + hA_i^\top ZB_i \\ A_{1i}^\top P - Y^\top + hA_{1i}^\top ZA_i & -Q + hA_{1i}^\top ZA_{1i} & -C_{1i}^\top + hA_{1i}^\top ZB_i \\ B_i^\top P - C_i + hB_i^\top ZA_i & -C_{1i} + hB_i^\top ZA_{1i} & -(D_i^\top + D_i + \gamma I) + hB_i^\top ZB_i \end{bmatrix}, \tag{34}
$$

where $(1,1) = A_i^\top P + PA_i + hX + Y^\top + Y + Q + hA_i^\top ZA_i$. From the Proof of Theorem 1, we know that we need $\dot{V}(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) < 0$ to prove the passivity of system (1). From (33) we know if $M_i < 0$ for all i, then $\dot{V}(x(t)) - 2y^\top(t)w(t) - \gamma w^\top(t)w(t) < 0$. From the Schur Complement (Lemma 1), we know that $M_i < 0$ is equivalent to the LMIs (25). This completes the proof of Theorem 3.

Similar to the the delay-independent case in last section, we can also design state feedback controller to make the closed-loop fuzzy system passive, and obtain delay-dependent passification result. Limited to the length of this paper, we omit it here.

5. NUMERICAL EXAMPLES

Consider a delayed fuzzy system of the following form.

Plant Rules

Rule 1. IF $x_1(t)$ is M_1, THEN

$$
\begin{aligned}
\dot{x}(t) &= A_1x(t) + A_{11}x(t-\tau) + B_1w(t) + B_{11}u(t), \\
y(t) &= C_1x(t) + C_{11}x(t-\tau) + D_1w(t).
\end{aligned}
$$

Rule 2. IF $x_1(t)$ is M_2, THEN

$$
\begin{aligned}
\dot{x}(t) &= A_2x(t) + A_{12}x(t-\tau) + B_2w(t) + B_{12}u(t), \\
y(t) &= C_2x(t) + C_{12}x(t-\tau) + D_2w(t).
\end{aligned}
$$

with

$$
\begin{aligned}
&A_1 = \begin{bmatrix} -1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, && A_{11} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix}, && B_1 = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, \\
&C_1 = [1, 0.2], && C_{11} = [-0.1, 0.2], && D_1 = 0.5, \\
&A_2 = \begin{bmatrix} -1 & 0 \\ 0.1 & -0.5 \end{bmatrix}, && A_{12} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, && B_2 = \begin{bmatrix} -0.2 \\ 0 \end{bmatrix}, \\
&C_2 = [0.1, 1], && C_{12} = [0.1, -0.1], && D_2 = 0.1.
\end{aligned}
\tag{35}
$$

First, we consider the above system without control input. Applying Theorem 1 and using the MATLAB LMI toolbox, we can obtain the following feasible solution:

$$P = \begin{bmatrix} 10.2317 & -4.1082 \\ -4.1082 & 19.9200 \end{bmatrix},$$

$$S = \begin{bmatrix} 10.0813 & -4.1640 \\ -4.1640 & 3.4802 \end{bmatrix},$$

and

$$\gamma = 15.4430,$$

which means that the above fuzzy system without control input is passive for all time delay in the sense of Definition 1.

Next, we consider the closed-loop system with

$$B_{11} = \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix},$$

$$B_{12} = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}.$$

Applying Theorem 2, we can obtain the following feasible solution:

$$X = \begin{bmatrix} 54.7595 & -1.5785 \\ -1.5785 & 41.1853 \end{bmatrix},$$

$$Q = \begin{bmatrix} 45.9380 & -6.8533 \\ -6.8533 & 11.0699 \end{bmatrix},$$

$$Y_1 = [8.6906, 13.2635],$$

$$Y_2 = [8.6906, 13.2635],$$

$$\gamma = 110.7354.$$

Subsequently, we can obtain the feedback gain $K_1 = K_2 = [0.1682, 0.3285]$. Thus, we can construct a fuzzy controller with feedback gains K_1, K_2 to make the closed-loop fuzzy system passive.

If in (35), we change A_{11} to

$$A_{11} = \begin{bmatrix} 0.1 & 0.6 \\ 0.2 & -0.1 \end{bmatrix},$$

then we can't find feasible solutions for LMIs (8). That is, by using the delay-independent criterion Theorem 1, we can't test the passivity of this fuzzy system. But by using the delay-dependent criterion Theorem 3, we can obtain an upper bound of the time delay $h \approx 8$ guaranteeing the passivity of this fuzzy system. For example, for $\tau = 7.5$, we can get the following feasible solution for LMIs (25) and (26),

$$P = \begin{bmatrix} 48.3388 & -15.6002 \\ -15.6002 & 128.8284 \end{bmatrix},$$

$$Q = \begin{bmatrix} 46.0013 & -14.3916 \\ -14.3916 & 20.0007 \end{bmatrix},$$

$$X = \begin{bmatrix} 1.4605 & -1.3696 \\ -1.3696 & 1.3954 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0.1727 & 7.6396 \\ -0.2281 & -7.6368 \end{bmatrix},$$

$$Z = \begin{bmatrix} 2.8228 & 2.2091 \\ 2.2091 & 43.3962 \end{bmatrix},$$

and

$$\gamma = 604.9523.$$

6. CONCLUSION

The passivity and state feedback passification of T-S fuzzy systems with time delays have been considered in this paper. All the theoretical results were given in terms of LMIs, which can be easily solved by using convex optimization technique. Numerical examples have also been presented to illustrate the effectiveness of the theoretical results. To our knowledge, this is the first investigation of the passivity and passification of delayed T-S fuzzy systems. The results presented in this paper can be easily extended to fuzzy systems with time-varying delays as well as with parametric uncertainties.

Since passivity has strong relation with stability, in future works we will study passivity-based stability analysis and stabilization of delayed nonlinear systems.

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