



Testing Higgs triplet model and neutrino mass patterns

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Abstract

The observed neutrino oscillation data might be explained by new physics at a TeV scale, which is testable in the future experiments. Among various possibilities, the low-energy Higgs triplet model is a prime candidate of such new physics since it predicts clean signatures of lepton flavor violating processes directly related to the neutrino masses and mixing. It is discussed how various neutrino mass patterns can be discriminated by examining the lepton flavor violating decays of charged leptons as well as the collider signatures of a doubly charged Higgs boson in the model.

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1. Introduction

The atmospheric, solar and reactor neutrino experiments [1–4] have firmly established the picture of three active neutrino oscillations, and provided us important information on two neutrino mass-squared differences and three mixing angles. Taking the most favorable parameter region of the solar neutrino oscillation (so-called LMA I), we have

$$\begin{aligned} \Delta m_{\text{atm}}^2 &= (1.1\text{--}4.8) \times 10^{-3} \text{ eV}^2, & \sin^2 \theta_{\text{atm}} &= 0.3\text{--}0.7, \\ \Delta m_{\text{sol}}^2 &= (0.5\text{--}1.0) \times 10^{-4} \text{ eV}^2, & \sin^2 \theta_{\text{sol}} &= 0.24\text{--}0.44, \end{aligned} \quad (1)$$

and the limit of $\sin^2 \theta_{\text{chooz}} < 0.038$ coming from the non-observation of $\nu_{\mu} \rightarrow \nu_e$ oscillation in the CHOOZ and atmospheric neutrino data [1,3].

Given such new experimental inputs, we could hope for uncovering new physics beyond the standard model, which must explain the observed neutrino data. In this regard, a “low-energy” model for neutrino masses and mixing is of particular interest since it may be tested in the future experiments observing lepton flavor violating processes in accelerators. A typical example of such a model would be the supersymmetric standard model with *R-parity violation* in which the flavor structure of neutrino mass matrix could be probed through the decay of

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the lightest supersymmetric particle [5]. Another example is the Zee model and its variations [6] which rely on *radiative mechanism* of neutrino mass generation.

In this Letter, we consider the *Higgs triplet model* [7] in which a triplet scalar field $\Delta = (\Delta^{++}, \Delta^+, \Delta^0)$ with the mass M is introduced to have the following renormalizable couplings:

$$\mathcal{L}_\Delta = \frac{1}{\sqrt{2}} [f_{ij} L_i L_j \Delta + \mu \Phi \Phi \Delta + \text{h.c.}] - M^2 |\Delta|^2, \quad (2)$$

where $L_i = (v_i, l_i)_L$ is the left-handed lepton doublet and $\Phi = (\phi^0, \phi^-)$ is the standard model Higgs doublet. Due to the “ μ ” term in the above equation, the neutral component Δ^0 of the triplet gets the vacuum expectation value (VEV), $v_\Delta = \mu v_\phi^2 / 2M^2$ where $v_\phi = \langle \Phi^0 \rangle = 246$ GeV. This leads to the neutrino mass matrix,

$$M_{ij}^v = f_{ij} v_\Delta. \quad (3)$$

We are interested in the possibility of the light triplet Higgs bosons, namely $M \sim v_\phi$, so that observations of various lepton flavor violating processes can provide a probe for the neutrino masses and mixing through the relation (3), and thus a direct test of the model. In this “low-energy triplet Higgs model”, the small parameters f and $\xi \equiv v_\Delta / v_\phi$ are required:

$$f_{ij} \xi \sim 10^{-12}, \quad (4)$$

for $M_{ij}^v \sim 0.3$ eV. We will see later that such a smallness could be understood by a radiative mechanism. Here, let us note that we are interested in the case of very small ξ , say $\xi \lesssim 10^{-6}$, so that the condition of $\rho = m_Z^2 / m_W^2 c_W^2 \simeq 1$ is simply satisfied in our consideration.

Phenomenological consequences of low-energy triplet Higgs bosons have been studied extensively in the past, in particular, centering around the exotic signatures of a doubly charged Higgs boson, $\Delta^{\pm\pm}$ [8–13]. The main purpose of this work is to investigate how the observation of such phenomena can test the pattern of the neutrino masses and mixing. For this to happen, we will mostly assume that $f \gtrsim \xi$ to detect the lepton flavor violating processes induced by the coupling f . This Letter is organized as follows. In Section 2, we derive the flavor structure of the bileptonic couplings f_{ij} depending on the acceptable neutrino mass patterns, based on which the observability of rare lepton decays such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\tau \rightarrow 3l$ will be discussed. In Section 3, we will consider the production and decays of doubly charged Higgs bosons in colliders from which some information on the couplings f can be obtained. We will see when the collider effects of the coupling f can be observed in relation to the above discussion. Then, we examine how the neutrino mass patterns can be discriminated through the observation of $\Delta^{\pm\pm}$ decays. In Section 4, we present a model in which the smallness of the couplings f and μ is explained by a radiative generation at two-loop level. We conclude in Section 5.

2. Neutrino mass patterns and low-energy lepton flavor violation

Current neutrino data (1) give us the following neutrino mixing matrix;

$$U \approx \begin{pmatrix} c_3 & s_3 & s_2 \\ -\frac{s_3}{\sqrt{2}} & \frac{c_3}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_3}{\sqrt{2}} & -\frac{c_3}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (5)$$

in the leading approximation where we put $c_2 \simeq 1$, $c_1 \simeq s_1 \simeq 1/\sqrt{2}$. Note that the mixing angles in Eq. (1) can be identified as $\theta_{\text{atm}} \approx \theta_1$, $\theta_{\text{sol}} \approx \theta_3$ and $\theta_{\text{chooz}} \approx \theta_2$. Then, the flavor structure of the coupling f can be determined simply by $f \propto M^v \approx U \text{diag}(m_1, m_2, m_3) U^T$. In the below, we will show the ratios;

$$[ff^\dagger] \equiv (ff^\dagger)_{11} : (ff^\dagger)_{22} : (ff^\dagger)_{33} : (ff^\dagger)_{12} : (ff^\dagger)_{13} : (ff^\dagger)_{23} \quad \text{and} \quad [f] \equiv f_{11} : f_{22} : f_{33} : f_{12} : f_{13} : f_{23}.$$

Given the information on Δm^2 (1), one has a variety of possibilities for the neutrino mass eigenvalues. Assuming CP conservation, the following different patterns can be allowed:

(i) Hierarchy with $m_1 < m_2 < m_3$ which gives

$$[ff^\dagger] = (s_2^2 + r s_3^2) : \frac{1}{2} : \frac{1}{2} : \frac{1}{\sqrt{2}} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : \frac{1}{\sqrt{2}} \left(s_2 - \frac{r}{2} \sin 2\theta_3 \right) : \frac{1}{2}, \quad (6)$$

$$(HI) \quad [f] = (s_2^2 + \sqrt{r} s_3^2) : \frac{1}{2} : \frac{1}{2} : \frac{1}{\sqrt{2}} \left(s_2 + \frac{\sqrt{r}}{2} \sin 2\theta_3 \right) : \frac{1}{\sqrt{2}} \left(s_2 - \frac{\sqrt{r}}{2} \sin 2\theta_3 \right) : \frac{1}{2}, \quad (7)$$

where $r \equiv \Delta m_{\text{atm}}^2 / \Delta m_{\text{sol}}^2$ which is in the range of [0.01–0.1] as in Eq. (1).

(ii) Inverse hierarchy with $m_1 \simeq m_2 \gg m_3$ (IN1) and $m_1 = -m_2 \gg m_3$ (IN2) resulting in

$$[ff^\dagger] = 1 : \frac{1}{2} : \frac{1}{2} : \frac{1}{\sqrt{2}} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : \frac{1}{\sqrt{2}} \left(s_2 - \frac{r}{2} \sin 2\theta_3 \right) : \frac{1}{2}, \quad (8)$$

$$(IN1) \quad [f] = 1 : \frac{1}{2} : \frac{1}{2} : \frac{1}{\sqrt{2}} \left(s_2 - \frac{r}{4} \sin 2\theta_3 \right) : \frac{1}{\sqrt{2}} \left(s_2 + \frac{r}{4} \sin 2\theta_3 \right) : \frac{1}{2}, \quad (9)$$

$$(IN2) \quad [f] = \cos 2\theta_3 : \frac{1}{2} (\cos 2\theta_3 - s_2 \sin 2\theta_3) : \frac{1}{2} (\cos 2\theta_3 + s_2 \sin 2\theta_3) : \frac{1}{\sqrt{2}} \sin 2\theta_3 : \frac{1}{\sqrt{2}} \sin 2\theta_3 : \frac{1}{2} \cos 2\theta_3. \quad (10)$$

(iii) Degeneracy with $m_1 \simeq m_2 \simeq m_3$ (DG1), $m_1 \simeq m_2 \simeq -m_3$ (DG2), $m_1 \simeq -m_2 \simeq m_3$ (DG3), $m_1 \simeq -m_2 \simeq -m_3$ (DG4) yielding

$$[ff^\dagger] = 1 : 1 : 1 : \frac{R}{\sqrt{2}} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : \frac{R}{\sqrt{2}} \left(s_2 - \frac{r}{2} \sin 2\theta_3 \right) : \frac{R}{2}, \quad (11)$$

$$(DG1) \quad [f] = 1 : 1 : 1 : \frac{R}{2\sqrt{2}} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : \frac{R}{2\sqrt{2}} \left(s_2 - \frac{r}{2} \sin 2\theta_3 \right) : \frac{R}{4}, \quad (12)$$

$$(DG2) \quad [f] = 1 : s_2^2 + \cos 2\theta_1 - \frac{R}{4} : s_2^2 - \cos 2\theta_1 - \frac{R}{4} : \sqrt{2} \left(s_2 - \frac{r}{4} \sin 2\theta_3 \right) : \sqrt{2} \left(s_2 + \frac{r}{4} \sin 2\theta_3 \right) : 1, \quad (13)$$

$$(DG3) \quad [f] = \cos 2\theta_3 : s_3^2 + s_2 \sin 2\theta_3 : s_3^2 - s_2 \sin 2\theta_3 : \frac{1}{\sqrt{2}} (\sin 2\theta_3 - 2s_2 s_3^2) : \frac{1}{\sqrt{2}} (\sin 2\theta_3 + 2s_2 s_3^2) : c_3^2, \quad (14)$$

$$(DG4) \quad [f] = \cos 2\theta_3 : c_3^2 - s_2 \sin 2\theta_3 : c_3^2 + s_2 \sin 2\theta_3 : \frac{1}{\sqrt{2}} (\sin 2\theta_3 + 2s_2 c_3^2) : \frac{1}{\sqrt{2}} (\sin 2\theta_3 - 2s_2 c_3^2) : s_3^2, \quad (15)$$

where $R \equiv \Delta m_{\text{atm}}^2 / m_1^2$. Since the recent WMAP results put a limit of $m_1 < 0.23$ eV [14], the ratio R has to be larger than about 0.02.

The schematic form of the bilepton couplings (2) can be written explicitly as

$$\mathcal{L} = \frac{1}{\sqrt{2}} f_{ij} \bar{l}_i^c i \tau_2 \mathbf{\Delta} L_j + \text{h.c.} = -\frac{1}{2} f_{ij} [\sqrt{2} \bar{l}_i^c P_L l_j \Delta^{++} + (\bar{l}_i^c P_L \nu_j + \bar{l}_j^c P_L \nu_i) \Delta^+ - \sqrt{2} \bar{\nu}_i^c P_L \nu_j \Delta^0 + \text{h.c.}], \quad (16)$$

where we used the matrix form of the triplet field;

$$\mathbf{\Delta} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}.$$

The above Lagrangian induces the tri-leptonic and radiative decays of a charged lepton at tree and one-loop level, respectively [13]. Let us now discuss the observational possibilities of such lepton flavor violating decays of muon

Table 1

The experimental limits on the branching ratios of various modes and the corresponding upper bounds on the product of couplings taking $x_\Delta = (M_\Delta/200 \text{ GeV})^2$

Mode	Current limit [15,16]	Future sensitivity [16,17]	Bound on the couplings
$\mu \rightarrow e\gamma$	1.2×10^{-11}	$\sim 10^{-14}$	$(ff^\dagger)_{12} < 1.2 \times 10^{-4} x_\Delta$
$\tau \rightarrow e\gamma$	2.7×10^{-6}	$\sim 10^{-8}$	$(ff^\dagger)_{13} < 1.3 \times 10^{-1} x_\Delta$
$\tau \rightarrow \mu\gamma$	0.6×10^{-6}	$\sim 10^{-8}$	$(ff^\dagger)_{23} < 6.1 \times 10^{-2} x_\Delta$
$\mu \rightarrow \bar{e}ee$	1.0×10^{-12}	$\sim 10^{-15}$	$f_{11}f_{12} < 9.3 \times 10^{-7} x_\Delta$
$\tau \rightarrow \bar{e}ee$	2.7×10^{-7}	$\sim 10^{-8}$	$f_{11}f_{13} < 1.1 \times 10^{-3} x_\Delta$
$\tau \rightarrow \bar{e}e\mu$	2.4×10^{-7}	$\sim 10^{-8}$	$f_{12}f_{13} < 1.5 \times 10^{-3} x_\Delta$
$\tau \rightarrow \bar{e}\mu\mu$	3.2×10^{-7}	$\sim 10^{-8}$	$f_{22}f_{13} < 1.2 \times 10^{-3} x_\Delta$
$\tau \rightarrow \bar{\mu}ee$	2.8×10^{-7}	$\sim 10^{-8}$	$f_{11}f_{23} < 1.2 \times 10^{-3} x_\Delta$
$\tau \rightarrow \bar{\mu}e\mu$	3.1×10^{-7}	$\sim 10^{-8}$	$f_{12}f_{23} < 1.7 \times 10^{-3} x_\Delta$
$\tau \rightarrow \bar{\mu}\mu\mu$	3.8×10^{-7}	$\sim 10^{-8}$	$f_{22}f_{23} < 1.4 \times 10^{-3} x_\Delta$

or tau in the triplet Higgs model. Table 1 shows the current limits on the products of couplings for various decay modes, and their future experimental sensitivities. For the discovery of some lepton flavor violating decay modes, one needs

$$\begin{aligned}
 f_{11}f_{12} &> 3.0 \times 10^{-8} x_\Delta, & \text{for } \mu \rightarrow 3e, \\
 (ff^\dagger)_{12} &> 3.5 \times 10^{-6} x_\Delta, & \text{for } \mu \rightarrow e\gamma, \\
 f_{ij}f_{k3} &\gtrsim 2.3 \times 10^{-4} x_\Delta, & \text{for } \tau \rightarrow 3l,
 \end{aligned} \tag{17}$$

where $i, j, k = 1, 2$ as indicated in Table 1.

In the cases of (IN2), (DG3) and (DG4), neither $\mu \rightarrow e\gamma$ nor $\tau \rightarrow 3l$ can be observed as the strong constraint from the $\mu \rightarrow 3e$ pushes them outside the future experimental sensitivity. To see this, let us note that $f_{11}f_{12} \propto \sin 2\theta_3 \cos 2\theta_3 / \sqrt{2}$ from Eqs. (10), (14) and (15), and $\cos 2\theta_3 > 0.1$ from Eq. (1), which shows that

$$\begin{aligned}
 f_{ij}f_{k3} &< \frac{f_{11}f_{12}}{\cos 2\theta_3} < 10^{-5} x_\Delta, \\
 (ff^\dagger)_{12} &= \frac{(R)s_2}{\cos 2\theta_3 \sin 2\theta_3} f_{11}f_{12} < 2 \times 10^{-6} x_\Delta,
 \end{aligned} \tag{18}$$

where R has to be included in the (DG) case. The situation can be different in other cases where one has the following relations for the ratio $f_{11}f_{12} : (ff^\dagger)_{12} : f_{ii}f_{23}$;

$$\begin{aligned}
 \text{(HI)} \quad & 2\sqrt{2} \left(s_2 + \frac{\sqrt{r}}{2} \sin 2\theta_3 \right) \sqrt{r} s_3^2 : 2\sqrt{2} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : 1, \\
 \text{(IN1)} \quad & \sqrt{2} \left(s_2 - \frac{r}{4} \sin 2\theta_3 \right) : \sqrt{2} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : 1, \\
 \text{(DG1)} \quad & \sqrt{2} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : 2\sqrt{2} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : 1, \\
 \text{(DG2)} \quad & \sqrt{2} \left(s_2 - \frac{r}{4} \sin 2\theta_3 \right) : \frac{R}{\sqrt{2}} \left(s_2 + \frac{r}{2} \sin 2\theta_3 \right) : 1,
 \end{aligned} \tag{19}$$

where $f_{ii} = f_{22}$ for (HI) and f_{11} otherwise. From this, one can see that the decay modes other than $\mu \rightarrow 3e$ can be seen only if the coupling f_{12} is made small and thus the following relation is fulfilled; $s_2 \approx -\sqrt{r} \sin 2\theta_3 / 2$ (HI),

$s_2 \approx r \sin 2\theta_3/4$ (IN1), $s_2 \approx -r \sin 2\theta_3/2$ (DG1) or $s_2 \approx r \sin 2\theta_3/8$ (DG2). In this case, one predicts

- (HI) $B(\tau \rightarrow \bar{\mu}\mu\mu) : B(\mu \rightarrow e\gamma) = 1 : 8.6 \times 10^{-3} r \sin^2 2\theta_3;$
- (IN1) $B(\tau \rightarrow \bar{\mu}ee) : B(\tau \rightarrow \bar{\mu}\mu\mu) : B(\mu \rightarrow e\gamma) = 1 : 0.5 : 4.8 \times 10^{-3} r^2 \sin^2 2\theta_3;$
- (DG1) $B(\tau \rightarrow \bar{\mu}ee) : B(\tau \rightarrow \bar{\mu}\mu\mu) = 1 : 1;$
- (DG2) $B(\tau \rightarrow \bar{\mu}ee) : B(\mu \rightarrow e\gamma) = 1 : 4.8 \times 10^{-3} R^2 r^2 \sin^2 2\theta_3.$

An ideal case is to observe both $\tau \rightarrow 3l$ and $\mu \rightarrow e\gamma$ decays which will enable us to discriminate the different mass patterns.

3. Collider test: production and decays of Higgs triplet

Some of striking collider signals in the triplet Higgs model comes from the decays of a doubly charged Higgs boson, such as $\Delta^{--} \rightarrow l_i l_j$, $W^- W^-$, which have been studied extensively in the past years [8–13]. We are interested in the situation that the decays $\Delta^{--} \rightarrow l_i l_j$ are sizable so that the neutrino mass structure can be tested in colliders. Depending on the masses of the triplet components, the fast decay process like $\Delta^{--} \rightarrow \Delta^- W^{*-}$ through gauge interactions can happen to over-dominate any other processes of our interest. The mass splitting among the triplet components arises upon the electroweak symmetry breaking and thus is of the order M_W . In order to study the mass spectrum and decay processes of the triplet Higgs bosons, let us first consider the most general scalar potential for a doublet and a triplet Higgs boson:

$$V = m^2(\Phi^\dagger \Phi) + \lambda_1(\Phi^\dagger \Phi)^2 + M^2 \text{Tr}(\Delta^\dagger \Delta) + \lambda_2[\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 \text{Det}(\Delta^\dagger \Delta) \\ + \lambda_4(\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5(\Phi^\dagger \tau_i \Phi) \text{Tr}(\Delta^\dagger \tau_i \Delta) + \frac{1}{\sqrt{2}}\mu(\Phi^T i \tau_2 \Delta \Phi) + \text{h.c.} \quad (20)$$

Note that the triplet VEV is given by $v_\Delta = \mu v_\Phi^2 / 2M_{\Delta^0}^2$. In this theory, the mass eigenstates consist of Δ^{++} , H^+ , H^0 , A^0 and h^0 . Under the condition that $|\xi| \ll 1$, the first five states are mainly from the triplet sector and the last from the doublet sector. The approximate mass diagonalizations are given as follows. For the neutral pseudoscalar and charged scalar parts,

$$\phi_I^0 = G^0 - 2\xi A^0, \quad \phi^+ = G^+ + \sqrt{2}\xi H^+, \\ \Delta_I^0 = A^0 + 2\xi G^0, \quad \Delta^+ = H^+ - \sqrt{2}\xi G^+, \quad (21)$$

where G^0 and G^+ are the Goldstone modes, and for the neutral scalar part,

$$\phi_R^0 = h^0 - a\xi H^0, \quad \Delta_R^0 = H^0 + a\xi h^0, \quad (22)$$

where $a = 2 + 4(4\lambda_1 - \lambda_4 - \lambda_5)m_W^2/g^2(m_{H^0}^2 - m_{h^0}^2)$. The masses of the Higgs bosons are

$$M_{\Delta^{\pm\pm}}^2 = M^2 + 2\frac{\lambda_4 - \lambda_5}{g^2}M_W^2, \quad M_{H^\pm}^2 = M_{\Delta^{\pm\pm}}^2 + 2\frac{\lambda_5}{g^2}M_W^2, \\ M_{H^0, A^0}^2 = M_{H^\pm}^2 + 2\frac{\lambda_5}{g^2}M_W^2. \quad (23)$$

The mass of h^0 is given by $m_{h^0}^2 = 4\lambda_1 v_\Phi^2$ as usual.

When $\lambda_5 > 0$, we have $M_{\Delta^{\pm\pm}} < M_{H^\pm} < M_{H^0, A^0}$, so that the doubly charged Higgs boson Δ^{--} can only decay to $l_i l_j$ or $W^- W^-$ through the following interactions;

$$\mathcal{L} = \frac{1}{\sqrt{2}}[f_{ij}\bar{l}_i^c P_L l_j + g\xi M_W W^- W^-]\Delta^{++} + \text{h.c.} \quad (24)$$

The corresponding decay rates are

$$\begin{aligned}\Gamma(\Delta^{--} \rightarrow l_i l_j) &= S \frac{f_{ij}^2}{16\pi} M_{\Delta^{\pm\pm}}, \\ \Gamma(\Delta^{--} \rightarrow WW) &= \frac{\alpha_2 \xi^2}{32} \frac{M_{\Delta^{\pm\pm}}^3}{M_W^2} (1 - 4r_W + 12r_W^2)(1 - 4r_W)^{1/2},\end{aligned}\quad (25)$$

where $S = 2$ (1) for $i \neq j$ ($i = j$) and $r_W = M_W^2/M_{\Delta^{\pm\pm}}^2$. In this case, the heavier states H^+ , H^0 and A^0 can have the decay modes; $H^0, A^0 \rightarrow H^+ W^{(*)-}$ and $H^+ \rightarrow \Delta^{++} W^{(*)-}$ leading to the production of $\Delta^{\pm\pm}$.

When $\lambda_5 < 0$, one has $M_{\Delta^{\pm\pm}} > M_{H^\pm} > M_{H^0, A^0}$. In this case, the decay processes of $\Delta^{--} \rightarrow H^- W^-$ and $H^- \rightarrow H^0(A^0)W^-$ can be allowed through the usual gauge interactions;

$$\mathcal{L} = i g W^+ \left[H^+ \overset{\leftrightarrow}{\partial} \Delta^{--} + \frac{1}{\sqrt{2}} H^0 \overset{\leftrightarrow}{\partial} H^- + \frac{i}{\sqrt{2}} A^0 \overset{\leftrightarrow}{\partial} H^- \right] + \text{h.c.}, \quad (26)$$

giving rise to the decay rate

$$\Gamma(\Delta^{--} \rightarrow H^- W^-) = \frac{g^2}{8\pi} M_W \left[1 + \frac{2y^2 - y - 1}{2} r_W \right] \left[\frac{(y+1)^2}{4} r_W - 1 \right]^{1/2}, \quad (27)$$

where $y \equiv 2|\lambda_5|/g^2$. This can be rewritten as $\Gamma(\Delta^{--} \rightarrow H^- W^-) = (5\sqrt{2} g^2/8\pi) M_W \delta^{1/2}$ in the limit of $\delta \equiv (M_{\Delta^{\pm\pm}} - M_{H^\pm} - M_W)/M_W \rightarrow 0$ that is, $y+1 \rightarrow 2r_W^{-1/2}$. To suppress the decay mode of Eq. (27), we will require $M_{\Delta^{\pm\pm}} < M_{H^\pm} + M_W$, that is, $M_{\Delta^{\pm\pm}} > \frac{(y+1)}{2} M_W$. For $M_{\Delta^{\pm\pm}} = 200$ GeV, it implies $|\lambda_5| < 0.89$. Thus, the decay $\Delta^{--} \rightarrow H^- W^-$ is forbidden unless the coupling λ_5 is extremely large. Now, the off-shell production of W , $\Delta^{--} \rightarrow H^- W^{*-}$, is allowed to have the rate;

$$\Gamma(\Delta^{--} \rightarrow H^- W^{*-}) \approx \frac{3G_F^2}{40\pi^3} \frac{y^5 M_W^{10}}{M_{\Delta^{\pm\pm}}^5} \quad (28)$$

in the leading term of yM_W^2 . With the further requirement of $\Gamma(\Delta^{--} \rightarrow H^- W^{*-}) < \Gamma(\Delta^{--} \rightarrow l_i l_j)$, we limit ourselves in the parameter space satisfying

$$|\lambda_5| < 0.16 \left(\frac{M_{\Delta^{\pm\pm}}}{200 \text{ GeV}} \right)^{6/5} \left(\frac{f_{ij}}{10^{-3}} \right)^{2/5}. \quad (29)$$

Here, let us remark that, after the diagonalization in Eqs. (21) and (22), we also get couplings for the interactions, $H^+ \rightarrow u\bar{d}, h^0 W^+, ZW^+$ and $H^0, A^0 \rightarrow f\bar{f}, W^+ W^-, ZZ, h^0 h^0, Zh^0$, all proportional to ξ , and thus they should be considered as well if $f \sim \xi$.

Before going to our main discussion, let us note that the triplet Higgs decay is short enough to occur inside colliders. Assuming Eq. (25) as the main decay rates and recalling $\sum_{ij} f_{ij}^2 \propto \text{Tr}(M_\nu^2)$ where $M_{ij}^\nu = f_{ij} \xi v_\phi$, one obtains the following form of the total decay rate:

$$\Gamma_{\Delta^{\pm\pm}} = M_{\Delta^{\pm\pm}} \left(\frac{1}{16\pi} \frac{\bar{m}^2}{\xi^2 v_\phi^2} + \frac{\alpha_2 \xi^2}{32 r_W} (1 - 4r_W + 12r_W^2)(1 - 4r_W)^{1/2} \right), \quad (30)$$

where $\bar{m}^2 \equiv \sum_i m_i^2$. When $M_{\Delta^{\pm\pm}} > 2M_W$, one finds the minimum value of the total decay rate given by

$$\Gamma_{\Delta^{\pm\pm}}|_{\min} = \frac{1}{8\pi} \frac{M_{\Delta^{\pm\pm}} \bar{m}^2}{\hat{\xi}^2 v_\phi^2},$$

where $\hat{\xi}^2 \equiv (2\sqrt{2}/g)r_W^{1/2}(\bar{m}/v_\phi)(1 - 4r_W + 12r_W^2)^{-1/2}(1 - 4r_W)^{-1/4}$. Taking $\bar{m} = 0.05$ eV and $M_{\Delta^{\pm\pm}} = 200$ GeV, we obtain $\Gamma_{\pm\pm}|_{\min} \approx 6 \times 10^{-13}$ GeV and $\hat{\xi} \approx 6 \times 10^{-7}$, leading to $\tau|_{\max} \approx 0.03$ cm. When $M_{\Delta^{\pm\pm}} < 2M_W$, only

the first term in Eq. (30) contributes and the total decay rate is then $\Gamma > 8 \times 10^{-14}$ GeV for $M_{\Delta^{\pm\pm}} = 100$ GeV and $\xi < 10^{-6}$. Thus, as far as $\Delta^{--} \rightarrow l_i l_j$ are the main decay modes of the doubly charged Higgs boson, its decay signal should be observed in colliders.

• Single production of $\Delta^{\pm\pm}$: $e^+e^- \rightarrow e^\pm l^\pm \Delta^{\mp\mp}$

In the e^+e^- colliders, an energetic virtual photon emitted from e^\pm leads to the enhanced $e^\mp\gamma$ scattering producing $l_i^\pm \Delta^{\mp\mp}$ when a coupling f_{1i} is sizable. Adopting the result of Ref. [10] with the p_T cut ($p_T = 10$ GeV) and neglecting the final state lepton masses, we obtain the following pairs of $M_{\Delta^{\pm\pm}}$ and f_{1i}^2 :

$M_{\Delta^{\pm\pm}}$ (GeV)	100	400	600	700	800	850	900
$f_{1i}^2 (10^{-6} x_\Delta)$	2.8	3.4	5.4	7.6	12	17	29

(31)

to get the cross-section of $\sigma = 0.01$ fb at $\sqrt{s} = 1$ TeV. This corresponds to $N = 10$ events for the integrated luminosity $L = 1000/\text{fb}$. The cross section of course scales with f_{1i}^2 given the mass $M_{\Delta^{\pm\pm}}$.

Let us first consider the cases of (IN2), (DG3) and (DG4) where the couplings f_{1i}^2 are strongly constrained as seen in Eq. (17). In each case, we get

$$(f_{11}^2, f_{12}^2, f_{13}^2) \approx \left(\cot 2\theta_3, \frac{1}{2} \tan 2\theta_3, \frac{1}{2} \tan 2\theta_3 \right) \sqrt{2} f_{11} f_{12}, \quad (32)$$

neglecting a small deviation due to the contribution of s_2 . Thus, if $\mu \rightarrow 3e$ decay is found near the current experimental limit and θ_3 is close to 45° , the final states $\mu^\pm \Delta^{\mp\mp}$ and $\tau^\pm \Delta^{\mp\mp}$ could be observed with

$$N(\mu\Delta) = N(\tau\Delta),$$

for smaller values of the triplet mass, say $M_{\Delta^{\pm\pm}} < 700$ GeV.

In the cases of (IN1), (DG1) and (DG2), one has $f_{12}^2 \ll f_{11} f_{12}$ and $f_{13}^2 \ll f_{11}^2$ and thus the characteristic signature is a copious production of the final state, $e^\pm \Delta^{\mp\mp}$. If the low energy decay $\tau \rightarrow 3l$ or $\mu \rightarrow e\gamma$ is observed, the value of f_{11}^2 is determined by the following comparison with $f_{11} f_{23}$ and $(ff^\dagger)_{12}$ triggering the decays $\tau \rightarrow \bar{\mu}ee$ and $\mu \rightarrow e\gamma$, respectively:

$$f_{11}^2 = \left[2, \frac{4}{R}, 1 \right] f_{11} f_{23} \quad \text{or} \quad f_{11}^2 = \left[\frac{8\sqrt{2}}{3r \sin 2\theta_3}, \times, \frac{4\sqrt{2}}{3Rr \sin 2\theta_3} \right] (ff^\dagger)_{12}, \quad (33)$$

for the cases of (IN1), (DG1) and (DG2), respectively. Here, \times cannot be specified as $(ff^\dagger)_{12}$ can be vanishingly small in the case (DG1). This shows that $f_{11}^2 \gg 10^{-6}$ and thus the production of $e^\pm \Delta^{\mp\mp}$ can be detected even for $M_{\Delta^{\pm\pm}} \sim 1$ TeV. Even in the case that only $\mu \rightarrow 3e$ decay is observed, there is some allowed parameter space for the production of $e^\pm \Delta^{\mp\mp}$ as we have

$$f_{11}^2 = \left[\frac{\sqrt{2}}{s_2 - \frac{r}{4} \sin 2\theta_3}, \frac{2\sqrt{2}}{R(s_2 + \frac{r}{2} \sin 2\theta_3)}, \frac{1}{\sqrt{2}(s_2 - \frac{r}{4} \sin 2\theta_3)} \right] f_{11} f_{12}. \quad (34)$$

For the case of (HI), we have

$$(f_{11}^2, f_{13}^2) = (t_3^2, 2)r \sin^2 2\theta_3 f_{22} f_{23} \quad \text{or} \quad (f_{11}^2, f_{13}^2) = (t_3^2, 2) \sqrt{\frac{r}{2}} \sin 2\theta_3 (ff^\dagger)_{12} \quad (35)$$

when f_{12} is made small to suppress the decay $\mu \rightarrow 3e$. This shows that the decay $\tau \rightarrow 3\mu$ and $\mu \rightarrow e\gamma$ could be observed together with the collider signals of producing the events $e\Delta$ and $\tau\Delta$ satisfying the relation

$$N(e\Delta) : N(\tau\Delta) \approx t_3^2 : 2.$$

Let us note that no signal of $l\Delta$ production can be observed if only the decay $\mu \rightarrow 3e$ is observable in the case (HI).

- Pair production of $\Delta^{\pm\pm} : \gamma^*, Z^* \rightarrow \Delta^{++} \Delta^{--}$.

When the couplings f_{ij} are much smaller than the electroweak gauge couplings, which is always the case except for (DG1), pairs of doubly charged Higgs bosons can be produced through the gauge interactions exchanging γ or Z , if allowed kinematically. Then, the produced $\Delta^{\pm\pm}$ may decay mainly to a pair of same-sign charged leptons through the couplings f . In this case, we can measure the relative sizes of the branching ratios $B(\Delta^{--} \rightarrow l_i l_j)$ and thus the ratios of f_{ij} , which enables us to confirm what neutrino mass texture is realized in nature. Let us show the expected ratio of $B(ee) : B(\mu\mu) : B(\tau\tau) : B(e\mu) : B(e\tau) : B(\mu\tau)$ calculated from Eqs. (6)–(15);

- (HI) $2r \sin^4 \theta_3 : \frac{1}{2} : \frac{1}{2} : \frac{1}{2} r \sin^2 2\theta_3 : \frac{1}{2} r \sin^2 2\theta_3 : 1;$
- (IN1) $1 : \frac{1}{4} : \frac{1}{4} : \frac{1}{16} r^2 \sin^2 2\theta_3 : \frac{1}{16} r^2 \sin^2 2\theta_3 : \frac{1}{2};$
- (IN2) $\cot^2 2\theta_3 : \frac{1}{4} \cot^2 2\theta_3 : \frac{1}{4} \cot^2 2\theta_3 : 1 : 1 : \frac{1}{2} \cot^2 2\theta_3;$
- (DG1) $1 : 1 : 1 : \frac{1}{16} R^2 r^2 \sin^2 2\theta_3 : \frac{1}{16} R^2 r^2 \sin^2 2\theta_3 : \frac{1}{8} R^2;$
- (DG2) $\frac{1}{2} : \frac{1}{32} R^2 : \frac{1}{32} R^2 : \frac{1}{8} r^2 \sin^2 2\theta_3 : \frac{1}{8} r^2 \sin^2 2\theta_3 : 1;$
- (DG3) $\cot^2 2\theta_3 : \frac{1}{4} \tan^2 \theta_3 : \frac{1}{4} \tan^2 \theta_3 : 1 : 1 : \frac{1}{2} \cot^2 \theta_3;$
- (DG4) $\cot^2 2\theta_3 : \frac{1}{4} \cot^2 \theta_3 : \frac{1}{4} \cot^2 \theta_3 : 1 : 1 : \frac{1}{2} \tan^2 \theta_3.$

(36)

In the above expressions, we assumed that s_2 is negligible.

In the linear collider with $\sqrt{s} = 1$ TeV, the pair production cross section is $\sigma \approx (100\text{--}10)$ fb for $M_{\Delta^{\pm\pm}} = (100\text{--}450)$ GeV [10]. Thus, taking $L = 1000/\text{fb}$, the number of the produced $\Delta^{\pm\pm}$ will be $N = (10^5\text{--}10^4)$. In LHC with $L = 1000/\text{fb}$, the number of the reconstructed pair production events is expected to be $N = (10^5\text{--}10^3)$ for $M_{\Delta^{\pm\pm}} = (100\text{--}450)$ GeV and it becomes down to $N = 10$ for $M_{\Delta^{\pm\pm}} = 1000$ GeV [11]. Thus, both the linear collider and LHC can produce enough numbers of $\Delta^{\pm\pm}$ to probe the neutrino mass pattern if $M_{\Delta^{\pm\pm}} \lesssim 450$ GeV. In this case, the precise measurement of the branching ratios can also determine the neutrino oscillation parameters such as r , R or θ_3 . It is amusing to note that LHC has a good potential to confirm the triplet Higgs model as the source of neutrino mass matrix up to the triplet mass around 1 TeV. For this, the observation of the leading decay modes will be enough to discriminate the neutrino mass patterns as follows:

- (HI) $B(\mu\mu) : B(\tau\tau) : B(\mu\tau) = \frac{1}{2} : \frac{1}{2} : 1;$
- (IN1) $B(ee) : B(\mu\mu) : B(\tau\tau) : B(\mu\tau) = 1 : \frac{1}{4} : \frac{1}{4} : \frac{1}{2};$
- (IN2) $B(e\mu) : B(e\tau) = 1 : 1;$
- (DG1) $B(ee) : B(\mu\mu) : B(\tau\tau) = 1 : 1 : 1;$
- (DG2) $B(ee) : B(\tau\tau) = 1 : 1;$
- (DG3) $B(e\mu) : B(e\tau) : B(\mu\tau) = 1 : 1 : \frac{1}{2} \cot^2 \theta_3;$
- (DG4) $B(\mu\mu) : B(\tau\tau) : B(e\mu) : B(e\tau) = \frac{1}{4} \cot^2 \theta_3 : \frac{1}{4} \cot^2 \theta_3 : 1 : 1.$

(37)

Here we assumed that $\cot 2\theta_3$ and $\tan \theta_3$ sit at their lowest allowed values and thus give a sub-leading effect.

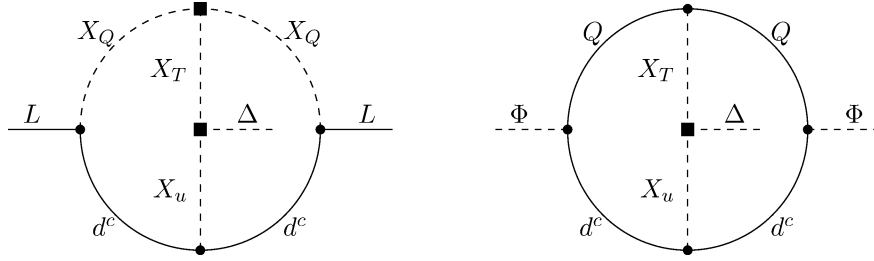


Fig. 1. Two loop diagrams generating the operators $LL\Delta$ and $\Phi\Phi\Delta$. Black squares represent vertices with $\langle S \rangle$.

4. A model: two-loop generation of $LL\Delta$ and $\Phi\Phi\Delta$

An unnatural feature of the Higgs triplet model generating the neutrino mass is that the model requires another hierarchy of couplings; the smallness of f or μ . This would have the same origin as the hierarchies of the usual quark and lepton Yukawa couplings, which is one of the difficult problems in particle physics. In this section, we separate the neutrino sector from the other and try to explain the smallness of f or μ through a radiative mechanism. In the case of $f \gg \mu$, a way to get the small μ has been explored in Ref. [18] in which the operator $\Phi\Phi\Delta$ has been obtained at two loop. A variant of such a scheme can be found to explain the smallness of both f and μ . For this, let us introduce the following new scalar fields and a Z_3 discrete symmetry:

X_T	X_Q	X_u	S
$(3, 3, -\frac{1}{3})_1$	$(3, 2, \frac{1}{6})_{\alpha^2}$	$(\bar{3}, 1, -\frac{2}{3})_1$	$(1, 1, 0)_\alpha$

(38)

where the $SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_3$ charge of each field is specified in the second line and $\alpha = e^{2\pi/3}$. We assign the Z_3 charge α to L and α^2 to e^c and Δ . All the other fields are neutral under Z_3 . The allowed couplings are

$$QQX_T, \quad Ld^cX_Q, \quad d^cd^cX_u, \quad X_QX_QX_T S^*, \quad \Delta X_T X_u S. \quad (39)$$

Then the operators $LL\Delta S^2$ and $\Phi\Phi\Delta S$ arise from the two-loop diagrams as in Fig. 1 and thus the small values of f and μ can be obtained when S gets a VEV of the order v_Φ .

5. Conclusion

We have investigated the testability of the low-energy Higgs triplet model and the resulting neutrino masses and mixing in the future collider experiments. The bileptonic couplings f_{ij} can be large enough to yield observable lepton flavor violating decays of a charged lepton such as $\mu \rightarrow 3e$, $\mu \rightarrow e\gamma$ or $\tau \rightarrow 3l$ depending on the neutrino mass patterns. For this to happen, the coupling f_{12} needs to be vanishingly small in order to satisfy the current bound on the $\mu \rightarrow 3e$ decay. Another effect of the bileptonic couplings is the production of a doubly charged Higgs boson accompanied by a charged lepton l_i in the e^+e^- collider. In this case, we have identified the characteristic flavor structure of the final state, $l_i^\mp \Delta^{\pm\pm}$, for each neutrino mass pattern. We have shown that copious production of the doubly charged Higgs boson pairs through the gauge interactions in the linear collider and LHC provides a promising way to test not only the triplet Higgs model but also the resulting neutrino mass matrix even when f is very small. In LHC, in particular, we expect sufficient production of the doubly charged Higgs bosons up to the mass ~ 1 TeV which will enable us to determine the neutrino mass pattern only by observing the leading decay channels. A problem in the low-energy triplet Higgs model is how to understand the smallness of the couplings f and μ . We have also worked out a radiative mechanism as one of possible solutions.

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