# Effect of neutrino trapping on the three flavor FFLO phase of QCD

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## Abstract

We compute the effect of a non-zero lepton chemical potential on the structure of the three flavor Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase of QCD at finite temperature. We show that, as in the BCS case, the lepton chemical potential favors two-species color superconductivity and disfavors the three species pairing. We stress that this study could be relevant for the cooling of a proto-neutron star with a FFLO core, if the temperatures are higher than the un-trapping temperature.
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## 1. Introduction

Investigations on the phase diagram of Quantum-Chromo-Dynamics (QCD) in the high quark density and low temperature regime has attracted a lot of interest in the last years; apart the purely theoretical speculations, these studies might be relevant for a deepest understanding of the physics of compact stellar objects, where cold and dense quark matter could be present. In this regime QCD predicts Cooper pairing of quarks, due to the existence of an attractive quark interaction in the color antisymmetric channel, see [1,2] and for reviews [3]. Formally, one introduces a bilinear quark expectation value (namely a di-quark condensate) in order to describe the collective pairing; since a pair of quarks is not a color singlet under $SU(3)_{color}$, the condensate spontaneously breaks the color symmetry. This phenomenon is similar to the Cooper pairing in ordinary (electromagnetic) BCS superconductors [4], where the role of the quarks is played by the electrons and $SU(3)_{color}$ is replaced by the electromagnetic $U(1)$. Because of this analogy the quark condensation phenomenon is known as color superconductivity. The spectrum

of the color superconductive phases is usually characterized by gapped fermions, massive gluons and Goldstone bosons related to the breaking of some the global symmetries of the QCD Lagrangian.

At asymptotic densities and zero temperature, the ground state of color superconductivity with massless up ($u$), down ($d$) and strange ($s$) quarks is the Color-Flavor-Locking (CFL) phase [5]. In the intermediate densities regime, as could be found in the interior of a compact stellar object, one cannot neglect neither the strange quark mass nor the differences $\delta\mu$ in the quark chemical potentials, induced by $\beta$ equilibrium and electric and color neutrality constraints. Therefore in the pre-asymptotic regime the CFL phase could be replaced by another ground state. Several ground states have been proposed in the literature as the candidates for the "pre-CFL" phases; we recall here the 2SC phase [2], and the gapless phases g2SC [6] and gCFL [7,8] (see [9] for recent studies). In all the above phases the Cooper pairs are characterized by a vanishing total momentum. This allows the whole Fermi sphere to partecipate to the pairing. Moreover, the pairs have vanishing total spin. However, the gapless phases present chromo-magnetic instability [10–14] as they show imaginary gluon Meissner masses: this should be intimately connected to the existence of the gapless modes in these phases. An instability is present also in the 2SC phase [10] (in [15–20] are discussed some antidotes to cure

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the chromo-magnetic instability of the 2SC and of the gapless phases).

For appropriate values of $\delta\mu$, it can be advantageous for quarks to form pairs with non-vanishing total momentum $2\boldsymbol{q}$. This state, introduced for the first time in the sixties in the contest of electromagnetic superconductors, is known as Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase [26]. Its relevance in two flavor QCD has been discussed in [27,28] (see [29] for a review). By virtue of the non-vanishing pair momentum, only a small region of the Fermi sphere is interested in the pairing phenomenon. This results in condensates that are smaller than the BCS ones. Moreover, the quark condensate in the FFLO phase is space-dependent. As a consequence, the translational and rotational symmetries are spontaneously broken and the spectrum of the low energy excitations is enriched by the presence of the respective Goldstone bosons, namely the phonons. As far as instability is concerned, the authors in [30] have shown that, with two flavors, the instability of 2SC implies that the FFLO phase is energetically favored. However, the problem of the chromomagnetic instability of the FFLO phase is under debate [31,32].

At densities relevant for the physics of the compact stellar objects the three flavors $u$, $d$ and $s$ can form FFLO pairs. In [33] a first attempt to the study of the three flavor FFLO phase (at zero temperature) has been presented, based on a Ginzburg–Landau (GL) expansion of the pressure. The assumed pairing ansatz is

$$\langle \psi_{i\alpha} C \gamma_5 \psi_{\beta j} \rangle = \sum_{I=1}^{3} \Delta_I(\boldsymbol{r}) \epsilon^{\alpha\beta I} \epsilon_{ijI} \qquad (1)$$

with

$$\Delta_I(\boldsymbol{r}) = \Delta_I \exp(2i\boldsymbol{q}_I \cdot \boldsymbol{r}), \qquad (2)$$

and the $\Delta_I$'s are the gap parameters. In the above equation $2\boldsymbol{q}_I$ represents the momentum of the Cooper pair. For values of the strange quark mass such that the FFLO phase is energetically favored with respect to the homogeneous phases, and among the different structures considered, it was found that $\Delta_1 = 0$ and $\Delta_2 = \Delta_3$, $\boldsymbol{q}_2 = -\boldsymbol{q}_3$ is the configuration with the highest pressure.

Recent interest has been devoted to the study of the relevance of a lepton chemical potential on the phase diagram of QCD [21–23]. This problem is directly connected to the phenomenon of the neutrino trapping, which could occur in the first cooling "era" of a proto-neutron star. In more detail, it is well known that the cooling of a neutron star occurs via the emission of neutrinos [24,25] (there is also a black body contribution due to photon emission, but it is irrelevant for our scopes); if temperatures are not lower than $\simeq 1$ MeV then there is a spherical inner region in the star, the neutrinosphere, from where only a very small fraction of neutrinos escapes (typically the star needs a few tens of seconds to reach the internal temperature of $\simeq 1$ MeV). The radius of the neutrinosphere (measured from the center of the star) depends on the temperature, and decreases as the star cools. Therefore the trapping interests regions closer and closer to the center of the star as its temperature lowers. As a consequence of the trapping there exists a (quasi-)conserved

charge, the lepton number, so one can introduce a lepton chemical potential $\mu_L$ associated to it. If color superconductive quark matter is present in the cooling star, it would be interesting to investigate about the effect of the neutrino trapping on its structure. Recent works along this line show that a non-zero $\mu_L$ has the effect to favor the 2SC phase, disfavoring the CFL phase [22,23].

In this short Letter we extend the results of [33], investigating the role of a non-zero $\mu_L$ on the three flavor FFLO phase of QCD. This study could be relevant if one conjectures that in the cooling era of a proto-neutron star, when neutrinos can be trapped, a FFLO phase is present in the star. Since the neutrino trapping requires a non-zero temperature, we work at finite temperature in all this Letter (we should mention here that the critical temperature of the FFLO phase is expected to be lower than the homogeneous one [34]).

## 2. The model

We consider an electrical and color neutral system of massless $u$, $d$ and massive $s$ quarks, in $\beta$ equilibrium with massless electrons and their neurinos. The lepton sector is described by the Dirac Lagrangian

$$\mathcal{L}_l = \bar{\psi}(i\slashed{\partial} + \mu_l \gamma_0)\psi, \qquad (3)$$

where we have collected the electron and the neutrino fields into the doublet $\psi = (e, \nu)$, and the chemical potential matrix is $\mu_l \equiv \mathrm{diag}(\mu_e, \mu_\nu) = \mathrm{diag}(-\mu_Q + \mu_L, \mu_L)$; $\mu_Q$ is the chemical potential associated to the conserved electric charge of the system. From the Lagrangian (3) one derives the pressure [35]

$$p_l = \frac{T}{2\pi^2} \sum_{a=e,\nu} g_l \int_0^\infty dk\, k^2 \log\big(1 + e^{\frac{\mu_a - k}{T}}\big), \qquad (4)$$

where $g_l$ is the degeneration factor (that counts the spin degrees of freedom): it is equal to 2 for electrons and 1 for neutrinos. One can check the correct normalization of $p_l$, that in the limit $T \to 0$ becomes $p_l = \mu_e^4/12\pi^2 + \mu_\nu^4/24\pi^2$.

Next we consider the quark sector. The quark Lagrangian is

$$\mathcal{L}_q = \bar{\psi}(i\slashed{\partial} - M + \mu\gamma_0)\psi - \frac{3}{8} G \bar{\psi}\gamma^\mu \lambda_a \psi\, \bar{\psi}\gamma^\mu \lambda_a \psi, \qquad (5)$$

where $M_{ij}^{\alpha\beta} = \delta^{\alpha\beta} \mathrm{diag}(0, 0, M_s)$ is the current mass matrix; $\mu_{\alpha\beta}^{ij}$ is the matrix of the chemical potentials: they depend in general on $\mu$ (the baryon chemical potential), $\mu_Q$, and $\mu_3$, $\mu_8$, related to the conserved color charges:

$$\mu_{\alpha\beta}^{ij} = (\mu\delta_{ij} + \mu_Q Q_{ij})\delta^{\alpha\beta} + \Big(\mu_3 T_3^{\alpha\beta} + \mu_8 \frac{2}{\sqrt{3}} T_8^{\alpha\beta}\Big)\delta^{ij}, \quad (6)$$

where $T_3$ and $T_8$ are the usual $SU(3)$ generators. Actually one should consider eight color chemical potentials, one for each conserved color charge $n_a = \langle\psi T_a \psi\rangle$ [36]; we have checked explicitly that only $n_3$ and $n_8$ can be non-zero. This motivates the choice in Eq. (6).

The interaction term in Eq. (5) is a Nambu–Jona-Lasinio inspired four fermion interaction, that mimics the one gluon

exchange of QCD. Here $G$ is a coupling constant, with dimension mass$^{-2}$; $\lambda_a$ are color matrices and a sum over flavors is understood. In the mean field approximation, after a Fierz rearrangement, the interaction term becomes

$$-\frac{1}{2}\epsilon_{\alpha\beta I}\epsilon^{ijI}\left(\psi_i^\alpha\psi_j^\beta\Delta_I(\mathbf{r}) + \text{c.c.}\right)$$
$$+ (L \to R) - \frac{1}{G}\Delta_I(\mathbf{r})\Delta_I^*(\mathbf{r}), \tag{7}$$

where the assumed pairing ansatz is in Eq. (1). In getting Eq. (7) we have neglected the chiral condensate: its effect is the dressing of the bare quark masses; we expect that at intermediate densities (namely $\mu \sim 500$ MeV) the chiral condensate is small if compared to the quark–quark condensate. Therefore we can safely neglect the constituent $u$ and $d$ masses. As for the strange quark, we should write a gap equation for its constituent mass; it should to be solved simultaneously to the gap equations for the di-quark condensate. A similar analysis has been performed recently in [9]. For simplicity we do not solve the chiral gap equation and we assume the strange quark mass as an external parameter.

We now discuss the other approximations used in the quark sector. First, we consider only the leading order effect of the strange quark mass, namely a shift in its chemical potential $\mu_s \to \mu_s - M_s^2/2\mu$. Second, to ensure color and electrical neutrality of the system, the chemical potentials related to the conserved charges have to satisfy the stationarity relations

$$\frac{\partial p}{\partial\mu_Q} = \frac{\partial p}{\partial\mu_3} = \frac{\partial p}{\partial\mu_8} = 0, \tag{8}$$

where $p = p_l + p_{\text{quarks}}$ (see below, Eq. (10)). As in Ref. [33], we work in the approximation $\mu_3 = \mu_8 = 0$. This assumption is justified because the phase transition from the superconductive to the normal phase is second order. Therefore, near the phase transition we expect $\mu_3$, $\mu_8$ not too different from their values in the normal phase, namely zero. In this way one has

$$\mu_u = \mu + \frac{2}{3}\mu_Q, \qquad \mu_d = \mu - \frac{1}{3}\mu_Q,$$
$$\mu_s = \mu - \frac{1}{3}\mu_Q - \frac{M_s^2}{2\mu}. \tag{9}$$

Moreover we assume $\Delta_1 = 0$. This is justified because at $T = 0$ and $\mu_L = 0$ one has $\mu_Q = -M_s^2/4\mu$. As can be inferred from Eq. (9) this implies that the difference of the chemical potentials $\delta\mu_{ds}$ between $d$ and $s$ quarks is greater than $\delta\mu_{ud}$ and $\delta\mu_{us}$. As a consequence, the pairing between $d$ and $s$ is disfavored. This is true also if $T \neq 0$ and $\mu_L \neq 0$ (see below). Finally, the interaction contribution to the pressure is derived in the Ginzburg–Landau (GL) expansion. Keeping this in mind the pressure of quark matter is

$$p_q = p_0 - \left(\frac{\alpha_2}{2}\Delta_2^2 + \frac{\beta_2}{4}\Delta_2^4 + \frac{\alpha_3}{2}\Delta_3^2\right.$$
$$\left. + \frac{\beta_3}{4}\Delta_3^4 + \frac{\beta_{23}}{2}\Delta_2^2\Delta_3^2 + O(\Delta^6)\right), \tag{10}$$

where $p_0$ is the pressure of the normal phase. In Eq. (10) the coefficients are given in terms of the functions [28,29,33]

$$\alpha(q,\delta\mu,T) = \frac{4\mu^2}{\pi^2}\left[\log\left(\frac{4\pi T}{\Delta_0}\right)\right.$$
$$\left. + \text{Re}\int\frac{d\mathbf{n}}{4\pi}\psi\left(\frac{1}{2} + i\frac{\delta\mu - \mathbf{q}\cdot\mathbf{n}}{2\pi T}\right)\right], \tag{11}$$

$$\beta(q,\delta\mu,T) = -\frac{\mu^2}{64\pi^4 T^2}\text{Re}\int\frac{d\mathbf{n}}{4\pi}\psi^{(2)}\left(\frac{1}{2} + i\frac{\delta\mu - \mathbf{q}\cdot\mathbf{n}}{2\pi T}\right) \tag{12}$$

as follows

$$\alpha_2 = \alpha\left(q_2, \frac{\mu_u - \mu_s}{2}, T\right), \qquad \alpha_3 = \alpha\left(q_3, \frac{\mu_d - \mu_u}{2}, T\right),$$
$$\beta_2 = \beta\left(q_2, \frac{\mu_u - \mu_s}{2}, T\right), \qquad \beta_3 = \beta\left(q_3, \frac{\mu_d - \mu_u}{2}, T\right);$$

moreover

$$\beta_{23} = -\frac{3\mu^2}{2\pi^2}\frac{1}{8\pi T^2}\int_0^1 dy_1\,dy_2\,\delta(1 - y_1 - y_2)$$
$$\times \int_{-1}^1 dz\,\text{Re}\,\psi^{(2)}\left(\frac{1}{2} + \frac{iA}{2\pi T}\right). \tag{13}$$

In the above relations $\mu$ is the baryon chemical potential as introduced in Eq. (6); the functions $\psi^{(n)}(z)$ are defined as $n$th-derivatives of the Euler $\psi(z)$ function, where $\psi(z) = \Gamma'(z)/\Gamma(z)$. In Eq. (13) we have introduced the function

$$A = y_1\frac{\mu_u - \mu_s}{2} - y_2\frac{\mu_d - \mu_u}{2} + z|y_1\mathbf{q}_2 + y_2\mathbf{q}_3|.$$

The angular integrals in Eqs. (11), (12) and (13) can be done exactly, but their expressions are uninformative so we prefer to leave them in implicit form. As for the integrals in $y_1$, $y_2$ in Eq. (13), one is performed by the $\delta(1 - y_1 - y_2)$ and the remaining integral can be performed numerically.

The magnitudes of the wave vectors $\mathbf{q}_I$ are determined by the variational condition $\frac{\partial p}{\partial q_I} = 0$; as in Ref. [33], in this Letter we use this relation at the lowest order, $\frac{\partial\alpha_I}{\partial q_I} = 0$. If $T = 0$ this approximation leads, as in the two flavor case, to the well-known relation $q_I = 1.1997|\delta\mu_I|$.

## 3. Results and conclusions

In this section we discuss our results; the coupling constant $G$ can be eliminated in favor of $\Delta_0$, the gap parameter of the homogeneous three flavor superconductor at $T = 0$ and $\delta\mu = 0$ [33]. We show the results obtained for $\Delta_0 = 25$ MeV, $\mu = 500$ MeV and $T = 0.1\Delta_0$ (for different values of the parameters we get qualitatively similar results). In Fig. 1 we show the electrical chemical potential $\mu_Q$ that satisfies the stationarity condition (8) as a function of $M_s^2/\mu$ for four values of the lepton chemical potential $\mu_L$. We notice that for a fixed value of the strange quark mass, increasing $\mu_L$ results in the decreasing of $|\mu_Q|$. This has important consequences on the pairing.

Indeed one has from Eq. (9)

$$\mu_d - \mu_u = -\mu_Q, \qquad \mu_u - \mu_s = \mu_Q + \frac{M_s^2}{2\mu}.$$

For $\mu_L = 0$ one has $\mu_Q = -M_s^2/4\mu$; therefore the mismatch between the $u$ and the $d$ Fermi surfaces is the same of the $u$ and the $s$ one. For $\mu_L \neq 0$ this is no longer true: in particular from Fig. 1 one gets $\mu_u - \mu_s > \mu_d - \mu_u$. This favors the pairing in the $u$–$d$ channel while disfavors the $u$–$s$ pairing.

This is better expressed in Fig. 2, where we show the results for the gap parameters $\Delta_2$, $\Delta_3$ as a function of $M_s^2/\mu$, for $\mu_L = 0$ (left panel) and $\mu_L = 200$ MeV (right panel). In the latter case we observe that for low values of the strange quark mass the gaps $\Delta_2$ and $\Delta_3$ have comparable magnitude. This means that $u$–$d$ and $u$–$s$ pairing are both active and the FFLO state is effectively a three flavor color superconductor. Increasing $M_s$ we find the window $130 \gtrsim M_s^2/\mu \lesssim 148$ MeV where both $u$–$d$ and $u$–$s$ pairing are active, but $\Delta_2 \ll \Delta_3$ meaning that the latter is disfavored if compared to the former. At $M_s^2/\mu \simeq 148$ MeV a second order phase transition takes place to the state with $\Delta_2 = 0$ and $\Delta_3 \neq 0$, that is a two flavor FFLO state with pairing in the $u$–$d$ channel. This is in perfect agreement to what is found in the homogeneous case [22,23]. Finally,
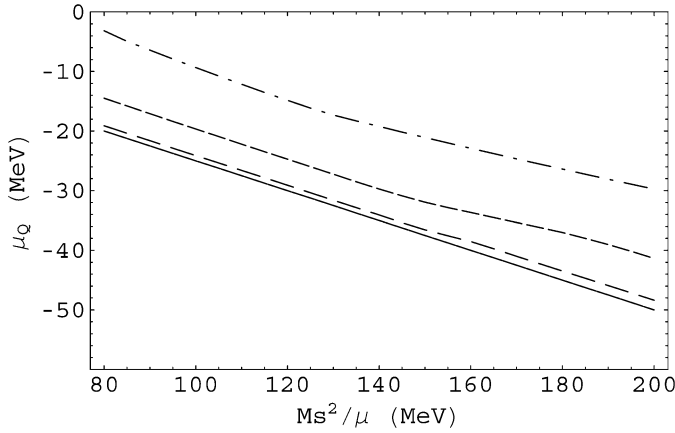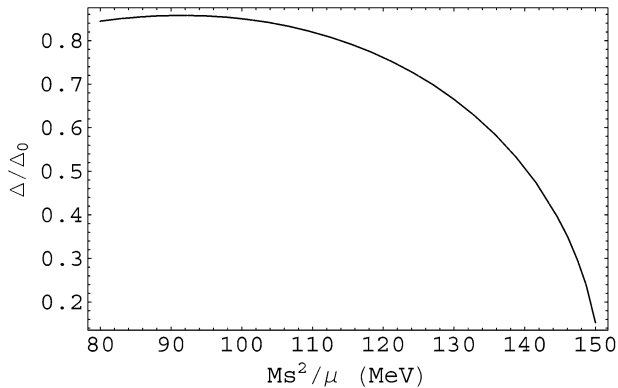
for $M_s^2/\mu \simeq 185$ MeV there is a second order phase transition to the non-superconductive phase. We find also that the larger is the value of $\mu_L$, the larger is window of $M_s^2/\mu$ where $\Delta_2 = 0$ and $\Delta_3 \neq 0$.

In conclusion, we have investigated the role of a lepton chemical potential on the structure of the three flavor FFLO phase of QCD. We find that a non-zero $\mu_L$ strongly favors two flavor pairing, in the channel $u$–$d$, while disfavoring the pairing $u$–$s$. This is a result of imposing electric neutrality in the system.

This study could be relevant for the cooling of a proto-neutron star. If one suppose that normal (that is, non-superconductive) quark matter is present in the star, then the neutrino emissivity will be dominated by direct Urca processes [24,25]; denoting by $\epsilon_\nu^{\text{Urca}}(T)$ the neutrino emissivity of normal quark matter in absence of trapping at the temperature $T$, the effect of trapping is an exponential suppression of the emissivity itself [37]

$$\bar{\epsilon}_\nu(r, \theta, T) = \exp\left(-\frac{l(r, \theta)}{\lambda(T)}\right) \times \epsilon_\nu^{\text{Urca}}(T), \tag{14}$$

here $\bar{\epsilon}_\nu(r, \theta, T)$ denotes the emissivity with the effect of the trapping included, $l(r, \theta)$ is the distance from the creation point of the neutrino to the surface of the star (more precisely, its projection along the $z$-axes). Finally, $\lambda(T)$ is the neutrino mean free path at the temperature $T$. The exponential factor in Eq. (14) takes into account the probability that a neutrino created at a distance $r$ from the center of the star can leave the star in the direction defined by the angle $\theta$. One computes the total luminosity for neutrino emission from the star by averaging Eq. (14) over all neutrino directions $\theta$ and integrating over all distances $r$ up to the star radius.

We turn to color superconductive quark matter. Un-trapping occurs for temperatures of order of 1 MeV (for higher temperatures, neutrinos are trapped); this imply that trapping effects should not be neglected in emissivity computations, if one considers the first cooling epoch of a neutron star (as explained in the introduction, typically such internal temperatures are reached on a time scale of a few tens of seconds). Investigations on the effects of color superconductive quark matter
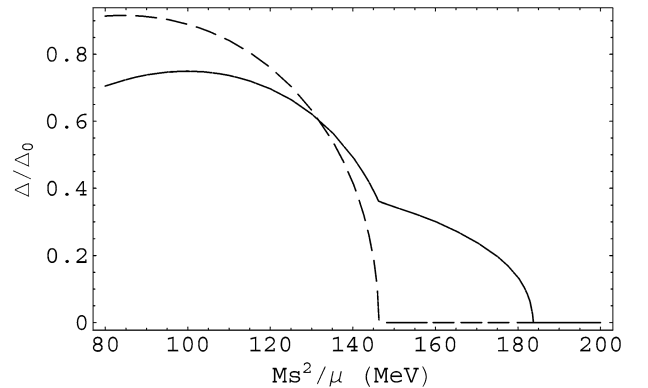


Fig. 1. Chemical potential $\mu_Q$ as a function of $M_s^2/\mu$ at $T = 0.1\Delta_0$. The cases $\mu_L = 0, 100, 200$ and $300$ MeV are represented respectively by the solid, long-dashed, short-dashed and dot dashed lines.



Fig. 2. Left panel: $\Delta_2/\Delta_0 = \Delta_3/\Delta_0$ against $M_s^2/\mu$, evaluated for $\mu_L = 0$. Right panel: $\Delta_2/\Delta_0$ (dashed line) and $\Delta_3/\Delta_0$ (solid line) as a function of $M_s^2/\mu$, computed for $\mu_L = 200$ MeV. In both pictures $T = 0.1\Delta_0$.

on the cooling evolution of a neutron star, neglecting neutrino trapping, have been performed in [38,39] (see also references therein). It will be interesting to see the effect of the FFLO phase on neutrino emissivity, if such a phase is actually present in the core of compact stars.

## Acknowledgements

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