Acoustic extraordinary refraction in layered sonic crystals with periodic slits

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A R T I C L E  I N F O
Article history:
Received 25 February 2016
Accepted 16 March 2016
Available online 25 March 2016

Keywords:
Sonic crystals
Backward effects
Forward effects
Beam splitting

A B S T R A C T
In this paper, we present a study of acoustic wave propagation in layered sonic crystals (SCs) with periodic slits, and demonstrate the extraordinary acoustic refractions induced by acoustic backward and forward wave effects in SCs. By tuning the parameters of geometry structures and incident angles of acoustic waves, the positive, negative and zero acoustic refractions are demonstrated respectively. Taking advantage of the exotic acoustic refraction, we present the splitting of acoustic beams through the layered SCs, which is a new method to achieve acoustic beam splitting.

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Introduction
In recent two decades, the acoustic artificial materials have attracted desirable attention due to their strong applications [1–4]. On the basis of sonic crystals (SCs), numerous applications and devices have been designed to achieve many abnormal acoustic phenomena and functions [5–8]. One of the most interesting and exotic phenomena and functions is acoustic negative refraction by flat sonic crystals [9–13]. Because of the extensive research interest and applications, the acoustic negative refraction based on sonic crystals has been comprehensively studied in these years [14–16], also incurred the research of acoustic zero refraction and positive refraction on the basis of sonic crystals [17,18]. The acoustic negative refraction of sonic crystals can be achieved both by the acoustic waves backward effect [8,15,16] and forward effect [4,9,14]. Recently, the layered metamaterial with periodic perforated plates has attracted much interesting due to its high anisotropy [16,19,20]. The acoustic negative refraction and focusing by forward and backward effects have been extensively studied based on the layered metamaterials [16], which are mainly restricted in relatively low frequency. In this paper, we study the acoustic refractions in a wide frequency range through the layered sonic crystals with periodic slitted plates. We demonstrate different acoustic refractions by tuning the parameters of the layered sonic crystals. Further, we study the beam splitting of acoustic waves through the sonic crystal by using the exotic refraction.

Results and discussion
The layered sonic crystal under study is schematically shown in Fig. 1, which consists of periodic steel plates with periodic slits immersed in water. The thickness of the plate is t, the width of the slits is d, the period of the plates is p_s (along z direction) and the period of the slits is p_f (along x direction). The material parameters are as follows: mass density ρ = 7760 kg/m³, longitudinal acoustic velocity v_L = 6010 m/s and transverse acoustic velocity v_T = 3320 m/s for steel; mass density ρ_d = 1000 kg/m³, longitudinal acoustic v_L = 1490 m/s for water. Specifically, for simplifying the structure parameters in the study, we set the slits period p_s = 1 mm, the slits width d = 0.5 mm, and the plates thickness t = 0.4 mm. The period p_f for plates can be tuned to achieve different acoustic refractions induced by acoustic forward and backward wave effects, which will be analyzed in detail below. Throughout the study, we use the Finite Element based software COMSOL Multiphysics to perform the band structures, transmissions and filed distribution calculations.

Firstly, we investigate band structures of two SCs with different structure parameters structure p_f/p_s = 1.88 and p_f/p_s = 1.10 respectively, and the results are shown in Fig. 2. As is well known, the propagation of acoustic waves can be determined by the phase velocity (v_p = |k|/|f|) and the group velocity (v_g = ∇φ/|f|). We select the frequency of acoustic waves f = 0.61(v_f/p_f) for example to demonstrate the forward and backward wave effects in the system, as shown in Fig. 2. For the system p_f/p_s = 1.88 [Fig. 2(a)], we can see that the wave vector along z−X_2 direction increases as the frequency increases near the frequency f = 0.61(v_f/p_f) (see the third
wave effect. However, for the system the propagating direction of group velocity and phase velocity of the wave-vector direction is also forward (positive). Therefore, we can obtain another picture of acoustic wave propagation in the layered SC. We can see that the wave vector along the EFC changes from outside. In this process, the parallel component of the phase velocity is conserved across the interface following the Snell’s law of refraction: \( k_2 \cdot \sin \theta_i = k_1 \cdot \sin \theta_r \). Here, \( k_1 \) is the incident vector, \( \theta_i \) is the angle of incidence, \( \theta_r \) is the angle of refraction [21]. As shown in Fig. 3(a) and (c), for the incident waves with 30°, the blue circles are the EFC in water, and the red curves are the EFCs of the sonic crystal. \( k_1 \) and \( S_1 \) are the wave vector and group velocity of the incident waves respectively, which have the same direction. Based on the parallel-k-conservation rule, the \( k_2 \) and \( S_2 \) in Fig. 3 are the wave vector and group velocity of the refracted Bloch waves respectively. We can see that, for the system with \( p_z/p_x = 1.88 \), the directions for the phase velocity and group velocity go forward (forward wave effect). But for the system with \( p_z/p_x = 1.10 \), the group velocity goes forward, while the phase velocity goes backward (backward wave effect). Interestingly, these two refractions are both negative. In order to verify the analysis above, we calculate the field distributions for a Gaussian beam incident into the sonic crystals with the incident angle of 30°, and show them in Fig. 3(b) and (d) respectively. We can see that the propagations of the acoustic waves in the sonic crystals are negatively refracted, as expected.

In Fig. 3(a), we can observe that the EFC in the upper part with \( k_x \) ranging from 0 to 0.25(2\( \pi/p_x \)) is convex. Importantly, the tangent of this part EFC changes from \( k_x \) positive direction into \( k_x \) negative direction. Based on the analysis above, we can conjecture that for the incident waves with \( k_x \) ranging from 0 to 0.25, the acoustic refraction induced by the sonic crystal would change from positive refraction into negative refraction under the forward effect. In order to verify the conjecture, we calculate the pressure field distributions with different incident angles. For a smaller incident angle (23°), a positive refraction is observed in Fig. 4(a). Whereas we increase the incident angle to 27°, the refractive angle becomes very small and is almost close to zero [Fig. 4(b)]. Then, continuing to increase the incident angle, the negative refraction would be occurred, as discussed in Fig. 3(a). Similarly, we can observe that, for the system with \( p_z/p_x = 1.10 \) and the EFC with frequency 0.61 (\( c_0/p_x \)) [Fig. 3(b)], the lower part with \( k_x \) ranging from 0 to 0.25 is convex. The tangent changes from \( k_x \) positive direction into \( k_x \) negative direction. Therefore, for the incident angles increasing from 0, the refraction of sonic crystal will change from positive refraction to zero refraction, and then negative refraction under the backward effects. In Fig. 4(c) and (d), we calculate the field distributions for incident angle 23° and 26°. As expected, we can observe the positive, nearly zero. Again, for continuing to increase the incident angle, the negative refraction is occurred as shown in Fig. 3(b). When the incident angle continually increases to the angle that

![Fig. 1. Schematic demonstration for the layered sonic crystal with periodic slit plates.](image1)

![Fig. 2. Band structures for SCs with two parameter set: (a) \((p_z, p_x) = (1.88, 1)\), and (b) \((p_z, p_x) = (1.10, 1)\). The insets represent the first Brillouin zone.](image2)
the incident wave vector \( k_x \) cannot match any Bloch mode in sonic crystal, the total reflection will occur.

Another exotic property for the system is the acoustic beam splitting, i.e., the presence of two or more output beams through the system with one beam inputted. We calculate the band structure for the SC with the parameter being \( pz/px = 1.52 \) and show the results in Fig. 5(a). Firstly, we consider a 30° incident beam along \( x \) direction [see Fig. 5(b) and (c)] at the normalized frequency \( f = 0.69 (c_0/px) \). EFC of the SC at frequency \( f = 0.69 (c_0/px) \) is calculated and shown in Fig. 5(b), where the EFC of water is also given as reference. Since the EFC of water (blue circle) is much bigger than the EFC for the system in the first Brillouin zone (red curves), we extend EFC of the system into the second Brillouin zones (green curves). One sees that the circular EFC in water is twice as large as the first Brillouin zone of the SC. It can be concluded from Fig. 5(b) that there are two Bloch modes in the SC excited by the incident waves, as marked with \( k_2 \) and \( k_1 \) in Fig. 5(b). Therefore, there would be two modes back to the background based on the matching of the wave vectors between the SC and the background, as marked with \( k_{r1} \) and \( k_{r2} \) in Fig. 5(b), which would lead to the presence of two beams. We calculate the field distribution to further validate the analysis, as shown in Fig. 5(c). We can see that the incident beam is split into two beams through the sonic crystal as expected, but there is one propagating path in the sonic crystal. The essence for the particular property is that the excited different modes inside the sonic crystal have the same group velocity but different wave vectors.

More output beams can also be realized by this system. As a demonstration, we consider a 0° incident beam along \( x \) direction [see Fig. 6(a) and (b)] at the normalized frequency \( f = 0.8 (c_0/px) \). The EFCs for SC and water at \( f = 0.8 (c_0/px) \) are shown in Fig. 6(a). We can see that there are two sets of EFCs for the SC in the first Brillouin zone: one looks like a pair of big round brackets ‘( )’ with
k_x ranging in (−0.32, 0.32); the other looks like a pair of small anti-round brackets "(" with k_x ranging in (−0.5, −0.45) (2π/px) and (0.45, 0.5). By analyzing the EFC, we can conclude that there would be six Bloch modes inside SC matching the incident wave vector. Due to the degeneracy in the matching of Bloch modes to the output modes in water, there comes out only three refracted beams after SC, as shown in Fig. 6(b). If the incidence angle is further increased to 14°, there will be no corresponding Bloch mode for the upper second Brillouin zone. Therefore, there are only four Bloch modes in SC that can match the incident wave vector, and there will be two output beams into the water after the SC due to the same degeneracy mentioned above, as demonstrated in Fig. 6(c).

Conclusions

In conclusion, we have studied the acoustic refractions in layered sonic crystals with periodic slit plates. We analyzed the backward and forward effects of acoustic waves in sonic crystal in detail and demonstrated the different acoustic refractions induced by the acoustic backward and forward effects. On the basis of the acoustic refraction, we studied the acoustic beam splitting through the sonic crystal. We achieved different output beams through the sonic crystal by tuning frequency and incident angle. These results have potential applications such as acoustic imaging and acoustic logic gate.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant No. 11464012, 11304119, 11564012 and 11564013), Natural Science Foundation of Education Department of Hunan Province, China (Grant No. 13A077), and Natural Science Foundation of Hunan Province, China (Grant No. 2016JJ2100). Aid program for Science and Technology Innovative Research Team in Higher Educational Institutions of Hunan Province.
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