Fatigue Strength Reduction Factor for Polycrystalline Nickel Base Superalloy with and without Non-Metallic Inclusions

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Abstract

Polycrystalline nickel base superalloy is popular for its wide applications; it is used in hot sections of power generation turbines, rocket engines and other challenging conditions due to its high strength and good creep, fatigue, and corrosion resistance at high temperature. However, the presence of inclusions introduced into the superalloy during the fabrication processes can significantly degrade the fatigue life. This paper utilizes a new probabilistic method which captures both the essence of microstructure and notch root stress gradient to determine the notch size and inclusions effects on the fatigue strength of notched nickel base superalloy with and without non-metallic inclusions. Notched cylindrical specimens of nickel base superalloy of varying notch root radii are modeled using microstructural-sensitive crystal plasticity finite element codes. The stresses extracted from the fatigue damage process zone around the notch are used in determining the fatigue strength reduction factor and the associated probability of failure of the notched specimens. The numerical results obtained are in direct correlation with the experimentally obtained value for the different notch root radii.

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1. Introduction

Nickel base superalloy IN 100 is widely used in the hot sections of the gas turbine engine due to its superior strength, high creep and corrosion resistance at high temperatures [1]. Inclusions play a major role in the fatigue...
strength reduction of the nickel base superalloy as the inclusions serve as zones of crack nucleation [2]. Popular inclusions commonly found in nickel base superalloy are carbides close to the grain boundaries or within a grain and pores within the polycrystal which results in incompatible deformation between the inclusions and the neighboring materials leading to localized plasticity when subjected to cyclic loading. In addition, the turbine engine operates at very high temperature and thus the blades have tiny pores for cooling purposes. The coolants are forced through the pores at high pressure for cooling of the system [3]. These pores serve as areas of stress concentration and are typical locations for localized micro-plasticity thus reducing the fatigue strength of the material when subjected to cyclic loading. The holes also serve as favorable zones for crack initiation leading to early retirement of the turbine blade and engine as shown in Fig. 1. Notches and inclusions form part of the major factors that dictate the fatigue strength of the aero engine material.

The fatigue strength reduction factor is used in the estimation of fatigue life and strength of notched structural components. For reliable fatigue life assessment and prevention of disastrous and calamitous fatigue failure of aero engines in service, it becomes of ultimate importance to develop robust methods of estimating the fatigue strength reduction factor of aero engine materials. In this paper, the concept of fatigue strength reduction factor, the ratio of unnotched to notched specimen fatigue strength in the high cycle fatigue (HCF) regime is effectively extended to incorporate microstructure sensitivity using probabilistic arguments. The uniqueness of this approach is that it combines elements from microstructure-sensitive crystal plasticity models for nickel-base superalloys with a nonlocal probabilistic mesomechanics framework for fatigue strength reduction and the probabilities of formation of small cracks at notches that depends on microstructure features including inclusions, grain size, compositions, etc.

2. Microstructure-Sensitive Constitutive Models and Finite Element Model

The crystal plasticity constitutive model developed by Mahesh [5] is used in this work. The shearing rate on each slip system is expressed as:

\[ \dot{\gamma}^\alpha = \dot{\gamma}_1 \left( \frac{\tau^\alpha - \chi^\alpha}{D^\alpha} - \kappa^\alpha \right)^{n_1} + \dot{\gamma}_2 \left( \frac{\tau^\alpha - \chi^\alpha}{D^\alpha} - \kappa^\alpha \right)^{n_2} \text{sgn} \left( \tau^\alpha - \chi^\alpha \right) \]  

(1)

where \( \dot{\gamma}_1 \) and \( \dot{\gamma}_2 \) are constants, the flow exponents are represented as \( n_1 \) and \( n_2 \), \( \kappa^\alpha \) is the slip resistance on each slip system, \( D^\alpha \) is the average drag resistance, \( \chi^\alpha \) is the back stress on each slip system. It is also important to note that the back stress is subtracted from the applied shear stress to reflect the net force responsible for driving the dislocation. The threshold stress accounts for the anomalous yield behavior of the \( \gamma \) - \( \gamma \) nickel base superalloy, i.e, the increase in yield stress resulting from increase in temperature in the intermediate temperature range. A total of 18 slip systems are used to model dislocation motion within the matrix of the nickel base super alloy; 12 octahedral slip systems and 6 cube slip systems. The 12 octahedral slip systems are the active at lower temperatures within the \( \gamma \) matrix which is typical of a FCC lattice structure. Taylor relation is assumed for the hardening of the threshold stress on each of the slip system and it is expressed as:

\[ \kappa^\alpha = \kappa_0^\alpha + \alpha_\rho \mu \sqrt[4]{\rho^\alpha} \]  

(2)

where each slip system dislocation density is represented as \( \rho^\alpha \) (for both cube and octahedral slip systems), \( \mu \) is the shear modulus and it is governed by a rule of mixtures, \( b \) is the effective burgers vector, \( \alpha_\rho \) is a coefficient which accounts for the statistical arrangement of the dislocation population. Based on the work of Qin et al. [6-7] and Shenoy et al. [8], the non-schmid stress, \( \tau_\alpha \), dependence of the octahedral slip systems is expressed as:
\[ \tau_{ss} = h_{ps} \tau_{ps} + h_{cb} \tau_{cb} + h_{se} \tau_{se} \]  

(3)

Here the resolved shear stress on the primary, secondary and cube slip systems are denoted by \( \tau_{ps}, \tau_{cb} \) and \( \tau_{se} \) respectively while \( h_{ps}, h_{cb}, h_{se} \) are constants. Back stress is used to capture the effects of Bauschinger effect and it is expressed as [9]:

\[ \chi_{s}^{a} = C_{s} \left\{ \eta \tilde{\mu} \tilde{b} \sqrt{\rho_{s}^{a}} \text{sgn}(\tau_{s}^{a} - \chi_{s}^{a}) - \chi_{s}^{a} \right\}^{\beta} \]  

(4)

where \( C_{s} \) is a fitting parameter and \( \eta \) reflects the relative proportion of geometrically necessary dislocations to total dislocation density.

The crystal plasticity model for nickel base superalloy is coded into ABAQUS 2006 UMAT. The notched geometry modeled in this work is a V-notched cylindrical component as shown in Figure 2. Different test cases are modeled which include: (a) nickel base specimen without inclusion, and (b) nickel base specimen with inclusions at varied distance from the notch root ranging from 4 to 10 times the grain size.

The metal carbide (MC) inclusions commonly found in nickel base superalloys are taken into consideration in this model. It is assumed that the shape of inclusions is elliptical with an aspect ratio \( a/b = 2 \), where \( a \) and \( b \) are the major and minor axis of the ellipse respectively. The MC inclusions are assigned linear elastic material properties with Young’s modulus 405 GPa and a Poisson’s ratio of 0.14 [5]. Here, the full geometry is modeled because there is no symmetry in the geometry due to presence of inclusions though symmetry is still maintained in load application. The V-notched specimen geometry has diameter \( D \) of 5 mm, notch root radius \( \rho \) of 0.213 mm and notch depth \( h \) of 0.267 mm with an applied stress of 568 MPa. The model is meshed using 3D stress four-node linear tetrahedron element type (C3D4) of size 64 \( \mu \)m as established by the mesh refinement study conducted.

**3. Weibull Weakest Link-based Probability of Failure and Fatigue Strength Reduction Factor Equations**

Based on Weibull’s weakest link theory, the probability of failure of a component having a fatigue damage process zone of volume \( V_d \), divided into small volume elements, \( dV \) is given as:

\[ P_f = 1 - \exp \left\{ -\frac{1}{V_d} \int_{V_d} \left( \frac{\sigma - \sigma_{th}}{\sigma_0} \right)^b \, dV \right\} \]  

(5)

where, \( b, \sigma_{th}, \sigma_0 \) represent a 3-parameter Weibull shape, location and scale parameters. \( \sigma \) is the stress distribution. To facilitate development of the expression for the fatigue strength reduction factor from Equation (5), the concept of stress homogeneity factor, is introduced in Equation (5) as:

\[ k = \frac{1}{V_d} \int_{V_d} \left( \frac{\sigma - \sigma_{th}}{\sigma_0} \right)^b \, dV \]  

(6)

Conventionally, the fatigue strength reduction factor is the ratio of unnotched to notched fatigue strength at the same probability of failure (usually 50%). Using Equations (5) and (6), the probability of failure of unnotched specimen and a notched specimen will be the same when
\[ \exp \left\{ -k_s \frac{V_s}{V_0} \left( \frac{\sigma_{\text{max},s}}{\sigma_0} \right) \right\} = \exp \left\{ -k_n \frac{V_n}{V_0} \left( \frac{\sigma_{\text{max},n}}{\sigma_0} \right) \right\} \]  

(7)

where the subscripts \( n \) and \( s \) represent the respective value of the variable for notched and smooth (unmatched) specimens respectively. For the life limiting case in which only one grain or element is critically stressed above the threshold, \( V_s = V_e \) (i.e. volume of element or grain) and \( k_s = 1 \); the fatigue strength reduction factor \( k_f \) can be obtained from thus Equation (7) as:

\[ k_f = \left[ \frac{1}{V_e} \right] \int \left( \frac{\sigma - \sigma_{\text{max},n}}{\sigma_{\text{max},n}} \right)^\gamma dV \left( \frac{V_e}{V} \right)^\gamma \]  

(8)

The yield stress, \( \sigma_{\text{y}} \), of the nickel base superalloy is 1045 MPa. The shape factor is determined to be 15.6 and the scale parameter is determined to be 2826.07 MPa.

4. Closed-Form Solution

Okeyoyin and Owolabi [10] obtained the closed-form solution for \( k_f \) of the form:

\[ k_f = \left[ \frac{1}{V_e} \right] \left( \frac{1}{2} \left( \frac{\rho}{a_c + \rho/2} \right)^\gamma \right)^\gamma \left( \frac{V_e}{V} \right)^\gamma \]  

(9)

where \( a_c \) is the critical distance.

5. Results and Discussion


Using Equation (5), the probability of failure was computed for ten different random grain orientations for each of the notch root radius and the results are plotted in Figure 3. The figure shows the combine effect of notch root radius and the grain orientation of nickel base superalloy on its probability of failure. It is observed that the probability of failure rapidly increases with increasing notch root radius. Also, it can be seen that the orientation of the nickel base superalloy grains plays a significant role in determining the susceptibility of the material to fatigue failure. For example, for nickel base superalloy with notch root of 0.284 mm, the probability of failure varies from 0.52 to 0.64. Thus, notches with favorably oriented grains exhibit higher probability of failure.

Using Equation (8), which is based on Weibull weakest link, the microstructure dependent, \( k_f \) for the estimated probability of failures are computed and the average \( k_f \) for each notch root radius is plotted as shown in Figure 4. Also, experimentally obtained \( k_f \) from Weiju and Nicholas [11] for nickel base superalloy are plotted for comparison purpose. The figure shows both the experimentally obtained \( k_f \) and that determined using the developed probabilistic framework. The results show that the \( k_f \) obtained follows the same trend as the experimentally obtained values. To show the general performance of the newly developed, the determined \( k_f \) using the new probabilistic approach and the closed form solution are plotted against \( k_f \) determined using existing conventional empirical methods including the Neuber, the Peterson, and the Heywood models are plotted in Figure 4. The percentage variance/deviation of each method from the experimental value is presented in Table 1. The results show that the \( k_f \) based on Weibull probabilistic exhibits relatively lower percentage variance from experimental values compared to other existing conventional methods.
Fig. 3. Probability of failure vs notch root radius for notched nickel base superalloy specimens without inclusion.

Fig. 4. Comparison of $k_f$ determined by the new model with existing conventional methods

Table 1: Percentage variance of $k_f$ from experimental value for different methods

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Experiment $K_f$</th>
<th>Weibull $K_f$ % Var</th>
<th>Neuber $K_f$ % Var</th>
<th>Peterson $K_f$ % Var</th>
<th>Heywood $K_f$ % Var</th>
<th>Closed form sol. $K_f$ % Var</th>
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<tr>
<td>0.150</td>
<td>1.56</td>
<td>1.54</td>
<td>1.28</td>
<td>1.61</td>
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<td>0.213</td>
<td>1.73</td>
<td>1.70</td>
<td>1.73</td>
<td>1.69</td>
<td>2.31</td>
<td>1.75</td>
</tr>
<tr>
<td>0.284</td>
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<td>1.74</td>
<td>-0.58</td>
<td>1.74</td>
<td>-0.58</td>
<td>1.78</td>
</tr>
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</table>
5.2 Fatigue Strength Reduction Factor for Nickel Base Super Alloy with Non-Metallic Inclusions

The $k_f$ is computed based on the stress distribution extracted from Abaqus for notch nickel base super alloy with inclusion. Table 2 shows how the computed $k_f$ compare with the $k_f$ of notched nickel base superalloy without inclusion. Table 2 shows that the introduction of inclusions in the matrix of the notched nickel base superalloy increases the $k_f$. Thus component with inclusions of any kind will tend to have lower fatigue strength than components without inclusion. This is expected based on the fact that inclusions serve as favorable sites of premature plastic deformation due to high stress concentration in the vicinity of the inclusion curvature.

Table 2: Fatigue strength reduction factor of notched nickel base superalloy with and without inclusion

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Without Inclusion</th>
<th>$k_f$</th>
<th>With Inclusion</th>
</tr>
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<tbody>
<tr>
<td>0.150</td>
<td>1.54</td>
<td>2.01</td>
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<tr>
<td>0.213</td>
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<td>0.284</td>
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6. Conclusions

A new probabilistic mesomechanics-based model was developed to obtain the fatigue strength reduction of notched nickel base superalloy materials with and without non-metallic inclusions. The result shows that the probability of failure and the fatigue strength reduction factor all increase with increasing notch root radius. Also it is noted that the grain orientation and the presence of inclusions play an important role in predicting the fatigue strength of the material. The presence of inclusions around the fatigue damage process zone increases the fatigue strength reduction factor of the notched nickel base superalloy.

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