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Variation P-set and its structure

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Abstract

Variation P-set is put forward from attribute viewpoint, and its structure is given. Variation P-set is composed of variation interior P-set and variation outer P-set. Duality theorems between variation P-set and P-set are proposed. The properties of variation P-set are discussed, and the interior P-theorem, outer P-theorem of variation P-set and discrete interval dynamic generation theorem of the packet degree are proposed. The relations between variation P-set and P-set are analyzed, and the interior P-decomposition theorem and outer P-decomposition theorem of variation P-set are obtained. Variation P-set is a new research direction of P-set.

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Keywords: P-set; Attribute set; Variation P-set; Duality theorem; Packet degree

1. Instruction

P-set (packet set) was put forward in [1-4]. It tells us the following fact: Given a finite general set $X=\{x_1,x_2,\cdots,x_m\}\subset U$, U is a finite element universe, X has its attribute set $\alpha=\{\alpha_1,\alpha_2,\cdots,\alpha_k\}\subset V$, V is a finite attribute universe. If r ($0<r<k$) attributes are deleted from α, α becomes $\alpha^F=\{\alpha_1,\alpha_2,\cdots,\alpha_{k-r}\}$, then the elements in set X increase, X generates outer P-set $X^F=\{x_1,x_2,\cdots,x_{m+p}\}$. If some attributes are supplemented to α, α becomes $\alpha^{\bar F}=\{\alpha_1,\alpha_2,\cdots,\alpha_k,\alpha_{k+1},\cdots,\alpha_n\}$, then the elements in X decrease, X generates internal P-set $X^{\bar F}=\{x_1,x_2,\cdots,x_{m-s}\}$. The sets $X^{\bar F}$, X and X^F satisfy $X^{\bar F}\subseteq X\subseteq X^F$. The sets pair ($X^{\bar F}$, X^F) which is composed of internal P-set $X^{\bar F}$ and outer P- sets X^F is

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called P-set generated by the general set X. From what is mentioned above, owing to the change of attribute set α, X becomes $X^{\overline{F}}$ or X^{F}, or X generates ($X^{\overline{F}}, X^{F}$). P-set has dynamic characteristics.

As is stated above analysis, internal P-set $X^{\overline{F}}$ corresponds to attribute set α^{F}, outer P-set X^{F} corresponds to attribute set $\alpha^{\overline{F}}$, X corresponds to attribute set α. Therefore, on the one hand, given attribute sets α, $\alpha^{\overline{F}}$ and α^{F}, we can get corresponding element sets X, X^{F} and $X^{\overline{F}}$, and on the other hand, given element sets X, X^{F} and $X^{\overline{F}}$, we can get corresponding attribute sets α, $\alpha^{\overline{F}}$ and α^{F}. Attribute sets $\alpha^{\overline{F}}$ and α^{F} are obtained from dynamic change of α. By the means of the concept of P-set, the sets pair ($\alpha^{\overline{F}}, \alpha^{F}$) which is composed of $\alpha^{\overline{F}}$ and α^{F} is the dual form of P-set ($X^{\overline{F}}, X^{F}$), or the variation form of P-set ($X^{\overline{F}}, X^{F}$). Therefore , we put forward variation P-set. We can know P-set from variation P-set viewpoint. Variation P-set expands the understanding of the essence of P-set. In the paper we put forward variation P-set and discuss its properties.

2. variation P-set and its structure

Suppose X is a finite set on the universe U, which cannot be empty. U is a finite element universe, V is a finite attribute universe.

Definition 2.1[1-13] Suppose $X \subset U$ is a subset on U. Call $F = \{f_1, f_2, \cdots, f_m\}$ and $\overline{F} = \{\overline{f_1}, \overline{f_2}, \cdots, \overline{f_n}\}$ elementary transfer families defined on U, if $f_i \in F$ and $\overline{f}_j \in \overline{F}$ satisfy:

$$\exists u \in U,\ \ u \overline{\in} X \Rightarrow f_i(u) = z \in X, \tag{1}$$

$$\exists x \in X \Rightarrow \overline{f_j}(x) = y \overline{\in} X. \tag{2}$$

Call $f_i \in F$ and $\overline{f}_j \in \overline{F}$ ($i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$) elementary transfers.

Elementary transfers can be applied to the attribute set $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_k\}$ [5-13]. We can express them in the following formulas:

$$\exists \beta_i \in V, \beta_i \overline{\in} \alpha \Rightarrow f(\beta_i) = \alpha_i' \in \alpha, \tag{3}$$

$$\exists \alpha_j \in \alpha \Rightarrow \overline{f}(\alpha_j) = \gamma_j \overline{\in} \alpha. \tag{4}$$

Obviously, the attribute set $\{\alpha_1, \alpha_2, \cdots, \alpha_k\}$ becomes $\{\alpha_1, \alpha_2, \cdots, \alpha_{j-1}, \alpha_{j+1}, \cdots \alpha_k, \beta_i\}$, where V is a finite attribute universe. Due to the effect of elementary transfers $f \in F$ and $\overline{f} \in \overline{F}$, some attributes are supplemented to the attribute set α and some attributes are deleted from α.

Definition 2.2 Given a general set $X = \{x_1, x_2, \cdots, x_m\} \subset U$, and $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_k\} \subset V$ is the attribute set of X. X^{F} is an outer P-set generated by X, if attribute set $\alpha^{\overline{F}}$ of X^{F} satisfies:

$$\alpha^{\overline{F}} = \alpha - \alpha^{-} \tag{5}$$

$\alpha^{\overline{F}}$ is called variation internal P-set generated by X, or $\alpha^{\overline{F}}$ is called variation internal P-set of X for short. α^{-} is called $\overline{F}$ -attribute element deleted set of α, moreover

$$\alpha^{-} = \{\alpha_i \mid \alpha_i \in \alpha, \overline{f}(\alpha_i) = \beta_i \notin \alpha, \overline{f} \in \overline{F}\}. \tag{6}$$

Definition 2.3 Given a general set $X = \{x_1, x_2, \cdots, x_m\} \subset U$, and $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_k\} \subset V$ is the attribute set of X, $X^{\overline{F}}$ is an internal P-set generated by X, if attribute set α^{F} of $X^{\overline{F}}$ satisfies:

$$\alpha^{F} = \alpha \cup \alpha^{+} \tag{7}$$

α^{F} is called variation outer P-set generated by X, or α^{F} is called variation outer P-set of X. α^{+} is called F -attribute element supplemented set of α, moreover

$$\alpha^{+} = \{\beta \mid \beta \in V, \beta \overline{\in} \alpha, f(\beta) = \alpha' \in \alpha, f \in F\}. \tag{8}$$

Definition 2.4 The set pair which is composed of variation internal P-set $\alpha^{\overline{F}}$ and variation outer P-sets α^{F} is called variation P-set generated by the general set X, moreover

$$(\alpha^{\overline{F}}, \alpha^{F}). \tag{9}$$

($\alpha^{\overline{F}}, \alpha^F$) is called variation P-set of X for short, the attribute set α is called the ground set of ($\alpha^{\overline{F}}, \alpha^F$).

From P-set[1-4] and Definition 2.2-2.4, we can get the following duality theorems.

Theorem 2.1 (Duality theorem of variation internal P-set) Let $\alpha^{\overline{F}}$ be the variation internal P-set of X, X^F be the outer P-set of X. Then $\alpha^{\overline{F}}$ and X^F are dual, moreover

$$\alpha^{\overline{F}} \text{ € } \quad X^F. \tag{10}$$

Theorem 2.2 (Duality theorem of variation outer P-set) Let α^F be the variation outer P-set of X, $X^{\overline{F}}$ be the internal P-set of X. Then α^F and $X^{\overline{F}}$ are dual, moreover

$$\alpha^F \text{ € } \quad X^{\overline{F}}. \tag{11}$$

Theorem 2.3 (Duality theorem of variation P-set) Let ($\alpha^{\overline{F}}, \alpha^F$) be the variation P-set of X, ($X^{\overline{F}}, X^F$) be the P-set of X. Then ($\alpha^{\overline{F}}, \alpha^F$) and ($X^{\overline{F}}, X^F$) are dual, moreover

$$(\alpha^{\overline{F}}, \alpha^F) \text{ € } \quad (X^{\overline{F}}, X^F). \tag{12}$$

Variation duality principle of P-set:

The existence of ($X^{\overline{F}}, X^F$) accompanies the generation of variation P-set ($\alpha^{\overline{F}}, \alpha^F$), they have the same structure, and have dual characteristics. They jointly express dual characteristics of P-set.

3. The properties of variation P-set

Definition 3.1 Suppose $\alpha^{\overline{F}}$ is the variation internal P-set of X, α is the attribute set of X. Call

$$\eta_{\alpha^{\overline{F}}} = \frac{|\alpha^{\overline{F}}|}{|\alpha|} \tag{13}$$

internal packet degree of $\alpha^{\overline{F}}$ about α, $\eta_{\alpha^{\overline{F}}}$ is called internal packet degree of $\alpha^{\overline{F}}$ for short, where $|\alpha^{\overline{F}}|$ is the cardinal number of $\alpha^{\overline{F}}$.

Definition 3.2 Suppose α^F is the variation outer P-set of X, α is the attribute set of X. Call

$$\eta_{\alpha^F} = \frac{|\alpha^F|}{|\alpha|} \tag{14}$$

outer packet degree of α^F about α, η_{α^F} is called outer packet degree of α^F for short.

Definition 3.3 The data pair which is composed of internal packet degree $\eta_{\alpha^{\overline{F}}}$ and outer packet degree η_{α^F} is called packet degree of variation P-set ($\alpha^{\overline{F}}, \alpha^F$), moreover

$$(\eta_{\alpha^{\overline{F}}}, \eta_{\alpha^F}). \tag{15}$$

Theorem 3.1 (Max-min theorem of outer-packet degree) Suppose η_{α^F} is the outer-packet degree of α^F, then $1 \le \eta_{\alpha^F} \le \frac{|V|}{|\alpha|}$.

Proof Suppose α^F is the variation outer P-set of X, when $F = \Phi$ (here Φ is an empty set), F-attribute element supplemented set $\alpha^+ = \Phi$, then $\alpha^F = \alpha \cup \alpha^+ = \alpha$, from Definition3.2, η_{X^F} has minimum 1. Due to the effect of elementary transfer family F, α^F can be maximum V, so η_{α^F} acquires maximum $\frac{|V|}{|\alpha|}$. Therefore we have $1 \le \eta_{\alpha^F} \le \frac{|V|}{|\alpha|}$.

Theorem 3.2 (Max-min theorem of internal-packet degree) Suppose $\eta_{\alpha^{\overline{F}}}$ is the internal packet degree of $\alpha^{\overline{F}}$, then $0 \le \eta_{\alpha^{\overline{F}}} \le 1$.

Theorem 3.3 (Internal-P theorem of $\alpha^{\overline{F}}$) Suppose $\alpha^{\overline{F}}$ is the variation internal P-set of X, then the necessary and sufficient condition of $0 \le \eta_{\alpha^{\overline{F}}} < 1$ is $\alpha - \alpha^{\overline{F}} \ne \Phi$.

Theorem 3.4 (Outer-P theorem of α^{F}) Suppose α^{F} is the variation outer P-set of X, then the necessary and sufficient condition of $\eta_{\alpha^{F}} > 1$ is $\alpha^{F} - \alpha \ne \Phi$.

Theorem 3.5 (Discrete interval dynamic generation theorem of the packet degree) Suppose $(\alpha_i^{\overline{F}}, \alpha_j^{F})$ is the variation P-set of X. $[\eta_{\alpha_i^{\overline{F}}}, \eta_{\alpha_j^{F}}]$ is the packet degree discrete interval generated by the packet degree $(\eta_{\alpha_i^{\overline{F}}}, \eta_{\alpha_j^{F}})$ of $(\alpha_i^{\overline{F}}, \alpha_j^{F})$, $i, j = 1, 2, \cdots, m$. Then there exists variation P-set $(\alpha_s^{\overline{F}}, \alpha_t^{F})$ of X, and the packet degree discrete interval $[\eta_{\alpha_s^{\overline{F}}}, \eta_{\alpha_t^{F}}]$ generated by the packet degree $(\eta_{\alpha_s^{\overline{F}}}, \eta_{\alpha_t^{F}})$ of $(\alpha_s^{\overline{F}}, \alpha_t^{F})$ satisfies $[\eta_{\alpha_s^{\overline{F}}}, \eta_{\alpha_t^{F}}] = \bigcap_{s \le i, t \le j} [\eta_{\alpha_i^{\overline{F}}}, \eta_{\alpha_j^{F}}]$.

The proof can be obtained from Definition 2.2-2.3 and Definition 3.1-3.2, omitted.

4. The relations between variation P- set ($\alpha^{\overline{F}}, \alpha^{F}$) and α

Theorem 4.1 (Discrete internal P-decomposition theorem of α) Suppose $\alpha_\lambda^{\overline{F}}$ is the variation internal P-set of X, $\lambda \in [0,1]$, α € X, then

$$\alpha = \bigcup_{\lambda \in [0,1]} \alpha_\lambda^{\overline{F}}, \tag{16}$$

where λ is the internal-packet degree of $\alpha_\lambda^{\overline{F}}$, and $\lambda = \eta_{\alpha_\lambda^{\overline{F}}} \in [0,1]$, $[0,1]$ is a real discrete interval composed of λ.

Theorem 4.2 (Discrete outer P-decomposition theorem of α) Suppose α_λ^{F} is the variation outer P-set of X, $\lambda \in [1, \xi]$, then

$$\alpha = \bigcap_{\lambda \in [1,\xi]} \alpha_\lambda^{F}, \tag{17}$$

where λ is the outer-packet degree of α_λ^{F}, and $\lambda = \eta_{\alpha_\lambda^{F}}$, $\xi = \dfrac{|V|}{|\alpha|} > 1$ is a given positive real number.

Theorem 4.3 Suppose $(\alpha^{\overline{F}}, \alpha^{F})$ is the variation P-set of X, α € X. Then $\alpha^{\overline{F}} \subseteq \alpha \subseteq \alpha^{F}$.

Theorem 4.4 Suppose $(\alpha^{\overline{F}}, \alpha^{F})$ is the variation P-set of X, α € X. Then α is the static kernel of $(\alpha^{\overline{F}}, \alpha^{F})$ if and only if $F = \overline{F} = \Phi$.

Where the word "static kernel " means that $\alpha^{\overline{F}}$ and α^{F} have no dynamic characteristics, i.e., $\alpha^{\overline{F}}$ doesn't shrink inward, and α^{F} doesn't expand outward.

Proof (1) If α is the static kernel of $(\alpha^{\overline{F}}, \alpha^{F})$, from the meaning of static kernel, we know that $\alpha^{\overline{F}}$ and α^{F} have no dynamic characteristics, then $F = \overline{F} = \Phi$.

(2) If $F = \overline{F} = \Phi$, from Definition 2.2, $\alpha^{-} = \{\alpha_i \mid \alpha_i \in \alpha, \overline{f}(\alpha_i) = \beta_i \notin \alpha, \overline{f} \in \overline{F}\} = \Phi$, then $\alpha^{\overline{F}} = \alpha - \alpha^{-} = \alpha$. Therefore $\alpha^{\overline{F}}$ has no dynamic characteristic. Similarly we can prove α^{F} has no dynamic characteristic. Therefore α is the static kernel of $(\alpha^{\overline{F}}, \alpha^{F})$.

Proposition 1 Under the dynamic-static condition, the attribute set α is a static form of the variation internal P-set $\alpha^{\overline{F}}$, and the variation internal P-set $\alpha^{\overline{F}}$ is an internal dynamic form of α.

Proposition 2 Under the dynamic-static condition, the attribute set α is a static form of the variation outer P-set α^{F}, and the variation outer P-set α^{F} is an outer dynamic form of α.

Proposition 3 Under the dynamic-static condition, the attribute set α is the special case of the variation P-set ($\alpha^{\overline{F}}$, α^{F}), and ($\alpha^{\overline{F}}$, α^{F}) is the general case of α .

5. Discussion

In information science and system science, the most sets have dynamic characteristics. Information system and identification system based on the information sets with dynamic characteristics have dynamic characteristics, too. In order to supplement dynamic characteristics to a general set, P-set ($X^{\overline{F}}$, X^{F}) which is generated by a general set X is put forward in [1-4]. From the concept of P-set, we have X € α , $X^{\overline{F}}$ € $\alpha^{\overline{F}}$ and X^{F} € α^{F} . Therefore, from attribute viewpoint, the concept of variation P-set ($\alpha^{\overline{F}}$, α^{F}) is put forward. In a certain dynamic system research, we can draw the same conclusion using variation P-set ($\alpha^{\overline{F}}$, α^{F}) or P-set ($X^{\overline{F}}$, X^{F}). Moreover, it is more convenient using variation P-set than P-set. For example, when we search for information on the internet, we just use attributes to search for it. Therefore, variation P-set will supply a new mathematical method and thought in many research fields, such as data mining and knowledge discovery.

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References

[1] Kaiquan Shi. P-set. *Journal of Shandong University (Natural Science)*; 2008, 43 (11): 77-84. (in Chinese).
[2] Kaiquan Shi. P-set and its applications . *An International Journal Advances in Systems Science and Applications*; 2009, 9(2): 209-219.
[3] Kaiquan Shi, Li Zhang. Internal P-set and data outer-recovery . *Journal of Shandong University (Natural Science)*; 2009, 44 (4): 8-14. (in Chinese).
[4] Kaiquan Shi. P-set and its applied characteristics. *Computer Science*, 2010, 37(8): 1-8.
[5] Kaiquan Shi, Bingxue Yao. Function S-rough sets and law identification. *Science in China (E: Information Sciences)*; 2008, 38 (4):553-564. (in Chinese).
[6] Kaiquan Shi, Bingxue Yao. Function S-rough sets and law identification . *Science in China (F: Information Sciences)*; 2008, 51(5): 499-510.
[7] Kaiquan Shi, Jianli Zhao. Function S-rough sets and security-authentication of hiding law. *Science in China (E: Information Sciences)*; 2008, 38(8): 1234-1243. (in Chinese).
[8] Kaiquan Shi, Jianli Zhao. Function S-rough sets and security-authentication of hiding law. *Science in China (F: Information Sciences)*; 2008, 51 (7): 924-935.
[9] Kaiquan Shi. Two direction S-rough sets . *International Journal of Fuzzy Mathematics*; 2005, 13(2): 335-349.
[10] Jiqin Liu. A dynamic rough communication model based on two-direction S-rough sets . *Computer Science*; 2008, 35 (1): 206-210. (in Chinese).
[11] Jiqin Liu, Haifeng Pei. Two-direction S-rough fuzzy set and its application. *Journal of Shandong University (Natural Science)*; 2008, 43(2): 19-22. (in Chinese).
[12] Jiqin Liu, Kaiquan Shi. A new dynamic rough communication model and its application. *The Journal of Fuzzy Mathematics*; 2009, 17(1): 145-159.
[13] Jiqin Liu. The properties of two-direction S-rough fuzzy sets. *The sixth International Conference on Fuzzy Systems and Knowledge Discovery*; 2009, 6: 469-473.