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Propagation of Mild Singularities in Higher Dimensional Thermoelasticity

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The propagation of mild singularities for the semilinear model of three-dimensional thermoelasticity is studied. It is shown that the propagation picture of such singularities of the solution to the semilinear model coincides with one of the solutions to the corresponding linear model. As a simple consequence of our method, a similar result for the full semilinear Cauchy problem of one-dimensional thermoelasticity is also presented. © 1999 Academic Press

1. INTRODUCTION

The system of thermoelasticity is a hyperbolic–parabolic coupled system describing the elastic and the thermal behaviour of elastic, heat conducted media. This system has been studied with respect to different (but typical) questions from the theory of systems of partial differential equations, e.g., the existence of global smooth solutions for small data [2, 4, 6, 8, 9, 14, 16] and the development of singularities in finite time for quasi-linear prob-

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lems with large data (see [2] and references in [7]). Comparing the results with those for hyperbolic and parabolic problems one understands which part of the system has a dominating influence on the properties of solutions.

In [5] it was proved that the solutions to the linear problem do not show a smoothing effect; i.e., in general, the H^s-regularity of the initial data will not be improved in opposition to the situation for the solutions of parabolic Cauchy problems. This hints to a dominating influence of the hyperbolic part. Consequently, there arises the hope to understand the propagation of singularities, of regularity, respectively. Indeed the paper [10] was devoted to the study of the Cauchy problem in one-dimensional semilinear thermoelasticity,

$$\begin{gathered} u_{tt} - \tau u_{xx} + \gamma\theta_x = f(u,\theta), \\ \theta_t - \kappa\theta_{xx} + \gamma u_{tx} = g(u), \\ u(t=0) = u_0, \qquad u_t(t=0) = u_1, \qquad \theta(t=0) = \theta_0, \end{gathered} \tag{1.1}$$

where f and g are smooth function satisfying $f(0,0) = g(0) = 0$. Under the assumptions

$$\begin{gathered} u_0, \theta_0 \in H^s(\mathbb{R}) \cap H^{s+1}(\mathbb{R} \setminus [a,b]), \\ u_1, \kappa\theta_{0,xx} - \gamma u_{1,x} \in H^{s-1}(\mathbb{R}) \cap H^s(\mathbb{R} \setminus [a,b]), \qquad s > 9/2, \end{gathered} \tag{1.2}$$

the precise propagation of H^{s+1}-regularity, H^s-singularities, respectively, (it depends on the private point of view) of the data given. A picture similar to that of wave equations ([1, 12, 13]) has been obtained. The characteristic lines are those from the wave operator $\partial_t^2 - \tau\partial_x^2$. We are interested in the propagation of H^s-singularities, too. These singularities are called *mild singularities* because the difference to the H^{s+1}-regularity is only one Sobolev order. The results from [10] motivate the following two questions:

- Can we study the full semilinear Cauchy problem in one-dimensional thermoelasticity; i.e., $g = g(u,\theta)$ instead of $g = g(u)$ in (1.1)?
- Is there a possibility to generalize the results to the higher dimensional case?

In the present paper we give a positive answer to both questions. Our main idea is inspired by [11], a paper about semilinear wave equations. There it was proved that the propagation of mild singularities for the semilinear wave equation coincides with one for the corresponding linear problem.

It seems to be interesting that recently in [3] such a result is proved for semilinear weakly hyperbolic equations. In this paper we prove that the propagation picture of mild singularities of the solution (U, θ) to the full semilinear Cauchy problem of three-dimensional thermoelasticity

$$\begin{gathered} U_{tt} + \mu\nabla \times \nabla \times U - \tau\nabla \operatorname{div} U + \gamma \nabla\theta = f(U, \theta), \\ \theta_t - \kappa \Delta \theta + \gamma \operatorname{div} U_t = g(U, \theta), \\ U(t = 0) = U_0, \qquad U_t(t = 0) = U_1, \qquad \theta(t = 0) = \theta_0, \end{gathered} \tag{1.3}$$

coincides with one of the solution $(\tilde{U}, \tilde{\theta})$ to the linear Cauchy problem

$$\begin{gathered} U_{tt} + \mu\nabla \times \nabla \times U - \tau\nabla \operatorname{div} U + \gamma \nabla\theta = 0, \\ \theta_t - \kappa \Delta \theta + \gamma \operatorname{div} U_t = 0, \\ U(t = 0) = U_0, \qquad U_t(t = 0) = U_1, \qquad \theta(t = 0) = \theta_0, \end{gathered} \tag{1.4}$$

where $U = (U_1, U_2, U_3)$, $f(U, \theta) = (f_1(U, \theta), f_2(U, \theta), f_3(U, \theta))$, $\mu > 0$, $\tau > 0$, and $\gamma \neq 0$ are constants. For simplicity, we suppose that $f_k, g \in C^\infty(\mathbb{R}^4)$ with $f_k(0,0) = g(0,0) = 0$, $k = 1,2,3$, throughout this paper. The main result of this paper shows that the nonlinearities in (1.3) have no influence on the propagation picture of mild singularities (cf. Corollary 2.3, Proposition 3.1, and Theorem 4.1).

At the end we formulate Theorem 4.2 which shows that our approach allows us to generalize the main result from [10] to the full semilinear Cauchy problem of one-dimensional thermoelasticity under the weaker assumption $s \geq 3$. If we study the semilinear model (1.3), then throughout this paper we suppose $s \geq 3$, which is the minimal order of mild singularities, can be treated by our approach. This order will be determined by Proposition 3.7.

Notations. We use standard notations for the Sobolev space $H^s(\mathbb{R}^n)$ and the Banach spaces $H^k, C^k([0,T], H^s(\mathbb{R}^n))$ with $k \in \mathbb{N}_0$ and $s \in \mathbb{R}$. For any $\Omega \subseteq \mathbb{R}_+ \times \mathbb{R}^n$, let $\Omega_\tau = \Omega \cap \{t = \tau\}$. We define $H^k, C^k([0, T], H^s(\Omega_t))$ as the spaces of function belonging to $H^k, C^k([T_1, T_2], H^s(D))$ for any rectangle $[T_1, T_2] \times D \subset \overline{\Omega} \cap \{0 \leq t \leq T\}$, and we omit the index t for simplicity. Moreover, to simplify the exposition, we denote by $C(T)$ the constant depending on $T > 0$, and $\|u\|_{C_T^k(H^s)}$ ($\|u\|_{H_T^k(H^s)}, \|u\|_{H^s}$ resp.) the norm of u in $C^k([0,T], H^s(\mathbb{R}^n))$ ($H^k([0,T], H^s(\mathbb{R}^n)), H^s(\mathbb{R}^n)$ resp.).

2. PROPAGATION OF MILD SINGULARITIES IN THE LINEAR CASE

In this section we study the propagation picture of mild singularities of the solution $(\tilde{U}, \tilde{\theta})$ to the linear problem (1.4). The decomposition of the displacement vector U into its curl-free part U^{po} and its divergence-free part U^{so} decouples (1.4) into the wave equation for the components of the solenoidal part U^{so},

$$U_{tt}^{\text{so}} - \mu \Delta U^{\text{so}} = 0, \qquad U^{\text{so}}(t=0) = U_0^{\text{so}}, \qquad U_t^{\text{so}}(t=0) = U_1^{\text{so}}, \quad (2.1)$$

and a simpler coupled system than (1.4) for the potential part U^{po} and the temperature difference θ,

$$\begin{gathered} U_{tt}^{\text{po}} - \tau \Delta U^{\text{po}} + \gamma \nabla \theta = 0, \qquad \theta_t - \kappa \Delta \theta + \gamma \operatorname{div} U_t^{\text{po}} = 0, \\ U^{\text{po}}(t=0) = U_0^{\text{po}}, \qquad U_t^{\text{po}}(t=0) = U_1^{\text{po}}, \qquad \theta(t=0) = \theta_0. \end{gathered} \quad (2.2)$$

First let us suppose for the data

$$\begin{gathered} U_0, \theta_0 \in H^s(\mathbb{R}^3) \cap H^{s+1}(\mathbb{R}^3 \setminus \{0\}), \\ U_1, \kappa \Delta \theta_0 - \gamma \operatorname{div} U_1^{\text{po}} \in H^{s-1}(\mathbb{R}^3) \cap H^s(\mathbb{R}^3 \setminus \{0\}), \qquad s \geq 1. \end{gathered} \quad (2.3)$$

Using the theory of wave equations we immediately obtain the following result for the solution of (2.1) (compare with Lemma 3.3).

LEMMA 2.1. *Under the assumptions* (2.3), *the solution* $\tilde{U}^{\text{so}} = (\tilde{U}_1^{\text{so}}, \tilde{U}_2^{\text{so}}, \tilde{U}_3^{\text{so}})$ *of* (2.1) *satisfies*

$$\begin{gathered} \tilde{U}^{\text{so}} \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3)) \\ \cap C^2([0,T], H^{s-2}(\mathbb{R}^3)), \\ \|\tilde{U}^{\text{so}}\|_{C_T^0(H^s) \cap C_T^1(H^{s-1}) \cap C_T^2(H^{s-2})} \leq C(T)\big(\|U_0\|_{H^s} + \|U_1\|_{H^{s-1}}\big). \end{gathered}$$

Moreover, we have (this is enough for the following considerations)

$$\tilde{U}^{\text{so}} \in C^0([0,T], H^{s+1}(I_1)) \cap C^1([0,T], H^s(I_1)),$$

where I_1 denotes the interior and the exterior of the forward light cone, that is,

$$I_1 = \big\{(x,t) \in (\mathbb{R}^3 \times [0,\infty)) \setminus \{|x| = \sqrt{\mu}\, t\}\big\}.$$

To study the problem (2.2) we know that after differentiation the components of U^{po} and θ satisfy the following fourth-order partial differential equation:

$$u_{ttt} - (\tau + \gamma^2) \Delta u_t - \kappa \Delta u_{tt} + \tau\kappa \Delta^2 u = 0. \quad (2.4)$$

For the Cauchy data $u(t=0)=u_0$, $u_t(t=0)=u_1$, and $u_{tt}(t=0)=u_2$ we have due to (2.3) the regularity assumptions

$$\begin{aligned} &u_0 \in H^s(\mathbb{R}^3) \cap H^{s+1}(\mathbb{R}^3 \setminus \{0\}), \\ &u_1 \in H^{s-1}(\mathbb{R}^3) \cap H^{s}(\mathbb{R}^3 \setminus \{0\}), \\ &u_2 \in H^{s-3}(\mathbb{R}^3) \cap H^{s-2}(\mathbb{R}^3 \setminus \{0\}) \qquad \text{if } u := \theta, \\ &u_2 \in H^{s-2}(\mathbb{R}^3) \cap H^{s-1}(\mathbb{R}^3 \setminus \{0\}) \qquad \text{if } u := U_k^{\mathrm{po}},\ k=1,2,3. \end{aligned} \tag{2.5}$$

The application of the partial Fourier transform $\mathscr{F}_p$ with respect to the spatial variables transforms (2.4) to

$$v_{ttt} + \kappa|\xi|^2 v_{tt} + (\tau+\gamma^2)|\xi|^2 v_t + \tau\kappa|\xi|^4 v = 0, \tag{2.6}$$

where $v = \mathscr{F}_p(u)$. For the representation of the solution of the Cauchy problem for (2.4) we need to know the behaviour of the roots $\beta_k = \beta_k(|\xi|)$ $(k=1,2,3)$ of the algebraic equation

$$\beta^3 - \kappa|\xi|^2\beta^2 + (\tau+\gamma^2)|\xi|^2\beta - \tau\kappa|\xi|^4 = 0.$$

This characteristic equation coincides with the equation (2.3) from [10] if we replace there ξ by $|\xi|$. This replacement characterizes the transition from the one-dimensional case to the higher dimensional one. It is not necessary to repeat all the calculations from [10]. We only formulate the results and sketch differences in the proof if they appear.

LEMMA 2.2. *Under the assumptions* (2.3) *there exists a uniquely determined solution* $(\tilde{U}^{\mathrm{po}}, \tilde{\theta})$ *to* (2.2) *satisfying*

$$\begin{aligned} &\tilde{U}^{\mathrm{po}}, \tilde{\theta} \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3)) \\ &\qquad \cap C^2([0,T], H^{s-3}(\mathbb{R}^3)), \\ &\big\|(\tilde{U}^{\mathrm{po}}, \tilde{\theta})\big\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})} \\ &\qquad \le C(T)\big(\|(U_0,\theta_0)\|_{H^s} + \|(U_1, \kappa\Delta\theta_0 - \gamma \operatorname{div} U_1^{\mathrm{po}})\|_{H^{s-1}}\big), \end{aligned}$$

and

$$\begin{aligned} &\tilde{U}^{\mathrm{po}} \in C^0([0,T], H^{s+1}(I_2)) \cap C^1([0,T], H^s(I_2)), \\ &\tilde{\theta} \in L^2([0,T], H^{s+1}(I_2)) \cap H^1([0,T], H^s(I_2)), \end{aligned}$$

where I_2 *denotes the interior and the exterior of the forward light cone,*

$$I_2 = \{(x,t) \in (\mathbb{R}^3 \times [0,\infty)) \setminus \{|x| = \sqrt{\tau}t\}\}.$$

Sketch of the Proof. Most of the formal calculations coincide with those from [10] for the one-dimensional case. Differences appear if we study the propagation behaviour of Fourier multipliers connected with the wave operator factor. Let us explain this difference in detail. In the one-dimensional case this step reduces to the understanding of the propagation behaviour of

$$\begin{aligned}\mathscr{F}_p^{-1}\big(\exp(-i\sqrt{\tau}\,\xi t)\mathscr{F}(u_0)\big) &= \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}^1} e^{i(x-\sqrt{\tau}t)\xi}\mathscr{F}(u_0)(\xi)\,d\xi \\ &= u_0\big(x-\sqrt{\tau}t\big).\end{aligned}$$

If $u_0 \in H^s(\mathbb{R}) \cap H^{s+1}(\mathbb{R}\setminus\{0\})$, then the mild H^s-singularity propagates only along one characteristic line. This brings additional regularity along the other characteristic line (cf. with the results from [10] and with Theorem 4.2).

In the three-dimensional case we have to study

$$\mathscr{F}_p^{-1}\big(\exp(-i\sqrt{\tau}|\xi|t)\mathscr{F}(u_0)\big) = (2\pi)^{-3/2}\int_{\mathbb{R}^3} e^{i(x\cdot\xi-\sqrt{\tau}|\xi|t)}\mathscr{F}(u_0)(\xi)\,d\xi.$$

If we assume $u_0 \in H^s(\mathbb{R}^3) \cap H^{s+1}(\mathbb{R}^3\setminus\{0\})$, then we only know $(0,\xi) \in WF_{s+1}(u_0)$ for all $\xi \in \mathbb{R}^3\setminus\{0\}$. But this gives that for any fixed $t > 0$,

$$WF_{s+1}\Big(\mathscr{F}_p^{-1}\big(\exp(-i\sqrt{\tau}|\xi|t)\mathscr{F}(u_0)\big)\Big) \subset \left\{\left(\sqrt{\tau}\,\frac{\xi}{|\xi|}t,\ \xi\right)\,\middle|\ \forall\xi \in \mathbb{R}^3\setminus\{0\}\right\}.$$

Consequently, we obtain H^{s+1}-regularity only in the interior and exterior of the forward light cone. Q.E.D.

Summarizing the statements of Lemmas 2.1 and 2.2 leads to the following result:

COROLLARY 2.3. *Under the assumptions*

$$U_0,\ \theta_0 \in H^s(\mathbb{R}^3) \cap H^{s+1}\big(\mathbb{R}^3\setminus\{0\}\big),$$

$$U_1,\ \kappa\Delta\theta_0 - \gamma\,\mathrm{div}\,U_1^{\mathrm{po}} \in H^{s-1}(\mathbb{R}^3) \cap H^s\big(\mathbb{R}^3\setminus\{0\}\big), \qquad s \geq 1,$$

there exists a uniquely determined solution $(\tilde{U}, \tilde{\theta})$ to the linear Cauchy problem of three-dimensional thermoelasticity

$$U_{tt} + \mu\nabla\times\nabla\times U - \tau\nabla\,\mathrm{div}\,U + \gamma\nabla\theta = 0,$$

$$\theta_t - \kappa\Delta\theta + \gamma\,\mathrm{div}\,U_t = 0,$$

$$U(t=0) = U_0, \qquad U_t(t=0) = U_1, \qquad \theta(t=0) = \theta_0,$$

satisfying

$$\tilde{U}, \tilde{\theta} \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3)),$$

and

$$\tilde{U}^{\text{so}} \in C^0([0,T], H^{s+1}(I_1)) \cap C^1([0,T], H^s(I_1)),$$
$$\tilde{U}^{\text{po}} \in C^0([0,T], H^{s+1}(I_2)) \cap C^1([0,T], H^s(I_2)),$$
$$\tilde{\theta} \in L^2([0,T], H^{s+1}(I_2)) \cap H^1([0,T], H^s(I_2)),$$

where I_l, $l = 1, 2$, *denote the interior and the exterior of the forward light cones,*

$$I_l = \left\{(x,t) \in (\mathbb{R}^3 \times [0,\infty)) \setminus \left\{|x| = \sqrt{\alpha_l}\, t\right\}\right\},$$
$$\alpha_1 = \mu, \qquad \alpha_2 = \tau.$$

3. REGULARITY RESULTS FOR CAUCHY PROBLEMS FROM THREE-DIMENSIONAL THERMOELASTICITY

To study the propagation of mild singularities in the linear case we had to reduce the system (2.2) to a decoupled system of fourth-order differential equations. The solution of this system possesses regularity properties $C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3))$. For the moment we only use this regularity with respect to the spatial variables on the right-hand side of the following Cauchy problem,

$$\begin{aligned} U_{tt} + \mu\nabla \times \nabla \times U - \tau\nabla \operatorname{div} U + \gamma \nabla\theta &= f(x,t), \\ \theta_t - \kappa \Delta \theta + \gamma \operatorname{div} U_t &= g(x,t), \\ U(t=0) = U_t(t=0) = \theta(t=0) &= 0. \end{aligned} \tag{3.1}$$

PROPOSITION 3.1. *Let us suppose that*

$$f \in L^2([0,T], H^s(\mathbb{R}^3)) \cap H^1([0,T], H^{s-1}(\mathbb{R}^3)),$$
$$g \in L^2([0,T], H^s(\mathbb{R}^3)),$$

with fixed $s \geq -1$. *Then there exists a uniquely determined solution of* (3.1) *satisfying*

$$U \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap C^1([0,T], H^s(\mathbb{R}^3)),$$
$$\theta \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap H^1([0,T], H^s(\mathbb{R}^3)).$$

To prove this proposition we need the following three lemmas for $n = 3$. The first two lemmas can be found in textbooks, e.g., [15].

LEMMA 3.2. *Consider the Cauchy problem for the heat equation on* $\mathbb{R}^n \times \mathbb{R}_+$,

$$\theta_t - \kappa \Delta \theta = g(x,t), \qquad \theta(t=0) = \theta_0(x),$$

under the assumptions $\theta_0 \in H^{s+1}(\mathbb{R}^n)$ *and* $g \in \bigcap_{j=0}^k H^j([0,T], H^{s-2j}(\mathbb{R}^n))$ *with fixed* $k \in \mathbb{N}$ *and* $s \geq 2k-1$. *Then there exists a uniquely determined solution*

$$\theta \in \bigcap_{j=0}^{k+1} H^j([0,T], H^{s+2-2j}(\mathbb{R}^n))$$

satisfying

$$\sum_{j=0}^{k+1} \|\theta\|_{H_T^j(H^{s+2-2j})} \leq C_0 \left(\sum_{j=0}^{k} \|g\|_{H_T^j(H^{s-2j})} + \|\theta_0\|_{H^{s+1}} \right). \qquad (3.2)$$

LEMMA 3.3. *Consider the Cauchy problem for the wave equation*

$$u_{tt} - \tau \Delta u = f(x,t), \qquad u(t=0) = u_0(x), \qquad u_t(t=0) = u_1(x),$$

under the assumptions $u_0 \in H^{s+1}(\mathbb{R}^n)$, $u_1 \in H^s(\mathbb{R}^n)$, *and* $f \in \bigcap_{j=0}^k H^j([0,T], H^{s-j}(\mathbb{R}^n))$ *with fixed* $k \in \mathbb{N}_0$ *and* $s \in \mathbb{R}$. *Then there exists a uniquely determined solution*

$$u \in \bigcap_{j=0}^{k+1} C^j([0,T], H^{s+1-j}(\mathbb{R}^n))$$

satisfying

$$\sum_{j=0}^{k+1} \|u\|_{C_T^j(H^{s+1-j})} \leq C(T) \left(\sum_{j=0}^{k} \|f\|_{H_T^j(H^{s-j})} + \|u_0\|_{H^{s+1}} + \|u_1\|_{H^s} \right). \quad (3.3)$$

LEMMA 3.4. *Consider the Cauchy problem*

$$u_{ttt} - (\tau + \gamma^2) \Delta u_t - \kappa \Delta u_{tt} + \tau\kappa \Delta^2 u = f(x,t),$$
$$u(t=0) = u_t(t=0) = 0, \qquad u_{tt}(t=0) = u_2(x).$$

If $u_2 \in H^{s-3}(\mathbb{R}^n)$ *and* $f \in L^2([0,T], H^{s-3}(\mathbb{R}^n))$ *with* $s \geq -1$, *then there exists a uniquely determined solution*

$$u \in C^0([0,T], H^s(\mathbb{R}^n)) \cap H^1([0,T], H^{s-1}(\mathbb{R}^n))$$

satisfying

$$\|u\|_{C_T^0(H^s) \cap H_T^1(H^{s-1})} \leq C(T)\big(\|f\|_{L_T^2(H^{s-3})} + \|u_2\|_{H^{s-3}}\big). \tag{3.4}$$

Proof. The statement of this lemma follows by a formal carrying over of the ideas from [10, Lemmas 2.10 to 2.12] for the one-dimensional case to the higher dimensional one. Q.E.D.

Proof of Proposition 3.1. Let (U, θ) be the solution to the linear Cauchy problem (3.1). Then U^{so} satisfies

$$U_{tt}^{\mathrm{so}} - \mu \Delta U^{\mathrm{so}} = f^{\mathrm{so}}(x,t), \qquad U^{\mathrm{so}}(t=0) = U_t^{\mathrm{so}}(t=0) = 0.$$

Using Lemma 3.3 we obtain

$$U^{\mathrm{so}} \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap C^1([0,T], H^s(\mathbb{R}^3)),$$

and

$$\|U^{\mathrm{so}}\|_{C_T^0(H^{s+1}) \cap C_T^1(H^s)} \leq C(T)\|f\|_{L_T^2(H^s)}. \tag{3.5}$$

The vector $(U^{\mathrm{po}}, \theta)$ satisfies

$$U_{tt}^{\mathrm{po}} - \tau \Delta U^{\mathrm{po}} + \gamma \nabla\theta = f^{\mathrm{po}}, \qquad \theta_t - \kappa \Delta \theta + \gamma \operatorname{div} U_t^{\mathrm{po}} = g,$$

$$U^{\mathrm{po}}(t=0) = U_t^{\mathrm{po}}(t=0) = \theta(t=0) = 0. \tag{3.6}$$

Decompose $(U^{\mathrm{po}}, \theta)$ into $(U^{\mathrm{po}}, \theta) = (U_1, \theta_1) + (U_2, \theta_2)$, where (U_1, θ_1) and (U_2, θ_2) satisfy the Cauchy problems

$$U_{1,tt} - \tau \Delta U_1 + \gamma \nabla\theta_1 = f^{\mathrm{po}}, \qquad \theta_{1,t} - \kappa \Delta \theta_1 + \gamma \operatorname{div} U_{1,t} = 0,$$

$$U_1(t=0) = U_{1,t}(t=0) = \theta_1(t=0) = 0, \tag{3.7}$$

$$U_{2,tt} - \tau \Delta U_2 + \gamma \nabla\theta_2 = 0, \qquad \theta_{2,t} - \kappa \Delta \theta_2 + \gamma \operatorname{div} U_{2,t} = g,$$

$$U_2(t=0) = U_{2,t}(t=0) = \theta_2(t=0) = 0, \tag{3.8}$$

respectively.

Differentiating equation for (U_1, θ_1) and (U_2, θ_2) it follows from (3.7) and (3.8),

$$\theta_{1,ttt} - (\tau + \gamma^2)\Delta\theta_{1,t} - \kappa\Delta\theta_{1,tt} + \tau\kappa\Delta^2\theta_1 = -\gamma \operatorname{div} f_t^{\mathrm{po}},$$

$$\theta_1(t=0) = \theta_{1,t}(t=0) = 0, \qquad \theta_{1,tt}(t=0) = -\gamma \operatorname{div} f^{\mathrm{po}}(t=0), \tag{3.9}$$

$$U_{2,ttt} - (\tau + \gamma^2)\Delta U_{2,t} - \kappa \Delta U_{2,tt} + \tau k \Delta^2 U_2 = -\gamma \nabla g,$$

$$U_2(t=0) = U_{2,t}(t=0) = U_{2,tt}(t=0) = 0. \tag{3.10}$$

The assumption for $f = f(x,t)$ allows us to apply Lemma 3.4 to the Cauchy problem (3.9). Due to (3.4) we immediately get the inequality

$$\|\theta_1\|_{C_T^0(H^{s+1}) \cap H_T^1(H^s)} \le C(T)\big(\|\operatorname{div} f_t^{\mathrm{po}}\|_{L_T^2(H^{s-2})} + \|\operatorname{div} f^{\mathrm{po}}(t=0)\|_{H^{s-2}}\big)$$
$$\le C(T)\|f\|_{H_T^1(H^{s-1})}. \tag{3.11}$$

On the other hand, from (3.7), the vector-function U_1 satisfies

$$U_{1,tt} - \tau \Delta U_1 = f^{\mathrm{po}} - \gamma \nabla\theta_1, \qquad U_1(t=0) = U_{1,t}(t=0) = 0,$$

which gives the estimate

$$\|U_1\|_{C_T^0(H^{s+1}) \cap C_T^1(H^s)} \le C(T)\|f\|_{L_T^2(H^s) \cap H_T^1(H^{s-1})}, \tag{3.12}$$

by using (3.3) and (3.11).

The assumption for $g = g(x,t)$ allows a repeated application of (3.4) to the Cauchy problem (3.10). Consequently,

$$\|U_2\|_{C_T^0(H^{s+2}) \cap H_T^1(H^{s+1})} \le C(T)\|g\|_{L_T^2(H^s)}. \tag{3.13}$$

The higher regularity of U_2 can be used to estimate θ_2. By (3.8) the component θ_2 satisfies the following Cauchy problem for the heat equation:

$$\theta_{2,t} - \kappa \Delta \theta_2 = g - \gamma \operatorname{div} U_{2,t}, \qquad \theta_2(t=0) = 0.$$

Applying (3.2) from Lemma 3.2 for $k = 0$, $n = 3$ in the above problem and using (3.13),

$$\|\theta_2\|_{L_T^2(H^{s+2}) \cap H_T^1(H^s)} \le C(T)\|g - \gamma \operatorname{div} U_{2,t}\|_{L_T^2(H^s)} \le C(T)\|g\|_{L_T^2(H^s)}, \tag{3.14}$$

which implies $\|\theta_2\|_{C_T^0(H^{s+1})} \le C(T)\|g\|_{L_T^2(H^s)}$, too, by using [15]. Combining (3.11) and (3.14) leads to the regularity result

$$\theta \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap H^1([0,T], H^s(\mathbb{R}^3)).$$

Using the Cauchy problem from (3.8) for U_2 and the regularity of θ_2 from (3.14) gives together with Lemma 3.3 for $k = 0$ and $n = 3$ that $U_2 \in C^1([0,T], H^{s+1}(\mathbb{R}^3))$. Hence, by using (3.12) we conclude the regularity result

$$U \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap C^1([0,T], H^s(\mathbb{R}^3)). \qquad \text{Q.E.D.}$$

The next proposition is devoted to the existence of solutions to the semilinear Cauchy problem (1.3).

PROPOSITION 3.5. *If the initial data satisfy*

$$(U_0, \theta_0) \in H^s(\mathbb{R}^3), (U_1, \kappa \Delta \theta_0 - \gamma \operatorname{div} U_1^{\mathrm{po}}) \in H^{s-1}(\mathbb{R}^3), \qquad s \geq 3,$$

then there is a constant $T_1 > 0$ such that the semilinear Cauchy problem (1.3) has a unique solution (U, θ) belonging to

$$C^0([0, T_1], H^s(\mathbb{R}^3)) \cap C^1([0, T_1], H^{s-1}(\mathbb{R}^3)) \cap C^2([0, T_1], H^{s-3}(\mathbb{R}^3)).$$

To prove this proposition we need the next two auxiliary results. The lemma can be shown by generalizing the Proposition 2.7, 2.9, and 2.14 from [10] to the higher dimensional case.

LEMMA 3.6. *Consider the Cauchy problem*

$$u_{ttt} - (\tau + \gamma^2) \Delta u_t - \kappa \Delta u_{tt} + \tau\kappa \Delta^2 u = f(x,t),$$
$$u(t=0) = u_t(t=0) = 0, \qquad u_{tt}(t=0) = u_2(x).$$

If $u_2 \in H^{s-3}(\mathbb{R}^3)$ and $f \in L^2([0,T], H^{s-3}(\mathbb{R}^3)) \cap H^1([0,T], H^{s-4}(\mathbb{R}^3))$ with $s \geq 1$, then there exists a uniquely determined solution

$$u \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3))$$
$$\cap\, C^2([0,T], H^{s-3}(\mathbb{R}^3))$$

satisfying

$$\|u\|_{C_T^0(H^s) \cap C_T^1(H^{s-1}) \cap C_T^2(H^{s-3})} \leq C(T)\left(\|f\|_{L_T^2(H^{s-3}) \cap H_T^1(H^{s-4})} + \|u_2\|_{H^{s-3}}\right). \tag{3.15}$$

PROPOSITION 3.7. *Let us suppose that*

$$f \in L^2([0,T], H^{s-1}(\mathbb{R}^3)) \cap H^1([0,T], H^{s-2}(\mathbb{R}^3))$$
$$\cap\, H^2([0,T], H^{s-3}(\mathbb{R}^3)),$$

and

$$g \in L^2([0,T], H^s(\mathbb{R}^3)) \cap H^1([0,T], H^{s-2}(\mathbb{R}^3))$$
$$\cap\, H^2([0,T], H^{s-4}(\mathbb{R}^3)),$$

with $s \geq 3$. Then there exists a uniquely determined solution of (3.1),

$$(U, \theta) \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3))$$
$$\cap\, C^2([0,T], H^{s-3}(\mathbb{R}^3))$$

satisfying

$$\|U\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})}$$
$$\leq C(T)\big(\|f\|_{L_T^2(H^{s-1})\cap H_T^1(H^{s-2})\cap H_T^2(H^{s-4})} + \|g\|_{L_T^2(H^{s-2})\cap H_T^1(H^{s-3})}\big), \tag{3.16}$$

$$\|\theta\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})}$$
$$\leq C(T)\big(\|f\|_{H_T^1(H^{s-2})\cap H_T^2(H^{s-3})} + \|g\|_{L_T^2(H^s)\cap H_T^1(H^{s-2})\cap H_T^2(H^{s-4})}\big). \tag{3.17}$$

Proof. As in the proof of Proposition 3.1, the solenoidal part of U satisfies

$$U_{tt}^{\mathrm{so}} - \mu\Delta U^{\mathrm{so}} = f^{\mathrm{so}}(x,t), \qquad U^{\mathrm{so}}(t=0) = U_t^{\mathrm{so}}(t=0) = 0.$$

The assumption for f and Lemma 3.3 imply

$$\|U^{\mathrm{so}}\|_{C_T^0(H^s)\cap C_T^1(H^{s-})\cap C_T^2(H^{s-3})} \leq C(T)\|f\|_{L_T^2(H^{s-1})\cap H_T^1(H^{s-2})}. \tag{3.18}$$

Differentiating (3.6) we obtain that U^{po} satisfies the Cauchy problem

$$U_{ttt}^{\mathrm{po}} - (\tau+\gamma^2)\Delta U_t^{\mathrm{po}} - \kappa\Delta U_{tt}^{\mathrm{po}} + \tau\kappa\Delta^2 U^{\mathrm{po}} = f_t^{\mathrm{po}} - \kappa\Delta f^{\mathrm{po}} - \gamma\nabla g,$$
$$U^{\mathrm{po}}(t=0) = U_t^{\mathrm{po}}(t=0) = 0, \qquad U_{tt}^{\mathrm{po}}(t=0) = f^{\mathrm{po}}(t=0).$$

By employing (3.15) from Lemma 3.6 with $f^{\mathrm{po}}(t=0) \in H^{s-2}(\mathbb{R}^3)$ and

$$f_t^{\mathrm{po}} - \kappa\Delta f^{\mathrm{po}} - \gamma\nabla g \in L^2([0,T],H^{s-3}(\mathbb{R}^3)) \cap H^1([0,T],H^{s-4}(\mathbb{R}^3)),$$

we get (3.16) by using (3.18). To estimate θ let us remember the decomposition for U and θ from (3.7) and (3.8). To solve the Cauchy problem (3.9) for θ_1 we can apply Lemma 3.6. Consequently, we get the estimate

$$\|\theta_1\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})} \leq C(T)\|f\|_{H_T^1(H^{s-2})\cap H_T^2(H^{s-3})}.$$

The Lemma 3.2 for $k=2$ and $n=3$ can be applied to estimate θ_2 as the solution to the Cauchy problem from (3.8). We get

$$\sum_{j=0}^{3}\|\theta_2\|_{H_T^j(H^{s+2-2j})} \leq C(T)\sum_{j=0}^{2}\|g - \operatorname{div} U_{2,t}\|_{H_T^j(H^{s-2j})}$$
$$\leq C(T)\left(\sum_{j=0}^{2}\|g\|_{H_T^j(H^{s-2j})} + \sum_{j=1}^{3}\|U_2\|_{H_T^j(H^{s+3-2j})}\right).$$

But the necessary norms for U_2 can be estimated after the application of Lemma 3.4 to the Cauchy problem (3.10) for U_2, and to the Cauchy problems for $U_{2,t}$ and $U_{2,tt}$ which are obtained after differentiation. It follows

$$\sum_{j=1}^{3} \|U_2\|_{H_T^j(H^{s+3-2j})} \le C(T) \sum_{j=0}^{2} \|g\|_{H_T^j(H^{s-2j})}.$$

All this together brings for θ_2 the desired estimate

$$\sum_{j=0}^{3} \|\theta_2\|_{H_T^j(H^{s+2-2j})} \le C(T) \sum_{j=0}^{2} \|g\|_{H_T^j(H^{s-2j})}.$$

Finally, we conclude with [15]:

$$\sum_{j=0}^{2} \|\theta_2\|_{C_T^j(H^{s+1-2j})} \le C(T) \sum_{j=0}^{2} \|g\|_{H_T^j(H^{s-2j})}.$$

The derived estimates for θ_1 and θ_2 lead to (3.17). The proposition is completely proved. Q.E.D.

Proof of Proposition 3.5. For (U, θ) we choose the ansatz $(U, \theta) = (\tilde{U}, \tilde{\theta}) + (V, \beta)$, where $(\tilde{U}, \tilde{\theta})$ solves (1.4). Then (V, β) solves

$$V_{tt} + \mu \nabla \times \nabla \times V - \tau \nabla \operatorname{div} V + \gamma \nabla \beta = f\big(\tilde{U} + V, \tilde{\theta} + \beta\big),$$
$$\beta_t - \kappa \Delta \beta + \gamma \operatorname{div} V_t = g\big(\tilde{U} + V, \tilde{\theta} + \beta\big),$$
$$V(t=0) = V_t(t=0) = \beta(t=0) = 0.$$

Using the Corollary 2.3 we have

$$(\tilde{U}, \tilde{\theta}) \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3)) \cap C^2([0,T], H^{s-3}(\mathbb{R}^3)),$$

and

$$\big\|(\tilde{U}, \tilde{\theta})\big\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})} \le C(T)\Big(\big\|(U_0, \theta_0)\big\|_{H^s} + \big\|(U_1, \kappa \Delta \theta_0 - \gamma \operatorname{div} U_1^{\mathrm{po}})\big\|_{H^{s-1}}\Big). \tag{3.19}$$

To solve the above semilinear Cauchy problem for (V, β) we construct a sequence of solutions $\{(V^\nu, \beta^\nu)\}_{\nu \ge 0}$ with $(V^0, \beta^0) = (0, 0)$, by the iteration scheme

$$V_{tt}^{\nu+1} + \mu\nabla \times \nabla \times V^{\nu+1} - \tau\nabla \operatorname{div} V^{\nu+1} + \gamma \nabla\beta^{\nu+1} = f\big(\tilde{U} + V^{\nu}, \tilde{\theta} + \beta^{\nu}\big),$$

$$\beta_t^{\nu+1} - \kappa \Delta \beta^{\nu+1} + \gamma \operatorname{div} V_t^{\nu+1} = g\big(\tilde{U} + V^{\nu}, \tilde{\theta} + \beta^{\nu}\big),$$

$$V^{\nu+1}(t=0) = V_t^{\nu+1}(t=0) = \beta^{\nu+1}(t=0) = 0.$$

1. *Step*. We show that there is a constant $T_0 > 0$ such that the sequence $\{(V^{\nu}, \beta^{\nu})\}_{\nu \geq 0}$ is bounded in $C([0,T_0], H^s(\mathbb{R}^3)) \cap C^1([0,T_0], H^{s-1}(\mathbb{R}^3)) \cap C^2([0,T_0], H^{s-3}(\mathbb{R}^3))$. Taking account of the estimates (3.16) and (3.17) from Proposition 3.7 it holds

$$\begin{aligned}
&\big\|\big(V^{\nu+1}, \beta^{\nu+1}\big)\big\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})} \\
&\quad \leq C(T)\left(\sum_{j=0}^{2}\Big(\big\|f\big(\tilde{U} + V^{\nu}, \tilde{\theta} + \beta^{\nu}\big)\big\|_{H_T^j(H^{s-1-j})}\right. \\
&\qquad\qquad \left. + \big\|g\big(\tilde{U} + V^{\nu}, \tilde{\theta} + \beta^{\nu}\big)\big\|_{H_T^j(H^{s-2j})}\Big)\right).
\end{aligned}$$

For any $T > 0$ and $\nu \geq 0$, let

$$M^{\nu}(T) := \big\|\big(\tilde{U} + V^{\nu}, \tilde{\theta} + \beta^{\nu}\big)\big\|_{C_T^0(H^s)\cap C_T^1(H^{s-1})\cap C_T^2(H^{s-3})}.$$

Then

$$M^1(T) \leq a\big(f, g, M^0(T)\big)\int_0^T M^0(t)\, dt,$$

$$M^2(T) \leq a\big(f, g, M^1(T)\big)\int_0^T M^1(t)\, dt,$$

and in general,

$$M^{\nu+1}(T) \leq a\big(f, g, M^{\nu}(T)\big)\int_0^T M^{\nu}(t)\, dt,$$

where $a = a(f, g, \cdot)$ is a smooth, positive, and increasing function. With a sufficiently small $T = T_0$ we get the boundedness of $\{(V^{\nu}, \beta^{\nu})\}_{\nu \geq 0}$ in $C([0,T_0], H^s(\mathbb{R}^3)) \cap C^1([0,T_0], H^{s-1}(\mathbb{R}^3)) \cap C^2([0,T_0], H^{s-3}(\mathbb{R}^3))$.

2. *Step*. There is a sufficiently small $T_1 \leq T_0$ such that the sequence $\{(V^{\nu}, \beta^{\nu})\}_{\nu \geq 0}$ is convergent in $C^0([0,T_1], H^s(\mathbb{R}^3)) \cap C^1([0,T_1], H^{s-1}(\mathbb{R}^3)) \cap C^2([0,T_1], H^{s-3}(\mathbb{R}^3))$.

We know that each element of the sequence $\{(W^\nu, \alpha^\nu)\}_{\nu \geq 0}$, where $W^\nu := V^{\nu+1} - V^\nu$ and $\alpha^\nu := \beta^{\nu+1} - \beta^\nu$, solves

$$W_{tt}^\nu + \mu \nabla \times \nabla \times W^\nu - \tau \nabla \operatorname{div} W^\nu + \gamma \nabla \alpha^\nu = f^\nu,$$
$$\alpha_t^\nu - \kappa \Delta \alpha^\nu + \gamma \operatorname{div} W_t^\nu = g^\nu,$$
$$W^\nu(t=0) = W_t^\nu(t=0) = \alpha^\nu(t=0) = 0,$$

with $f^\nu := f(\tilde{U} + V^\nu, \tilde{\theta} + \beta^\nu) - f(\tilde{U} + V^{\nu-1}, \tilde{\theta} + \beta^{\nu-1})$ and $g^\nu := g(\tilde{U} + V^\nu, \tilde{\theta} + \beta^\nu) - g(\tilde{U} + V^{\nu-1}, \tilde{\theta} + \beta^{\nu-1})$.

Similar arguments as in the first step together with the boundedness of $\{(V^\nu, \beta^\nu)\}_{\nu \geq 0}$ in $C([0, T_0], H^s(\mathbb{R}^3)) \cap C^1([0, T_0], H^{s-1}(\mathbb{R}^3)) \cap C^2([0, T_0], H^{s-3}(\mathbb{R}^3))$ and Hadamard's formula yield

$$\|(W^\nu, \alpha^\nu)\|^2_{C_T^0(H^s) \cap C_T^1(H^{s-1}) \cap C_T^2(H^{s-3})}$$
$$\leq C\Big(f, g, \sup_{\nu \geq 0} M^\nu(T)\Big) \int_0^T \|(W^{\nu-1}, \alpha^{\nu-1})\|^2_{C_t^0(H^s) \cap C_t^1(H^{s-1}) \cap C_t^2(H^{s-3})} \, dt$$

for any $\nu \geq 1$ and $T \leq T_0$. Hence, a small $T_1 \leq T_0$ guarantees the property of $\{(V^\nu, \beta^\nu)\}_{\nu \geq 0}$ to be a Cauchy sequence in the desired function space. The uniquely determined limit element (V, β) gives together with $(\tilde{U}, \tilde{\theta})$ the uniquely determined solution (U, θ) of (1.3) which belongs to $C^0([0, T_1], H^s(\mathbb{R}^3)) \cap C^1([0, T_1], H^{s-1}(\mathbb{R}^3)) \cap C^2([0, T_1], H^{s-3}(\mathbb{R}^3))$. The proof is complete. Q.E.D.

4. PROPAGATION OF MILD SINGULARITIES

THEOREM 4.1. *Let us consider the semilinear Cauchy problem of three-dimensional thermoelasticity*

$$U_{tt} + \mu \nabla \times \nabla \times U - \tau \nabla \operatorname{div} U + \gamma \nabla \theta = f(U, \theta),$$
$$\theta_t - \kappa \Delta \theta + \gamma \operatorname{div} U_t = g(U, \theta),$$
$$U(t=0) = V_0, \qquad U_t(t=0) = V_1, \qquad \theta(t=0) = \theta_0.$$

Let $\Omega \subset \mathbb{R}^3$ be a given closed domain. Then under the assumptions

$$V_0, \theta_0 \in H^s(\mathbb{R}^3) \cap H^{s+1}(\mathbb{R}^3 \setminus \Omega),$$
$$V_1, \kappa \Delta \theta_0 - \gamma \operatorname{div} V_1^{\mathrm{po}} \in H^{s-1}(\mathbb{R}^3) \cap H^s(\mathbb{R}^3 \setminus \Omega), \qquad s \geq 3,$$

there exist a positive constant T and a unique solution

$$(U, \theta) \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3)) \cap C^2([0,T], H^{s-3}(\mathbb{R}^3))$$

satisfying

$$U^{\text{so}} \in C^0([0,T], H^{s+1}(K_1)) \cap C^1([0,T], H^s(K_1)),$$
$$U^{\text{po}} \in C^0([0,T], H^{s+1}(K_2)) \cap C^1([0,T], H^s(K_2)),$$
$$\theta \in L^2([0,T], H^{s+1}(K_2)) \cap H^1([0,T], H^s(K_2)),$$

where K_l, $l = 1, 2$, *denote the sets* $K_l := \bigcap_{x_o \in \Omega} I_l(x_o)$ *with*

$$I_l(x_o) = \left\{(x,t) \in (\mathbb{R}^3 \times [0,T]) \setminus \left\{|x - x_o| = \sqrt{\alpha_l}\, t\right\}\right\},$$
$$\alpha_1 = \mu, \qquad \alpha_2 = \tau.$$

Proof. By using Proposition 3.5 the Cauchy problem has a unique solution $(U, \theta) \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3))$. Let $(\tilde{U}, \tilde{\theta})$ be the solution of the corresponding linear Cauchy problem with the same data and homogeneous right-hand side. Then (W, β) with $W := U - \tilde{U}$, $\beta := \theta - \tilde{\theta}$, solves

$$W_{tt} + \mu\nabla \times \nabla \times W - \tau\nabla \operatorname{div} W + \gamma\nabla\beta = f(U, \theta),$$
$$\beta_t - \kappa\Delta\beta + \gamma \operatorname{div} W_t = g(U, \theta),$$
$$W(t=0) = W_t(t=0) = \beta(t=0) = 0.$$

Since $s \geq 3$, it follows that

$$(f(U,\theta), g(U,\theta)) \in C^0([0,T], H^s(\mathbb{R}^3)) \cap C^1([0,T], H^{s-1}(\mathbb{R}^3)).$$

The application of Proposition 3.1 implies

$$W \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap C^1([0,T], H^s(\mathbb{R}^3)),$$
$$\beta \in C^0([0,T], H^{s+1}(\mathbb{R}^3)) \cap H^1([0,T], H^s(\mathbb{R}^3)).$$

Consequently, the propagation picture of mild H^s-singularities of (U, θ) coincides with that of $(\tilde{U}, \tilde{\theta})$, which is the same as desired by using Corollary 2.3. Q.E.D.

At the end of this paper we mention that our approach allows us to generalize the main theorem from [10] to the full semilinear case under the weaker assumption $s \geq 3$. Without new difficulties one can prove the next result.

THEOREM 4.2. *Let us consider the semilinear Cauchy problem of one-dimensional thermoelasticity*

$$u_{tt} - \tau u_{xx} + \gamma\theta_x = f(u,\theta), \qquad \theta_t - \kappa\theta_{xx} + \gamma u_{tx} = g(u,\theta),$$
$$u(t=0) = u_0, \qquad u_t(t=0) = u_1, \qquad \theta(t=0) = \theta_0.$$

Then under the assumptions

$$u_0, \theta_0 \in H^s(\mathbb{R}) \cap H^{s+1}(\mathbb{R} \setminus [a,b]),$$
$$u_1, \theta_{0,xx} - \gamma u_{1,x} \in H^{s-1}(\mathbb{R}) \cap H^s(\mathbb{R} \setminus [a,b]), \qquad s \geq 3$$

(*compare with* (1.2)), *there exist a positive constant* T *and a unique solution*

$$(u,\theta) \in C^0([0,T], H^s(\mathbb{R})) \cap C^1([0,T], H^{s-1}(\mathbb{R}))$$
$$\cap\, C^2([0,T], H^{s-3}(\mathbb{R}))$$

satisfying

$$\left(\partial_t + \sqrt{\tau}\,\partial_x\right)u \in C^0([0,T], H^s(I \cup III)),$$
$$\left(\partial_t - \sqrt{\tau}\,\partial_x\right)u \in C^0([0,T], H^s(I \cup II)),$$
$$\left(\partial_t + \sqrt{\tau}\,\partial_x\right)\theta \in L^2([0,T], H^s(I \cup III)),$$
$$\left(\partial_t - \sqrt{\tau}\,\partial_x\right)\theta \in L^2([0,T], H^s(I \cup II)),$$

where I, II, and III denote the three regions

$$I := \left\{(x,t) : -\infty < x < a - \sqrt{\tau}t, 0 < t \leq T\right\}$$
$$\cup \left\{(x,t) : b + \sqrt{\tau}t < x < \infty, 0 < t \leq T\right\}$$
$$\cup \left\{(x,t) : b - \sqrt{\tau}t < x < a + \sqrt{\tau}t, \frac{b-a}{2\sqrt{\tau}} < t \leq T\right\};$$

$$II := \left\{(x,t) : a - \sqrt{\tau}t \leq x < a + \sqrt{\tau}t, 0 < t < \min\left(T; \frac{b-a}{2\sqrt{\tau}}\right)\right\}$$
$$\cup \left\{(x,t) : a - \sqrt{\tau}t \leq x \leq b - \sqrt{\tau}t, \frac{b-a}{2\sqrt{\tau}} \leq t \leq T\right\};$$

$$III := \left\{(x,t) : b - \sqrt{\tau}t < x \leq b + \sqrt{\tau}t, 0 < t < \min\left(T, \frac{b-a}{2\sqrt{\tau}}\right)\right\}$$
$$\cup \left\{(x,t) : a + \sqrt{\tau}t \leq x \leq b + \sqrt{\tau}t, \frac{b-a}{2\sqrt{\tau}} \leq t \leq T\right\},$$

as in Fig. 1.1 *of* [10].

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