Double diffusive natural convective flow characteristics in a cavity

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Abstract

The influences of Soret and Dufour coefficients on free convection flow phenomena in a partially heated square cavity filled with water-Al$_2$O$_3$ nanofluid is studied numerically. The top surface has constant temperature $T_c$, while bottom surface is partially heated $T_h$ with $T_h > T_c$. The concentration in top wall is maintained higher than bottom wall ($C_c < C_h$). The remaining bottom wall and the two vertical walls are considered adiabatic. Water is considered as the base fluid. By Penalty Finite Element Method the governing equations are solved. The effect of the Soret and Dufour coefficients on the flow pattern and heat and mass transfer has been depicted. Comprehensive average Nusselt and Sherwood numbers, average temperature and concentration and mid-height horizontal and vertical velocities inside the cavity are presented as a function of the governing parameters. Results shows that both heat and mass transfer increased by Soret and Dufour coefficients.

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Keywords: Soret and Dufour coefficients; double-diffusive natural convection; finite element method; water-Al$_2$O$_3$ nanofluid.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$c$</td>
<td>Dimensional concentration (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$C$</td>
<td>Non-dimensional concentration</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure (kJ kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Concentration susceptibility</td>
</tr>
<tr>
<td>$D$</td>
<td>Solutal diffusivity (m$^2$ s$^{-1}$)</td>
</tr>
<tr>
<td>$D_f$</td>
<td>Dufour parameter</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration (m s$^{-2}$)</td>
</tr>
<tr>
<td>$h$</td>
<td>Local heat transfer coefficient (W m$^{-2}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity (W m$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Thermal diffusion ratio</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the enclosure (m)</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number,</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$Sc$</td>
<td>Schmidt number</td>
</tr>
<tr>
<td>$Sh$</td>
<td>Sherwood number</td>
</tr>
<tr>
<td>$S_r$</td>
<td>Soret parameter</td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
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1. Introduction

The natural convection in enclosures continues to be a very active area of research during the past few decades. While a good number of works have made significant contributions for the development of the theory, an equally good number of works have been devoted to many engineering applications that include electronic or computer equipment, thermal energy storage systems and etc.

Double diffusive convection of water has been studied by Nithyadevi and Yang [1] and Sivasankaran and Kandaswamy [2, 3]. Yet, most work done considers flow inside closed enclosures, the applications included, such as pollution dispersion in lakes, chemical deposition, and melting and solidification process. Diffusion of matter caused by temperature gradients (Soret effect) and diffusion of heat caused by concentration gradients (Dufour effect) become very significant when the temperature and concentration gradients are very large. Generally these effects are considered as second order phenomenon. These effects may become important in some applications such as the solidification of binary alloys, groundwater pollutant migration, chemical reactors, and geosciences. The importance of these effects has also seen in Mansour et al. [4], Platten [5] and Patha et al. [6].

Double diffusive and Soret induced convection in a shallow horizontal enclosure is studied numerically by Mansour et al. [4]. They found that the Nusselt number has decreases in general with the Soret parameter while the Sherwood number increases or decreases with this parameter depending on the temperature gradient induced by each solution.

In the above studies convection heat transfer is due to the imposed temperature gradient between the opposing walls of the enclosure taking the entire vertical wall to be thermally active. But in many naturally occurring situations and engineering applications it is only a part of the wall which is thermally active. For example in solar energy collectors due to shading, it is only the unshaded part of the wall that is thermally active. In order to have the results to possess applications, it is essential to study heat transfer in an enclosure with partially heated active walls. Only a few studies are reported in the literature concerning heat transfer in enclosures with partially heated side walls, by Oztop [7] and Erbay et al. [8].

Natural convection in an enclosure with partially active walls is studied by Nithyadevi et al. [9] and Kandaswamy et al. [10] without Soret and Dufour effects. Present study deals with the natural convection in a square enclosure filled with water and partially heated vertical walls for three different combinations of heating location in the presence of solute concentration with Soret and Dufour effects. The hot region is located at the top, middle and bottom of the left vertical wall of the enclosure.

boundary-layer flow of a nanofluid past a vertical plate where similarity solution was performed in order to obtain correlation formulas giving the reduced Nusselt number as a function of the various relevant parameters. The stability boundaries for both non-oscillatory and oscillatory cases had been approximated by simple analytical expressions. For the porous medium the Darcy model is employed.

Effects of Soret Dufour, chemical reaction and thermal radiation on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet was investigated by Pal and Mondal [16]. The author used shooting algorithm with Runge–Kutta–Fehlberg integration scheme to solve the governing equations. Natural convection heat transfer of nanofluids in a vertical cavity: Effects of non-uniform particle diameter and temperature on thermal conductivity was performed by Lin and Violi [17]. Moreover, Saleh et al. [18] studied natural convection heat transfer in a nanofluid-filled trapezoidal enclosure. They found that acute sloping wall and Cu nanoparticles with high concentration were effective to enhance the rate of heat transfer.

The present work discussed the effect of Soret and Dufour parameter on double diffusive natural convection in a partially heated cavity. The results are presented in the form of streamlines, isotherms, average Nusselt number Nu and average Sherwood number Sh, average temperature of the fluid and mid height velocity in the cavity for relevant parameter.

2. Physical model

Figure 1 shows a schematic diagram of a partially heated square enclosure. The fluid in the cavity is water-based nanofluid containing Al₂O₃ nanoparticles with Soret and Dufour coefficients. The nanofluid is assumed incompressible and the flow is considered to be laminar. It is taken that water and nanoparticles are in thermal equilibrium and no slip occurs between them. The top horizontal wall has constant temperature \( T_c \), while bottom wall is partially heated \( T_b \), with \( T_b > T_c \). The concentration in top wall is maintained higher than bottom wall \( (C_c < C_b) \). The remaining bottom wall and the two vertical walls are considered adiabatic. The thermophysical properties of the nanofluid are taken from Saleh et al. [18] and given in Table 1. The density of the nanofluid is approximated by the Boussinesq model.

![Fig. 1. Schematic diagram of the enclosure](image)

Table 1. Thermo physical properties of fluid and nanoparticles [18]

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Fluid phase (Water)</th>
<th>Al₂O₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kgK)</td>
<td>4179</td>
<td>765</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>997.1</td>
<td>3600</td>
</tr>
<tr>
<td>( k ) (W/mK)</td>
<td>0.6</td>
<td>46</td>
</tr>
<tr>
<td>( \beta \times 10^5 ) (1/K)</td>
<td>21</td>
<td>0.63</td>
</tr>
</tbody>
</table>

3. Governing equations

The governing equations for laminar natural convection in a cavity filled with water-alumina nanofluid in terms of the Navier-Stokes and energy equation (non dimensional form) are given as:
The corresponding boundary conditions take the following form:

- at all solid boundaries \( U = V = 0 \)
- at \( Y = 0, 0.3, 0.7 \), \( X = 0, 1 \), \( C = 0 \)
- at \( Y = 1, X = 0, C = 1 \)

at the remaining boundaries \( \frac{\partial \theta}{\partial N} = 0, \frac{\partial C}{\partial N} = 0 \)

the following dimensionless dependent and independent variables

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{UL}{v_f}, \quad V = \frac{VL}{v_f}, \quad P = \frac{PL^2}{\rho_f v_f^2}, \quad \theta = \frac{T - T_{r_e}}{T_h - T_c}, \quad C = \frac{C - C_c}{C_h - C_c}
\]

are used to make the above equations non-dimensional.

where, \( \rho_{sf} = (1 - \phi) \rho_f + \rho_s \) is the density,

\[
\left( \rho C_p \right)_{sf} = (1 - \phi) \left( \rho C_p \right)_f + \phi \left( \rho C_p \right)_s
\]

is the heat capacitance,

\( \beta_{sf} = (1 - \phi) \beta_f + \phi \beta_s \) is the thermal expansion coefficient,

\( \alpha_{sf} = k_{sf} / \left( \rho C_p \right)_{sf} \) is the thermal diffusivity,

the dynamic viscosity of Brinkman model [19] is \( \mu_{sf} = \mu_f \left( 1 - \phi \right)^{2.5} \)

and the thermal conductivity of Maxwell Garnett (MG) model [20] is \( k_{sf} = k_f + k_s + 2k_f - 2\phi (k_f - k_s) \)

Prandtl number \( Pr = \frac{v}{\alpha_f} \), Schmidt number \( Sc = \frac{v}{D_f} \), thermal Rayleigh number \( Ra_T = \frac{g \beta_f L^4 (T_h - T_c)}{\nu_f^2} \), solutal Rayleigh number \( Ra_c = \frac{g \beta_c L^4 (C_h - C_c)}{\nu_f} \), Dufour coefficient \( D_f = \frac{D}{\nu_f} \frac{k_f (C_h - C_c)}{C_f C_p (T_h - T_c)} \)

and Soret coefficient \( S_f = \frac{D}{\nu_f} \frac{k_f (T_h - T_c)}{T_m (C_h - C_c)} \) are used.

The average Nusselt and Sherwood numbers at the heated and concentrated surfaces of the enclosure may be expressed, respectively as

\[
Nu = \frac{1}{L_h} \int_0^L \frac{\partial \theta}{\partial Y} \, dX \quad \text{and} \quad Sh = \frac{1}{L_h} \int_0^L \frac{\partial C}{\partial Y} \, dX
\]

4. Numerical implementation

The Galerkin finite element method is used to solve the non-dimensional governing equations along with boundary conditions for the considered problem. The equation of continuity has been used as a constraint due to mass conservation.
and this restriction may be used to find the pressure distribution. The penalty finite element method [21] is used to solve the Eqs. (2) - (4), where the pressure $P$ is eliminated by a penalty constraint. The continuity equation is automatically fulfilled for large values of this penalty constraint. Then the velocity components ($U, V$), temperature ($\theta$) and concentration ($C$) are expanded using a basis set. The Galerkin finite element technique yields the subsequent nonlinear residual equations. Three points Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion $\varepsilon$ such that $|\psi^{n+1} - \psi^n| < 10^{-4}$, where $n$ is the number of iteration and $\psi$ is a function of $U, V, \theta$ and $C$.

4.1. Grid independent test

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for $Ra = 10^4$, $Pr = 6.2$, $Df = S_r = 0.5$, $Sc = 5$, $\phi = 5\%$ in the chamber. In the present work, we examine five different non-uniform grid systems with the following number of elements within the resolution field: 2569, 4730, 6516, 8457 and 10426. The numerical scheme is carried out for highly precise key in the average Nusselt ($Nu$) and Sherwood ($Sh$) numbers for the aforesaid elements to develop an understanding of the grid fineness as shown in Fig. 2. The scale of the average Nusselt and Sherwood numbers for 8457 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 8457 elements is preferred for the computation.

![Fig. 2. Grid test for the geometry](image)

![Streamlines Isotherms Concentration](image)

**Fig. 3. Comparison between present work and Nithyadevi and Yang using**

$Pr = 11.573$, $D_f = S_r = 0.5$, $Sc = 5$ and $Ra_T = 10^5$
4.2. Code validation

The present numerical solution is validated by comparing the current code results for streamlines, isotherms and concentration profiles using $D_f = S_r = 0.5$, $Sc = 5$, $Pr = 11.573$ and $Ra_T = 10^3$ with the graphical representation of Nithyadevi and Yang [2] which was reported for double diffusive natural convection in a partially heated enclosure with Soret and Dufour effects. Fig. 3 demonstrates the above stated comparison. As shown in Fig. 3, the numerical solutions (present work and Nithyadevi and Yang [2]) are in good agreement.

5. Results and discussion

In this section, numerical results of streamlines and isotherms for various values of Soret ($S_r$) and Dufour ($D_f$) coefficients and with Al$_2$O$_3$/water nanofluid in a square enclosure are displayed. $Ra = Ra_T = Ra_c$ is assumed for the present numerical calculation. The considered values of $D_f$ and $S_r$ are $D_f = S_r = (0, 0.5$ and $1)$. But the Prandtl number $Pr = 6.2$, the Rayleigh number $Ra = 10^4$, the Schmidt number $Sc = 5$ and solid volume fraction of the nanofluid $\phi = 5\%$ are kept fixed for this study. In addition, the values of the average Nusselt and Sherwood numbers, mean temperature and concentration as well as horizontal and vertical velocities at the middle of the cavity have been calculated for different mentioned parameters.

Figure 4 (a) - (c) exposes the effect of $S_r$ on the flow, thermal and concentration fields while $D_f = 0.5$ and $Sc = 5$. At the absence of the Soret coefficient ($S_r$) a primary anticlockwise circulating cell occupies the bulk of the chamber. The size of the inner vortex of this cell becomes larger with the increasing of the Soret coefficient. In addition for the largest value of $S_r$, the streamlines form rectangular pattern whereas initially they are circular. As well as another vortex is appeared near the left wall of the chamber. The isotherms and iso-concentrations are crowded around the active location on the bottom surface of the enclosure for ($S_r = 1$). In addition, the temperature lines corresponding to $S_r = 1$ become less bended. Decreasing Soret effect leads to deformation of the thermal and concentration boundary layers at the right part of the cold upper wall and middle of the bottom surface.

![Fig. 4. Effect of $S_r$ on (a) streamlines, (b) Isotherms and (c) Concentration at $D_f = 0.5$ and $Sc = 5$](image-url)
Fig. 5. Effect of $S_r$ on (i) $N_u$ and $Sh$ and (ii) $\theta_w$ and $C_{lw}$ at $D_f = 0.5$ and $Sc = 5$

Fig. 6. Mid height (i) horizontal and (ii) vertical velocities for different $S_r$ with $D_f = 0.5$ and $Sc = 5$

Fig. 7. Effect of $D_f$ on (a) streamlines, (b) Isotherms and (c) Concentration at $S_r = 0.5$ and $Sc = 5$
The average Nusselt (\(Nu\)) and Sherwood (\(Sh\)) numbers, average temperature (\(\theta_{av}\)) and concentration (\(C_{av}\)) along with the Soret coefficient (\(S_r\)) are depicted in Fig. 5(i)-(ii). It is seen from Fig. 5(i) that \(Nu\) enhances gradually whereas \(Sh\) remains almost invariant for mounting \(S_r\). Consequently Fig. 5(ii) shows that \(\theta_{av}\) devalues and \(C_{av}\) rises sequentially for all values of Soret coefficient \(S_r\).

Figure 6(i)-(ii) shows the mid-height horizontal and vertical velocity profiles inside the chamber for different \(S_r\) effect. It is observed that the fluid particle moves with greater velocity for the absence of Soret coefficient \(S_r\). The waviness devalues for higher values of \(S_r\).

The effect of \(D_f\) on the flow, thermal and concentration fields is presented in Fig. 7 (a) - (c) while \(S_r = 0.5\) and \(Sc = 5\). A primary anticlockwise recirculation cell occupying the whole cavity is found for the absence of the Dufour coefficient \((D_f)\). The fluid rises along the right wall and falls along the left wall. The size of the inner vortex of this cell becomes larger with the increasing of the Dufour coefficient. The strength of the flow circulation, the thermal current and concentration activities are much more activated with escalating \(D_f\). Increasing \(D_f\) the temperature and concentration lines at the middle part of the enclosure become vertical whereas initially they are almost horizontal. Due to rising values of \(D_f\), the temperature and concentration distributions become distorted resulting in an increase in the overall heat and mass transfer. It is worth noting that as the Dufour coefficient increases, the thickness of the thermal boundary layer near the horizontal surfaces rises which indicates a steep temperature and concentration gradients. Hence, an increase in the overall heat and mass transfer within the cavity is observed.

Figure 8(i)-(ii) displays the mean Nusselt and Sherwood numbers, average temperature (\(\theta_{av}\)) and concentration (\(C_{av}\)) for the effect of Dufour coefficient \(D_f\). Both \(Nu\) and \(Sh\) grow up for varying \(D_f\). The rate of heat transfer is found to be more effective than the mass transfer rate. On the other hand, \(\theta_{av}\) and \(C_{av}\) has notable changes with different values of \(D_f\). The value of mean concentration is always higher than that of average temperature at a particular value of Dufour coefficient.

The \(U\) and \(V\) velocities at the middle of the cavity for various \(D_f\) are depicted in Fig. 9 (i)-(ii). A small variation in velocity is found due to changing \(D_f\). Some perturbations are seen in the horizontal velocity graph for \(D_f = 0\) and in the vertical velocity profile for \(D_f = 1\).
6. Conclusion

The influence of nanoparticles on natural convection boundary layer flow inside a square cavity with water-Al₂O₃ nanofluid is accounted. Various Soret-Dufour coefficients and Schmidt number have been considered for the flow, temperature and concentration fields as well as the heat and mass transfer rate, horizontal and vertical velocities at the middle height of the enclosure while Pr, Ra and φ are fixed at 6.2, 10⁴ and 5% respectively. The results of the numerical analysis lead to the following conclusions:

- The structure of the fluid streamlines, isotherms and iso-concentrations within the chamber is found to significantly depend upon the Soret-Dufour coefficients.
- The Al₂O₃ nanoparticles with the highest $S_r$ and $D_f$ is established to be most effective in enhancing performance of heat transfer rate than the rate of mass transfer.
- Greater variation is observed in velocities at a particular point for the changes of $S_r$ with compared to that of $D_f$.
- Average concentration is higher than average temperature inside the chamber for the pertinent parameters.

Overall the analysis also defines the operating range where water-Al₂O₃ nanofluid can be considered effectively in determining the level of heat and mass transfer augmentation.

Acknowledgements

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References