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Numerical studies of the conventional impact damper with discrete frequency optimization and uncertainty considerations

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Abstract This paper presents the performance of a single horizontal conventional Impact Damper (ID) in both wide range frequency and resonance excitations. The effects of the coefficient of restitution, *e*, mass ratio, μ, and clearance, *d*, on the performance of ID are investigated. The optimal parameters are numerically found by discretely varying the clearance and excitation frequency. The performance of optimal ID is discussed, with respect to different parameters, in both resonance and off-resonance modes. In addition, it is shown how the efficiency of the optimal conventional ID is deteriorated as a result of mistuning in the amplitude and frequency of excitation. This is estimated by suggesting a new criterion of post processing data. It is shown that an ID designed to resist high amplitude excitation is able to perform well at lower amplitude. However, the opposite trend can significantly deteriorate the efficiency of optimal ID. In regard to excitation frequency, the ID, optimized with respect to a wide range of frequency, is less sensitive to frequency mistuning. Finally, the vulnerability of the optimized ID versus uncertainties in structural parameters is clearly determined and it is illustrated that less robustness occurs when the performance of the controller is more efficient.

1. Introduction

In recent decades, the use of passive energy devices has become more widespread in structural control, due to their advantages, such as easy application to both existing and new structures, easy maintenance and no energy supply requirement [1]. An impact damper is a relatively temporary passive control device, which has a free mass enclosed between two barriers and is able to attenuate undesirable vibrations as a result of transferring the structure momentum to the auxiliary mass, and changing the dynamic behavior of the controlled system after each collision [2]. Because of the nonlinear behavior of the impact damper, it is able to be effective over a wide frequency range (unlike tuned mass dampers) and, therefore, is potentially attractive for the protection of civil structures against environmental disturbances, such as wind and earthquake [3].

There are many advantages in using impact dampers over traditional passive devices: They are inexpensive, simple in design [4,5], robust, and effective in harsh environments with a wide range of frequencies [6]. Several practical applications of impact dampers, in addition to experimental and numerical investigations, have been reported in literature to reduce excessive vibrations of mechanical systems, such as turbine blades, light poles, printed circuit boards, robot arms and high-speed railway bridges [7–12].

Two main factors are important in the proper design of impact dampers; firstly, tuning the characteristics of impact dampers, such as the coefficient of restitution, *e*, the mass ratio, μ, and clearance, *d*, governing the impact pattern of the auxiliary mass. The amplitude and frequency of load in operational situations is the second factor to be considered. Trigui et al. proved that with a particle damper, a very high value of specific damping capacity can be achieved, compared

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Nomenclature

ξ	Damping ratio
Ω	Frequency of the external sinusoidal acceleration
α	Phase angle of steady response relative to the excitation
ω_n	Natural frequency of main system
$\dot{x}$	Velocity of main system
x_0	Initial main system displacement
$\dot{x}_0$	Initial main system velocity
$\dot{Y}$	Absolute particle velocity
$\dot{y}$	Relative particle velocity
μ	Mass ratio m/M
$\dot{X}_+$	Velocity of M just after impact
$\dot{y}_+$	Relative particle velocity after impact
$\dot{y}_-$	Relative particle velocity before impact
e	Constant coefficient of restitution
$\dot{x}_+$	Velocity of M just after impact
X	Absolute main system displacement
a	Amplitude of external acceleration
$\ddot{X}$	Absolute acceleration of main *system*
σ	Root mean square
M	Mass of main system
K	Stiffness of main system
C	Viscose damping constant
g	Gravity 9.81 m/s^2
m	Mass of particle
d	Allowable particle travel
D	Dimensionless of d divided by $\sigma_{\text{max-uncontrolled}}$ displacement
Δt	Time increment
$\Delta(\Omega/\omega_n)$	Frequency increment

with the intrinsic material damping of the majority of structural metals. The effect of clearance and acceleration on the evolution of specific damping capacity was analyzed, and its dependence on system parameters and excitation sources was revealed [13]. Certain studies have reported the effect of impact damper parameters on its performance, versus the resonance condition, and some others have demonstrated its efficiency against off-resonance loading with inadequate optimization details.

Popplewell and Liao, in 1991, suggested a simple analytical procedure to optimize the clearance of a rigid impact damper under resonance conditions, and illustrated the effect of e and μ [5]. Bapat and Sankarin, in 1984, studied the performance of impact dampers in resonance, and developed user oriented charts to obtain rough estimates of key parameters [4]. Also, they showed that the efficiency of the optimal impact damper under resonance circumstances is not necessarily optimal at other frequencies. It was observed that the effectiveness of the impact damper deteriorates at other frequencies, compared to its efficiency when the optimization is conducted in the resonance frequency [3,4,14–17]. So, it is vital to predict the operational situation of systems controlled by a single Impact Damper (ID) to calculate the optimal parameters. Also, it is essential to know how badly optimal impact dampers perform in off-optimum and uncertain situations. In fact, there has been little research undertaken on the performance of optimal impact dampers over a variety of uncertainties.

Several researchers have also investigated the performance of impact dampers in forced vibration with a wide band frequency. Butt, in 1994, conducted a numerical research to illustrate the performance of impact dampers at a discrete range of frequencies, and showed that the effectiveness of impact dampers depends on system amplitude, as well as impact pattern [18]. However, only the effectiveness of impact dampers in some cases has been estimated, not preferably, in the comprehensive range of e, μ and d. Like other research, it has been shown that the impact damper produces a negative effect at frequencies lower than the original natural frequency [18,19].

Although there have been many numerical investigations about traditional impact dampers, only a few of them have extensively shown its performance at a wide range of frequencies. Popplewell et al. [17] and Bapat and Sankar [4], showed that the optimum parameters of impact dampers in resonance can vary under off-resonance conditions. Masri and Ebrahim [20] and Semericigael and Popplewell [21] studied the effectiveness of impact dampers subjected to random vibrations. They expressed that the effectiveness of impact dampers increases when an equi-spaced-impact/cycle occurs. Papalou and Masriin, in 1996, presented the results of an experimental and analytical study of the performance of granular impact dampers under wide band random excitation [2], and investigated the influence of system parameters and the intensity of the excitation. In addition, some researchers have tried to study the effect of wide band disturbances, such as earthquakes [16,22,23]. Park et al., in 2009, observed deterioration in the performance of impact dampers in some frequencies less than the natural frequency of the primary system [15]. Butt studied the impact-damped harmonic oscillator by obtaining the time-history solution for primary and auxiliary masses at different frequencies to investigate the effect of impact damper parameters, especially in resonance [18]. Duncan et al. numerically investigated the damping performance of the single particle vertical impact damper over a wide range of excitation frequencies and amplitudes, to study the influence of key parameters, such as mass ratio, clearance, coefficient of restitution and structural damping ratio [6]. A discrepancy has also been discovered in regard to how the coefficient of restitution affects damper performance in the natural frequency of a system, but this discrepancy remains unclear when considering all frequencies. Lu et al. [24] evaluated the effects of a large number of system parameters, such as number, size and particle material, mass ratio, excitation frequency and amplitude level, coefficient of restitution, damping ratio of the primary system, and the coefficient of friction, using high-fidelity simulations based on the discrete-element method. It should be mentioned that these researchers have not compared optimal parameters and effectiveness under both resonance and off-resonance conditions. Besides, they have not included finding the optimal parameters of the impact damper and estimating the effect of amplitudes and frequencies in the deterioration of their performance in off-optimal circumstances. The first major part of this paper, thus, studies the optimal parameters of a traditional impact damper subjected to harmonic excitations with resonance and off-resonance considerations.

Another major part of this investigation is dedicated to the numerical estimation of the negative effects of different uncertainties on the performance of an optimal impact damper in order to provide a general insight into the robust design of impact dampers. In practice, structures are subjected to uncertainty in three main areas. First, there is an uncertainty in the dynamic characteristics of structures such as mass, damping ratio and, particularly, stiffness. This kind of uncertainty may present as a result of an inadequate modeling of boundary conditions at the structural joints, the effect of non-structural elements, degradation due to aging, and a fluctuation in structural mass [25]. Furthermore, the dynamic characteristics of structures change as a result of earthquake and wind excitations. Second, there are some uncertainties in the characteristics of impact dampers, especially the coefficient of restitution, because of destroying collisions [2]. The third and most inevitable, is the uncertainty in environmental loadings, such as intensity, dominant frequency, and wide or narrow band characteristics of excitations. To appropriately design impact damper parameters, it is essential to take uncertainties into account. Few, if any, researchers have investigated the reliability of optimal impact dampers. Certain studies have reported the vulnerability of impact dampers to efficient clearance. Bapat and Sankar indicated that changing the optimal clearance causes deterioration in the performance of impact dampers, due to changing the impact pattern [4]. They also showed the sensitivity of impact dampers exposed to frequencies less than the natural frequency of the controlled system. Papalou and Masriin, in 1996, demonstrated that utilizing granular impact dampers leads to less sensitivity in changes of parameters [2]. Chatterjee et al. illustrated the 'jump' region of performance, in which the amount of clearance is optimal. In other words, the effectiveness of an optimized impact damper is drastically worsened after a small change in clearance [26]. Duncan et al. illustrated the reduction in damping of a vertical impact damper by increasing the damping ratio of the structure and the amplitudes of excitations [6]. Masri and Ibrahim reported less sensitivity of impact dampers to changes in system parameters, compared to the conventional dynamic vibration absorber [22]. Shaw, in 2006, attenuated vibrations of flexible blades in a bladed disk assembly using impact absorbers tuned slightly below the excitation order. They expressed that, during impact operation, the absorber performance is highly insensitive to the tuning order, but dependent on absorber mass and impact properties [27]. Dehghan-Niri et al. demonstrated the significant robustness of a single impact damper subjected to 3% error in structural stiffness [28]. The particle damper is more robust when considering arbitrary levels of excitation in different directions [29]. Lu et al. showed the behavior of the particle damper is compared to a multi-unit impact damper to enhance an understanding of the two passive control devices. A systematic investigation of the performance of particle dampers (vertical and horizontal) attached to primary single-degree-of-freedom and multi-degree-of-freedom systems under different dynamic loads (free vibration, stationary random excitation, as well as non-stationary random excitation with single component or multi-component) was also conducted, and the optimum operating regions were all determined [30]. As mentioned, none of these researchers clearly addressed the performance of the optimal single impact damper subjected to different kinds of uncertainty.

The paper begins by briefly introducing the mathematical model of a traditional impact damper. Section 3 is dedicated to studying the optimal parameters of an impact damper. This is followed by Section 4, studying the effect of different uncertainties on the performance of an impact damper with optimal parameters. Finally, the conclusion is given in Section 5.

2. Mathematical model

A numerical simulation was carried out using MATLAB software in order to study the performance of the impact damper

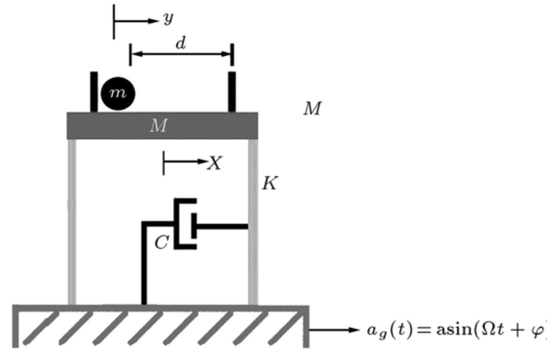

Figure 1: Schematic view of SDOF equipped by impact damper.

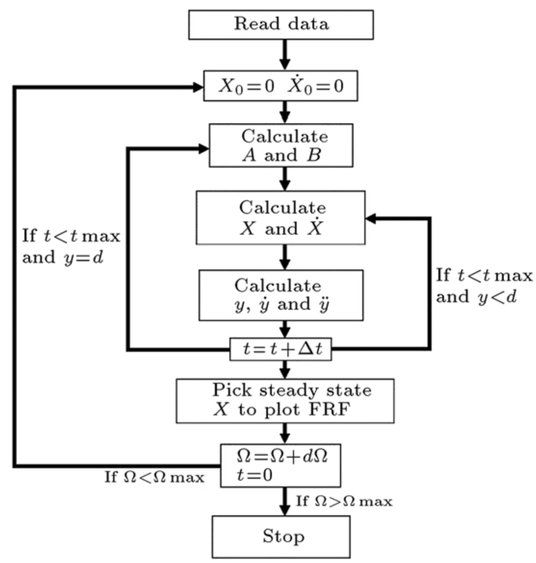

Figure 2: Flowchart of calculating the states of each mass [18].

considering its different parameters. This was accomplished by obtaining time history displacement responses for the primary mass. The primary system (SDOF) consists of a base oscillating harmonic acceleration with amplitude, *a*, and radian frequency, Ω. A single degree of freedom structure is attached to the oscillating base by means of a spring, which has stiffness K, and a dashpot which has damping C. The single conventional impact damper with its container is mounted on the structure. Properties of the impact damper are clearance, *d*, coefficient of restitution, *e*, and auxiliary mass, *m*. A schematic view of the system is shown in Figure 1.

The main assumptions and formulations in this section are adopted from [18,19] and summarized in this section. When the excitation is applied to the structure, the free particle is assumed to be resting against the left wall of the container and pressed against the wall. As the velocity of the primary mass decreases, the particle starts moving relative to it. It is assumed that the container's surface area that comes in contact with the particle is frictionless. Consequently, the absolute velocity of the particle stays constant between two collisions. The initial displacement and velocity values for the primary and auxiliary masses are assumed to be zero. The motion of the structure, subjected to a sinusoidal based acceleration between impacts, is given in Eqs. (1)–(3) [18,19,31]. A flowchart of calculating the states of each mass is shown in Figure 2.

$$B = \frac{\dot{x}_0 + (x_0 - U\sin(\Omega t - \alpha))\,\omega_d \tan\omega_d t + (x_0 - U\sin(\Omega t - \alpha))\,\xi\omega_n - U\Omega\cos(\Omega t - \alpha)}{e^{\xi\omega_n t}\omega_d \sin\omega_d t + e^{\xi\omega_n t}\omega_d\cos\omega_d t} \tag{3a}$$

$$A = \frac{\frac{x_0 - U\sin(\Omega t - \alpha)}{e^{\xi\omega_n t}} - B\sin\omega_d t}{\cos\omega_d t} \tag{3b}$$

$$U = \frac{a}{\sqrt{\left(\omega_n^2 - \Omega^2\right)^2 + \ (2\xi\omega_n\Omega)^2}} \tag{3c}$$

$$\alpha = \tan^{-1}\left(\frac{2\xi\omega_n\Omega}{\omega_n^2 - \Omega^2}\right). \tag{3d}$$

Box I:

$$\ddot{X} + 2\xi\omega_n\dot{X} + \omega_n^2 X = a\sin\Omega t, \tag{1}$$

where, $\omega_d = \omega_n\sqrt{(1-\xi^2)}$ and the solution of the differential equation (1) is:

$$x(t) = e^{\xi\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right) + U\sin(\Omega t - \alpha). \tag{2a}$$

$$\begin{aligned}\dot{x}(t) \ = \ & e^{\xi\omega_n t}\left(-A\omega_d\sin\omega_d t + B\omega_d\cos\omega_d t\right) \\ & - \xi\omega_n e^{\xi\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right) \\ & + U\Omega\cos(\Omega t - \alpha).\end{aligned} \tag{2b}$$

Using initial conditions, $x_0 = x(t)$ and $\dot{x}_0 = \dot{x}(t)$, the constants, A and B, are obtained (see equations in Box I).

After each collision, constants A and B are updated by using the values of displacement and the velocity of the primary mass.

The relative velocity of the auxiliary mass at time t is defined in Eq. (4).

$$\dot{y}(t) = \dot{Y} - \dot{x}(t), \tag{4}$$

where $\dot{Y}$ is the absolute velocity of the auxiliary mass after the previous impact and $\dot{x}(t)$ is the velocity of the primary mass at time t.

The total displacement $s(t)$ traveled by the particle since the previous impact is expressed as:

$$s(t) = \sum s(t-\tau) + s(\tau). \tag{5}$$

The total displacement of the particle in each time step is equal to $\sum s(t-\tau)$, except for the last time step, which would be equal to $s(\tau)$. Each collision occurs when the total displacement of the particle equals the allowable clearance. The time of the impact and the velocity before the impact can be determined within a reasonable error range by utilizing a very small time interval (τ) less than 10^{-4} s, and a linear interpolation between two points of displacement in the time history (immediately before and after the barrier).

As far as the equations at the event of the impact are concerned, the relative velocity of the auxiliary mass after each collision is expressed in Eq. (6), based on the restitution model:

$$\dot{y}_+ = -e\dot{y}_-. \tag{6}$$

Considering the conservation of momentum [18,26], Eq. (7) is obtained:

$$\dot{X}_+ + \mu\left(\dot{X}_+ + \dot{y}_+\right) = \dot{X}_- + \mu\left(\dot{X}_- + \dot{y}_-\right). \tag{7}$$

After substituting Eq. (6) into Eq. (7), the velocity of the primary mass after each collision is determined as:

$$\dot{X}_+ = \dot{X}_- + \frac{\mu(1+e)\,\dot{y}}{(1+\mu)}. \tag{8}$$

Table 1: Characteristics of benchmark SDOF structure.

M (kg)	Damping ratio ξ (%)	K (N/m)	$\sigma_{\text{max-uncontrolled}}$ for $a = 0.3g$ (cm)	$\sigma_{\text{max-uncontrolled}}$ for $a = 0.5g$ (cm)
1.35	2.16	865	7.29	12.15

σ: Root mean square.

By summation of the relative velocity of the auxiliary mass and that of the primary mass, the absolute velocity of the auxiliary mass can be stated as:

$$\dot{Y} = \dot{x}_+ + \dot{y}_+. \tag{9}$$

It is assumed that the impacts are instantaneous; each has zero duration, and the coefficient of restitution, e, is independent of impact velocity. This assumption might seem more realistic if there is no nonlinear behavior of the material during each impact, and the impact duration is short enough to be neglected. These assumptions were considered in most previous analytical and computational research studies [6,18]. The velocity and displacement of the primary mass right after a previous collision are used to update the initial conditions to obtain the time-history until the next impact occurs.

Because of the nonlinear behavior of the impact damper, it was inevitable to choose a benchmark system to initiate the parametric study. A SDOF system was selected based on previous research work experience [14]. The characteristics of the SDOF system are summarized in Table 1.

The Route Mean Square (RMS) of the displacement of the primary mass is assumed to be representative of the response of the system. To consider the effect of transient motion, it is suggested that the performance of the impact damper be calculated from the beginning of the motion. The auxiliary mass and main structure states were collected for 100 cycles. The number of cycles was increased from 100 to 200 cycles in separate runs of the model, where the calculated performance varied less than five percent. Two different values of a equal to 0.3 and 0.5g, were applied to study the effect of the amplitude of the sinusoidal base acceleration. The dimensionless value of clearance after 100 cycles is defined as:

$$D = d/\sigma_{\text{max-uncontrolled}}, \tag{10}$$

where $\sigma_{\text{max-uncontrolled}}$ is the maximum RMS value of the uncontrolled primary mass after 100 cycles with swiping the excitation frequency. Two criteria are utilized for performance evaluation of the impact damper:

$$J_1 = \sigma_{\text{max-controlled}}/\sigma_{\text{max-uncontrolled}}, \quad 0 \le \Omega \le \infty, \tag{11}$$

$$J_2 = \sigma_{\text{max-controlled}}/\sigma_{\text{max-uncontrolled}}, \quad \Omega/\omega_n = 1, \tag{12}$$

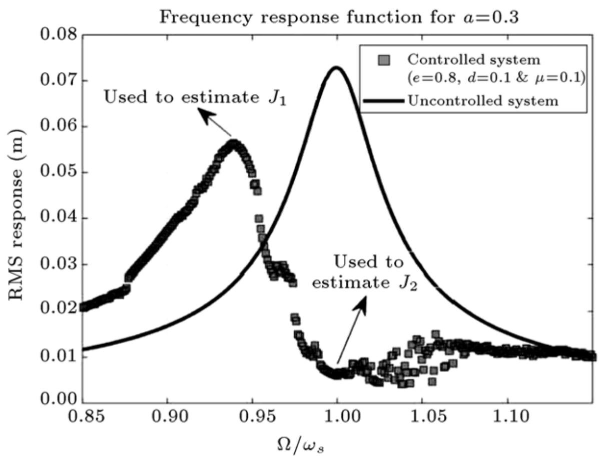

Figure 3: Frequency response function of structure.

where $\sigma_{\text{max-controlled}}$ is the maximum RMS of the primary mass displacement response, due to the sinusoidal base acceleration of the controlled system after 100 cycles. J_1 is able to evaluate the performance of the impact damper when excitation is more likely to be broadband, such as earthquakes and mechanical tools with varying operational speeds. To obtain the value of J_1, the frequency increment of $\Delta\Omega = 0.0253$ Rad/s($\Delta\,(\Omega/\omega n) = 0.001$) is used. Figure 3 illustrates two values that are used in J_1 and J_2 calculations in Eqs. (11) and (12).

To study the characteristics of the impact damper, $\mu = [0.02 \quad 0.04 \quad 0.08 \quad 0.1]$, $e = [0.2 \quad 0.4 \quad 0.6 \quad 0.8]$ and the clearance increase of $\Delta d = 5$ mm are used. Some of these values and the assumptions of the mathematical model are not always practical. The results may not be practically achievable but are included to show theoretical insight.

3. Numerical search for optimum point

To find the optimum parameters of ID, based on criteria J_1 and J_2, a vast search in a meshed domain of ID parameters is undertaken.

Figure 4(a) and (b) show the criteria (J_1) and (J_2) versus D for different mass ratio μ and $e = 0.8$. It is evident in these figures that the effectiveness of the impact damper, with respect to both criteria, increases initially with D, reaches the optimum point at each value of mass ratio (μ) and, afterward, starts decreasing. The reasons for this trend are:

(1) At very low values of D, the relative velocity of the auxiliary mass is so small that a small amount of energy is absorbed;
(2) At large values of D, few impacts occur. This behavior agrees with that of Bapat and Sankar, in 1985 [4]. Figure 8 in Lu et al. [30] shows this trend more clearly.

Although the larger mass ratio shows greater performance, it seems to be more sensitive to clearance. This can be demonstrated in Figures 4 and 5; the small change in optimal clearance can reduce the optimal efficiency of the higher mass ratio more than that of the lower mass ratio. It is noticeable that a little increment in optimal clearance has a more negative effect than a large reduction in it. This considerable drop in performance after the optimum point, especially when the amount of e is large, was also noticed in [26] as a 'jump' area. This observation is really important and needs taking into account in the design procedure, since in practice, just the values of clearance become larger because of some degradation

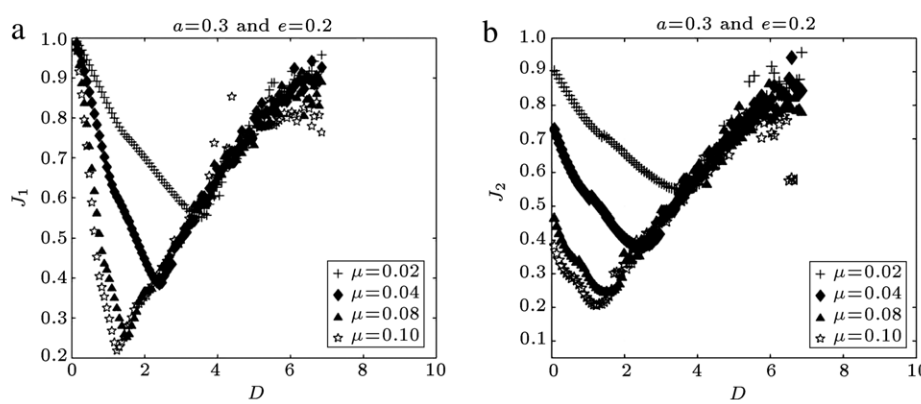

Figure 4: (a) Variation of J_1, and (b) variation of J_2 versus D for $e = 0.2$.

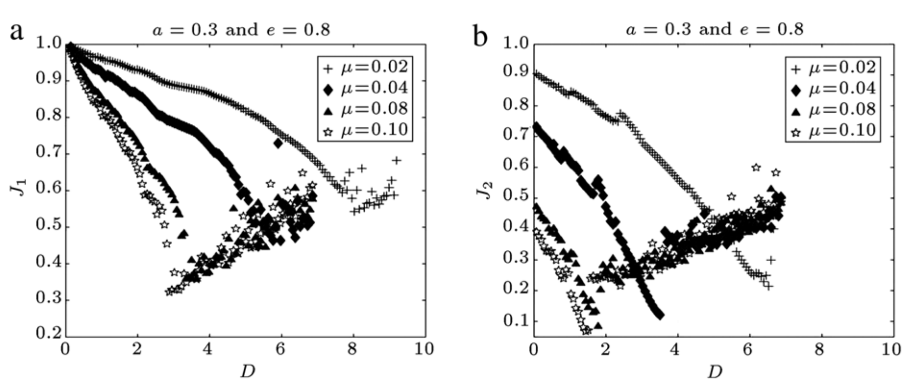

Figure 5: (a) Variation of J_1, and (b) variation of J_2 versus D for $e = 0.8$.

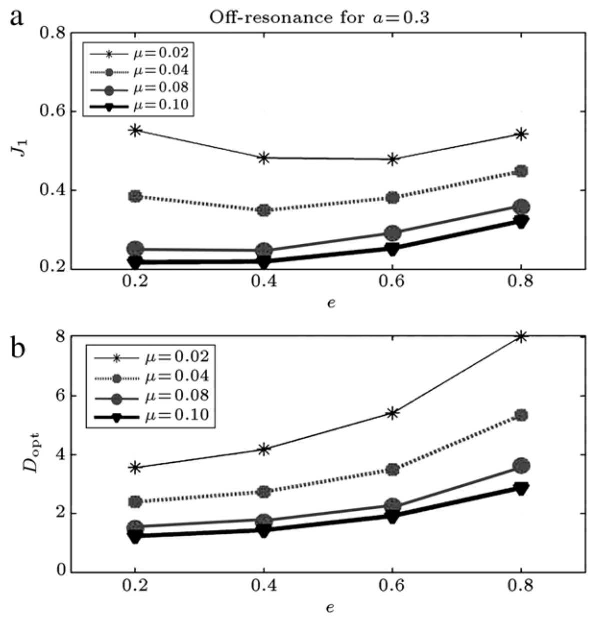

Figure 6: (a) Efficiency of optimized points, and (b) optimized clearance D for J_1 and $a = 0.3$.

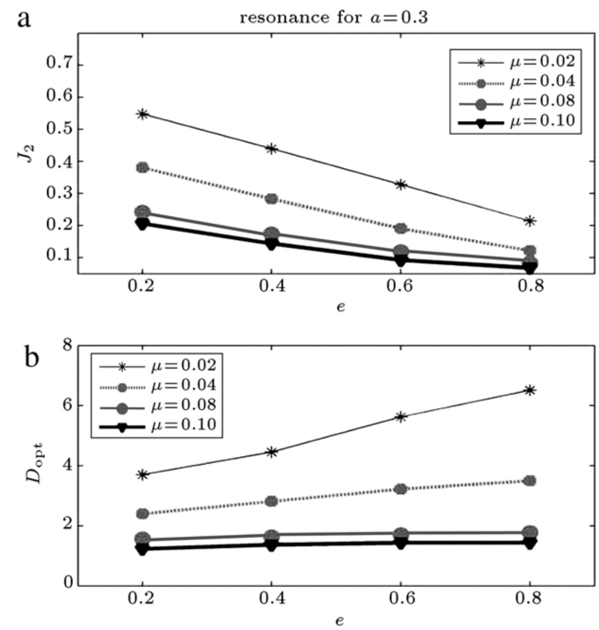

Figure 7: (a) Efficiency of optimized points, and (b) optimized clearance D for J_2 and $a = 0.3$.

of the barriers due to many collisions. Thus, for the designer, it is recommended to choose a value of D less than D_{opt}. All preceding results are similar to those of amplitude $a = 0.5g$ with $e = 0.4$ and 0.6.

3.1. User-oriented charts

After an extensive numerical search with two previous criteria, the values of D_{opt} were determined and different useful design charts were obtained (Figures 6 and 7).

3.1.1. Effect of e in optimized points

Figure 6(a) shows the variation of J_1 versus e values in optimal points; it is illustrated that an increase in e results in a decrease in the efficiency of the optimal ID. On the other hand, as shown in Figure 7(a), the efficiency of optimal ID, with respect to J_2, is enhanced with e. There has been a discrepancy, where some researchers, unlike others [32], believe in an escalation in the damping of the controlled system due to an increase in the value of e in the resonance mode [5,33]. Duncan et al. [6] demonstrated that larger values of the coefficient of restitution of e increase the damping of the system only where the maximum damping value occurs (optimal points). This fully agrees with results shown in Figure 7(a). On the contrary, this study, for the first time, demonstrates that under off-resonance conditions, the efficiency of optimum points is decreasing as e values increase (Figure 6(a)). This can be another reason for the mentioned discrepancy.

Figures 6(b) and 7(b) illustrate D_{opt} for two previous criteria and show the variation of D_{opt} with e and μ. In terms of J_2, The values of D_{opt} increase with e especially for smaller value of μ; if μ is large ($\mu \geq 0.08$), D_{opt} is mostly constant between $D = 1.7$ and 2.0 (Figure 7(b)). This trend remains the same for J_1, especially for smaller e. Referring to Figures 6(b) and 7(b), it is concluded that there is less difference between the values of D_{opt}, which are obtained from optimization of J_1 and J_2, when the values of the coefficient of restitution are approximately less than 0.4. This can also be proved by referring to Figures 8(b) and 9(b) and shall be argued in Section 5 as

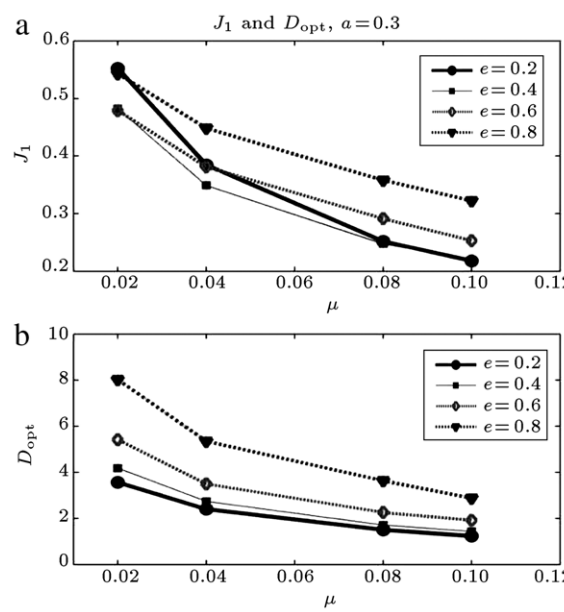

Figure 8: (a) Efficiency of optimized points, and (b) optimized clearance D for J_1 and $a = 0.3$.

the reason for the existence of lower performance deterioration of an optimal impact damper with respect to smaller e ($e \leq 0.4$). The previous results are similarly achieved, considering the amplitude of $0.5g$.

3.1.2. Effect of μ in optimized points

It is predictable that the efficiency of an impact damper is improved by utilizing larger values of μ [6]. This improvement is demonstrated in Figures 8(a) and 9(a). The performance of an impact damper is improved rapidly at small values of μ, for each value of e, and gradually, performance is saturated at large values of μ. This result can also be observed in Figures 6(a) and 7(a).

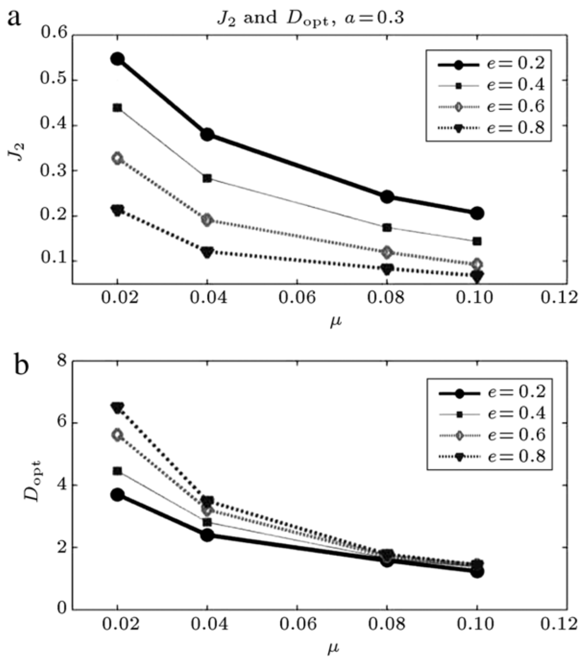

Figure 9: (a) Efficiency of optimized, and (b) optimized clearance D for J_2 and $a = 0.3$.

In both criteria, the D_{opt} decreases with respect to mass ratio, μ. For small values of e ($e \leq 0.4$), these figures are mostly similar; it means that consideration of mostly exact operational conditions (J_1 or J_2) for optimization with smaller values of e is not as important as that for larger values of e.

It can be inferred from Figure 9(b) that for all values of e, the values of D_{opt} are almost equal for large values of μ ($\mu = 0.1$) under the resonance circumstance. This also will be argued in Section 5.

4. Uncertainties

The only certainty in all branches of science is that no entity is certain. Thus, designers have to consider the negative effect of uncertainties on the reliability of control devices. Otherwise, it can cause irreparable damage to structures. In this research, deterioration in the performance of each optimized impact damper subjected to off-optimal situations (mistuning), such as the frequency and amplitude of load, and errors in structural dynamic parameters, such as stiffness and the damping of the structure, shall be illustrated.

4.1. Uncertainties in environmental situations (frequency and amplitude of excitation)

Disturbance is considered one of the environmental uncertainties and its effect should be taken into account for accurate design of impact dampers. Because of the nonlinear behavior of the system controlled by ID, the intensity and dominant frequency of the disturbance play a great role in the effectiveness of ID. Therefore, for optimal design of ID, the approximate intensity of the load and its dominant frequency should be estimated and considered. This, however, may become difficult, if not impossible, in the presence of unpredictable vibrations, such as earthquake excitations. So, we should estimate robustness, and design the impact damper as robust as possible. For this reason, the new post processing factor is suggested to determine the robustness of the impact damper by comparing the performance of optimized ID under certain conditions and exposed to greater or lower amplitudes and resonance or off-resonance situations with the controller, which must have been designed appropriately for the specified uncertain condition. In other words, this criterion can show the difference in effectiveness of the optimized system based on pre-assumed (unreal) excitation (frequency or amplitude), compared with that of an optimized system based on real excitation. Eqs. (13) and (14) are suggested to estimate the deterioration in the performance of the controlled structure in the presence of uncertainties in the vibration amplitude.

$$\mathbf{PA_1}_{\text{off-res } A \text{ to } B} = J_{1AB} - J_{1BB}, \quad (13)$$

$$\mathbf{PA_2}_{\text{res } A \text{ to } B} = J_{2AB} - J_{2BB}. \quad (14)$$

J_{2AB} indicates the performance of the system that is optimized in a resonance situation under A (for example $a = 0.3g$) condition, and encounter B condition (for example $a = 0.5g$). The Post Processing Amplitude index, **PA**, can estimate the effect of disregarding the dominant amplitude of vibration on the optimization of ID.

For example, the performance and d_{opt} of the system in resonance for $e = 0.4$ and $\mu = 0.04$, subjected to $a = 0.3g$, are 28.3% and 20.5 cm, respectively. Now, the performance of this system should be determined when the system encounters $a = 0.5g$ in resonance. With this consideration, the performance is changed to 43.7%. After that, this performance is compared with the effectiveness of a system optimized by considering $a = 0.5g$, $\mu = 0.04$ and $e = 0.4$, whose d_{opt} is equal to 34.53 cm and whose performance is 28.32%. By subtracting these two recent performances (according to Eq. (14), $\mathbf{PA_2}_{\,0.3\text{–}0.5} = 43.7 - 28.32 = 15.38\%$ refer to Figure 10(b)), we can find out, in the system optimized incorrectly, how much can change for the worse if, from the beginning of optimization, we did not consider the exact situation. In these two criteria, the impact damper is optimized for situation A and then exposed to situation B, and its performance compares with a system optimized considering situation B. For convenience, these criteria numerically estimate the consequences of dismissing the real/operational excitation in optimization of such a nonlinear system. These criteria are versatile, not only for impact dampers, but also for nonlinear systems optimization, having uncertainties in some properties of excitation that may have a profound effect on the behavior of nonlinear systems. The Post Processing Frequency **PF** index shall be introduced as well.

4.1.1. Uncertainties in amplitude

As far as amplitude is concerned, the **PA** is estimated for two opposite situations. Firstly, the system is optimized for low amplitude and, then, in an operational situation, is exposed to high amplitude. Secondly, the reverse trend (Figure 10(a) and (b)) depict the deterioration of system performance when the system is optimized, considering low amplitude and encountering high amplitude under two conditions (resonance and off-resonance optimization). It is obvious that by increasing e, the effect of accuracy in choosing the exact amplitude in optimization gets worse. This means that if designers use low amplitude to design an impact damper, in cases of low e and a, the difference between the performance extracted from assumed and real/operational amplitudes is not as large as if they use an exact amplitude for optimization from the beginning. So, it is preferable to use low e, or the concern is not important when e is less and the system may be exposed to

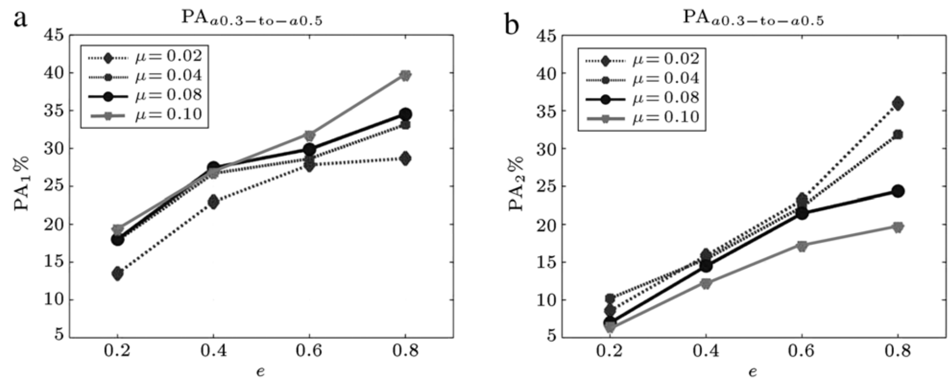

Figure 10: Variation of **PA** index in exposing optimized system according to low amplitude to high amplitude versus *e*. (a) Off-resonance, and (b) resonance.

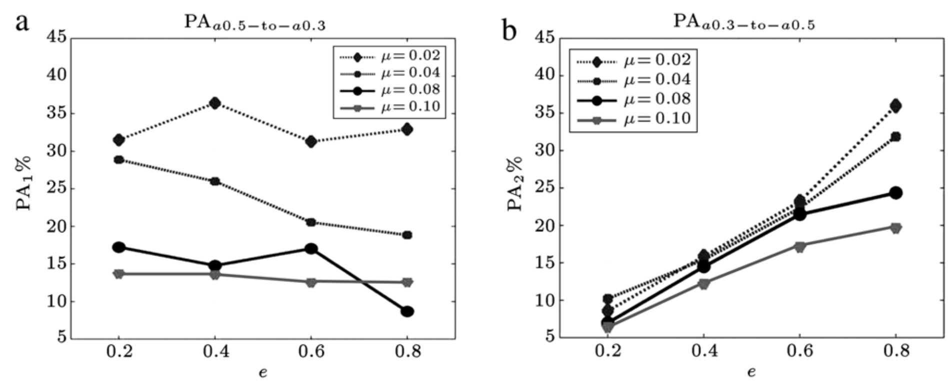

Figure 11: Variation of **PA** index in exposing optimized system according to high amplitude to low amplitude versus *e*. (a) Off-resonance, and (b) resonance.

larger amplitude than was expected under both resonance and off-resonance conditions. The logical reason for this behavior is that by increasing the amount of e, the difference between d_{opt} for low amplitude and d_{opt} for high amplitude grows drastically, and this leads to a significant change in the impact pattern of optimal ID.

For example, in cases of off-resonance, the d_{opt} s for ID with properties of $e = 0.2$ and $\mu = 0.1$ for two different intensities of $a = 0.3$ and $a = 0.5$ are 9 cm and 15.01 cm, respectively. So, the difference between them is 6 cm. This difference increases dramatically to 14.53 cm in cases where $e = 0.8$ and $\mu = 0.1$. As shown in Figure 5(a) and (b), a small increase in values of D leads to a large reduction in the performance of optimal ID, caused by changing the optimal impact pattern. It can be obtained from Figure 10(a) that in off-resonance optimization mode, the sensitivity of the system to transitions from low to high amplitude is higher when μ increases. This trend is the reverse when it comes to the resonance condition. It should be mentioned that in the latter situation, the significance of the effect of e is more important than that of μ.

Now, we shall argue the opposite trend, i.e. designing based on large amplitude (here $0.5g$) and comparing its efficiency with effectiveness, according to a practical condition (in this case, low amplitude of $0.3g$). Figure 11(a) and (b) depict the dependency of the optimal design on the accuracy of using practical amplitude, when ID may encounter low intensity in resonance and off-resonance optimization. The post processing criterion, $\mathbf{PA_2}$, varies steadily with e. Comparing Figures 10 and 11, the low to high amplitude may have a more negative effect than that in high to low cases. Also, it can be inferred that a larger value of μ is not only better for efficiency (referring to Figures 8(a) and 9(a)), but is also much more reliable in terms of uncertainties in the intensity of harmonic loads.

The reason for the reliability of large values of μ in this trend can be justified from Figures 8(b) and 9(b). We can see the variety of d_{opt} is decreased, due to growth in the value of μ, and so, the difference decreases between the d_{opt} of a system subjected to the amplitude pre-designed for $0.5g$ and the d_{opt} of a system subjected to $a = 0.3g$ to which the system is practically exposed. For instance, under an off-resonance condition, if $e = 0.4$ and $\mu = 0.02$ for $a = 0.5g$ and $a = 0.3g$, d_{opt} is 50.5 cm and 30.5 cm, respectively; accordingly, the difference is 20 cm. But, when μ increases to $\mu = 0.1$, this difference becomes 6.5 cm. Here, it can be concluded generally that it is more advantageous for both robustness and effectiveness to choose large amounts of μ and small amounts of e (if possible).

4.1.2. Uncertainty in frequency of excitation

As mentioned in recent investigations, optimality at the special frequency never ensures the efficiency of the system at other frequencies. But, in most cases, there has been a lack of estimation of this vital sensitivity, especially when considering optimality. This section aims at a numerical discussion of systems controlled with optimized IDs and then exposed to different frequencies, for which they are not designed. Two cases will be considered: first, if the system is optimized with the J_1 criterion but encounters only resonance, and second, if the system is optimized for the J_2 criterion and the excitation frequency consists of different frequencies.

In Figure 12, the J_1 values are shown for IDs optimized according to J_2. Figure 12 clearly shows that the efficiency of ID can worsen up to 75%, for high values of e, and increase from 20% to 60% for lower values of e, related to $\mu = 0.1$ and $\mu = 0.02$. To find out the sensitivity of the system in

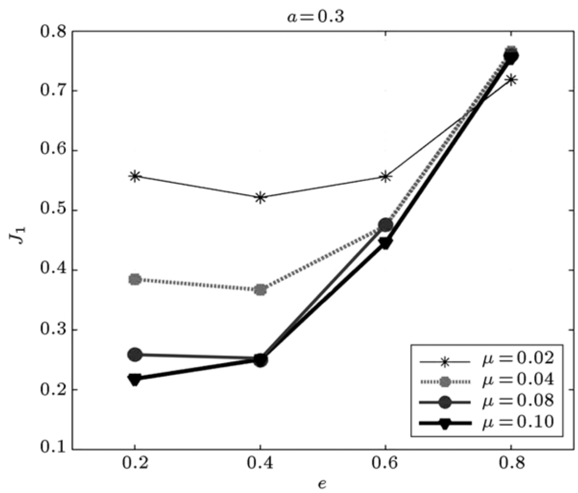

Figure 12: J_1 values for IDs optimized according to J_2 (performance of optimal points of J_2 encountered wide range of frequency).

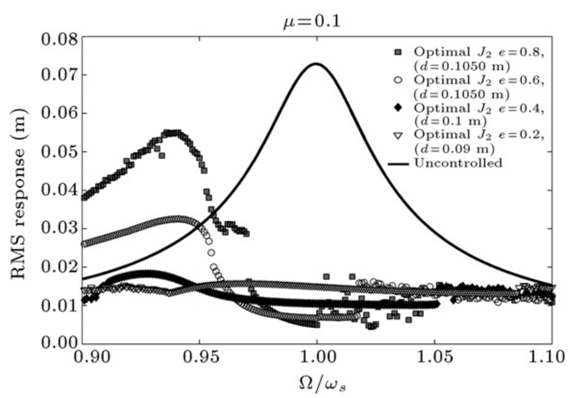

Figure 13: Frequency response function of different ID optimized by J_2 and $\mu = 0.1$ to show how Figure 12 for $\mu = 0.1$ is produced.

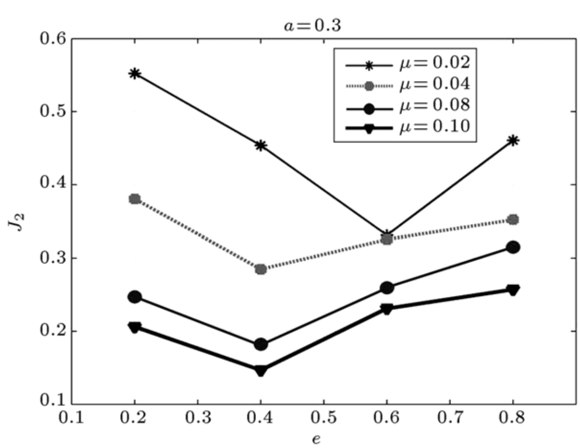

Figure 14: J_2 values for IDs optimized according to J_1 (performance of optimal points of J_1 encountered resonance).

this regard, one should compare Figure 12 with Figure 7(a), from which, it can be figured out that for $e \leq 0.4$, the worst response at all frequencies is approximately equal to that in the resonance mode and, as mentioned before, the associated D_{opt} s do not vary. This trend is more accurate if the value of μ is high. It can also be demonstrated according to Figure 13. Also, as shown in Figure 13, another reason for this behavior is that the critical frequency is close to the natural frequency of structure, where the value of e decreases. The critical frequency is the frequency at which the response maximizes. On the other hand, when it comes to high values of e, the gap between efficiency in resonance and at critical frequency varies exponentially.

In Figure 14, the J_2 values are shown for IDs optimized according to J_1. In terms of off-resonance (J_1) optimization, Figure 14 demonstrates that systems designed according to J_1 are more reliable when they encounter a resonance operational situation. So, it is preferable to use J_1 in the optimization of traditional ID parameters, when more uncertainty in the frequency content of excitation exists. This also can be proved by comparing Figure 14 to Figure 6(a).

Figures 13 and 14 show how efficiency of a controlled system can change for the worst in different off-optimality at frequency of excitation. It is evident in Figure 13 that if the system, designed based on J_2, is excited with lower frequencies than resonance, it may perform more inefficiently; this is more significant when the higher restitution coefficient, e, is used. Comparing Figures 7(a) and 6(a) with Figures 12 and 14 shows how the effectiveness of optimized ID, according to J_1 and J_2, worsens. Although Figures 12 and 14 can demonstrate the effect of off-optimality and uncertainty in the frequency content of excitation, they do not show the sensitivity to accuracy in the presumed frequencies that were set in the optimization. This leads to coming up with a new idea to introduce another post processing criterion, which can estimate this sensitivity numerically.

$$\mathbf{PF}_{\text{res. to off-res.}} = J_{21} - J_{11}, \tag{15}$$

$$\mathbf{PF}_{\text{off-res. to res.}} = J_{12} - J_{22}. \tag{16}$$

The first subscript indicates the optimization condition and the second indicates the operational condition. For example, J_{21} indicates the performance of the system optimized under a resonance condition and exposed to all frequencies.

This post processing frequency criterion, **PF**, is proposed to estimate the sensitivity of the optimal design of a particular ID in terms of the frequency content of excitation. This criterion is able to find out how big the difference is between the efficiency of the system designed based on resonance, but exposed to all frequencies, and that of a system designed according to off-resonance, but exposed to natural frequency. For example, assume that a particular ID is designed according to the resonance condition, but in reality is exposed to a wide range of frequencies with similar amplitude; it is useful to see how much improvement is made if it had been optimized initially based on exact reality (wide range of frequencies). Indeed, this factor is able to show the vulnerability of optimal ID due to the presence of uncertainty in frequency. To simply understand how Figure 15(a) and (b) are extracted from Eqs. (15) and (16), Figures 12 and 14 should be compared to Figures 7(a) and 6(a), respectively.

Figure 15(a) and (b) demonstrate that the assumed frequency applied to the design should be close to reality (operational situation), especially at higher e values and lower μ values. These figures are extracted from $a = 0.5g$. This trend is similar to $a = 0.3g$.

4.2. Uncertainty in dynamic characteristics of structure

In this section, the effect of $\pm 10\%$ error in two dynamic parameters of the structure (damping and stiffness) on the efficiency of optimal designed impact dampers is investigated.

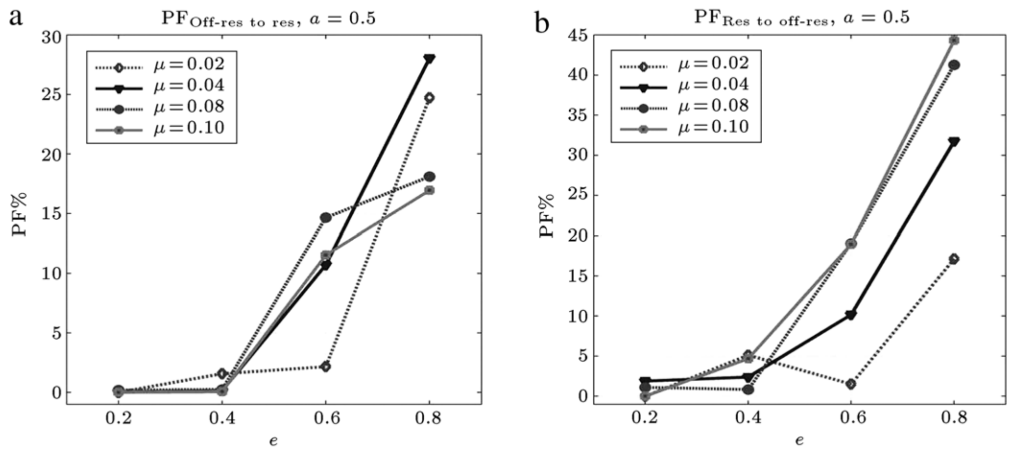

Figure 15: Variation of **PF** index due to increasing in value of e for $a = 0.5g$. (a) Off-resonance to resonance, and (b) resonance to off-resonance.

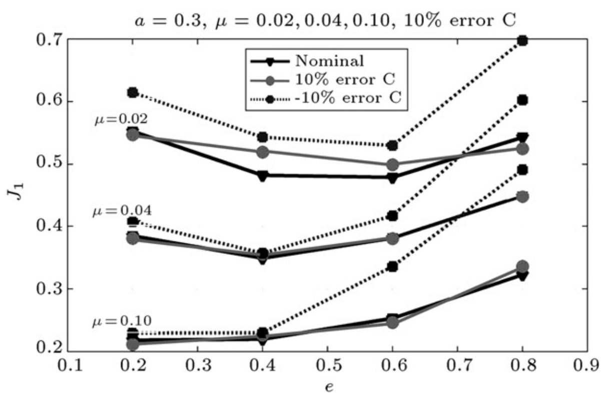

Figure 16: Deterioration of performance of optimized ID due to $\pm$10% error in C of structure with considering J_1.

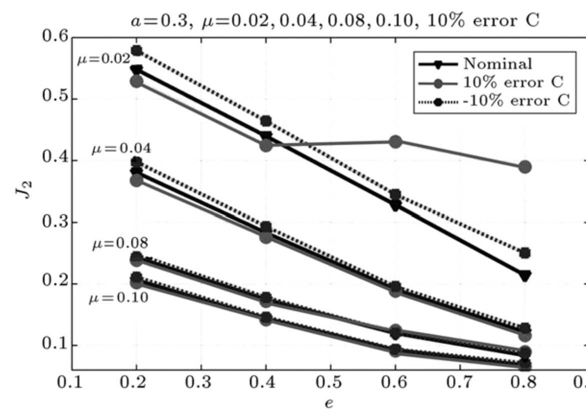

Figure 17: Deterioration of performance of optimized ID due to $\pm$10% error in C of structure with considering J_2.

4.2.1. Uncertainty in C

Figure 16 shows that in cases of an off-resonance condition, 10% negative error in the damping of the structure causes disability in the optimized system, which is worse than that of 10% positive error. The two main reasons for this behavior are: first, both negative and positive errors can lead to off-optimality, but negative error results in a larger displacement than the displacement for which the impact damper is designed; and second, negative error in the damping of the structure inherently increases the harmonic response of the primary system. In some cases, such as $\mu = 0.02$, in Figure 17, the negative effect of off-optimality dominates the inherent positive reduction due to increasing C, especially at large e values. This behavior can be justified physically, when C increases for small μ, the number of collisions with an inappropriate direction may increase, so that the reduction of response, due to an increase in C, is dominated by an accumulation of these undesirable collisions.

When a system is subjected to negative error in damping, the natural response of the system is magnified, so that d_{opt} is no longer appropriate and is less than the suitable d_{opt}. So, according to Figures 3 and 4, this deterioration is reasonable. It can be seen from Figure 16 that this deterioration gradually grows at large values of e, in off-resonance mode. Thus, it is again preferable for the designer to choose the material that has less coefficient of restitution, especially when the uncertainty in the frequency of the harmonic load is high, and the system should be controlled against the resonance excitation.

Figure 17 demonstrates that as far as the optimal design of a conventional impact damper subjected to a wide range of frequencies is concerned, the negative effect of an uncertainty of C under the resonance condition can be neglected.

Large values of μ cause the ID to become more robust, especially in cases of less amounts of e. This trend is similar under both resonance and off-resonance conditions. Figures 16 and 17 also prove that the sensitivity of a system to an increase in the displacement of the primary system, due to a reduction in C, decreases by increasing the value of μ. In this regard, it must be expressed that uncertainty in the value of C has less negative influence on the effectiveness of conventional ID than that due to uncertainty in the value of stiffness K (as will be shown in the following section).

4.2.2. Uncertainty in K

To investigate the vulnerability of an optimal conventional ID subjected to uncertainty in stiffness of the primary system, $\pm$10% error in stiffness is assumed.

Figure 18 illustrates the effect of error in stiffness on the performance of optimal points in off-resonance situations. It is noticeable that not only can a change in K lead to a variation in system responses, inherently, but can also result in deterioration in performance caused by the mistuning of optimized ID. As discussed, all parameters that are able to change the displacement response of the primary system can have a profound effect on the efficiency of ID tuned to an intact primary system. The most important issue that the designer

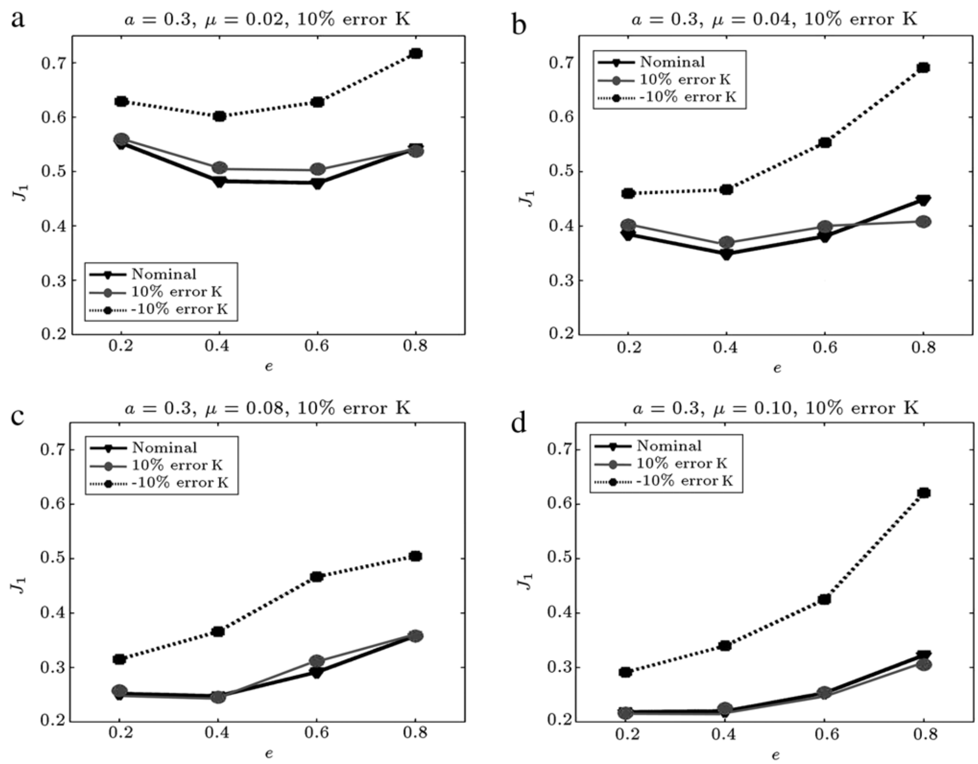

Figure 18: (a)–(d) Performance deterioration of optimized ID due to $\pm 10\%$ error in K of structure in off-resonance condition.

should pay more attention to is to utilize as large as possible values of μ and apply as low as possible values of e ($e \leq 0.4$).

Nevertheless, using less amounts of e is not optional, but there are some developed kinds of ID with less inherent e, such as granular, bean bag, and buffered ID, which allow us to avoid the negative consequences of high values of e. In addition, it can be stated that the improvement in effectiveness and robustness of traditional ID, associated with these kinds of ID, is related to the natural reduction in the value of e.

In Figure 19, it is obvious that in many cases, the effect of mistuning can dominate the inherent reduction of response caused by increasing values of K. Therefore, in the design of conventional ID, both factors able to increase or decrease the response of the primary system should be taken into account. Again, the physical justification of this behavior is that when displacement reduces, the number of collisions with inappropriate directions may increase, so that the reduction of response is dominated by an accumulation of these undesirable collisions. Figure 19 can prove this statement by showing more deterioration after increasing the value of K.

As a general expression, Figures 18(a)–(d) and 19(a)–(d) demonstrate the negative variation of 5%–20% of the maximum dynamic response of the primary system subjected to $\pm 10\%$ error in K, and this disability grows gradually by increasing the value of e and decreases by increasing μ.

5. Conclusions

In this paper, the performance of conventional ID, due to two major concerns: optimality (in resonance and off-resonance situations) and the reliability of optimized ID due to different kinds of uncertainty, is numerically studied. Firstly, the user oriented charts of the optimal design of an impact damper are gained according to a variety of parameters, such as μ, e and d, by a discrete numerical search in the selected design space.

Secondly, the deterioration of optimal effectiveness subjected to various kinds of uncertainty with different factors, such as impact parameters (μ, e and d), primary system parameters (K and C) and excitation (frequency and amplitude), are investigated, and the following conclusions are obtained:

1. According to initiative charts under both resonance and off-resonance conditions, the larger the μ, the more efficient and reliable is the ID.
2. By using the maximum frequency response function of an uncontrolled structure subjected to a particular amplitude of harmonic based acceleration for making the dimensionless response of the controlled primary system, the approximately unique space, considering C, K, and the amplitude of harmonic load, simultaneously, is established. Therefore, there is no need to investigate the effect of various amounts of amplitude vastly.
3. Not only is this paper helpful in explaining the reports of apparent discrepancies in how the coefficient of restitution affects the effectiveness of ID under a resonance condition, it also shows that by increasing the value of e, the performance of optimal ID is gradually worsened in off-resonance circumstances. This trend is completely in conflict with the behavior of optimal ID due to increasing the e value at the optimal point of resonance.
4. The d value, at which maximum efficiency occurs, decreases with increasing mass ratio and decreasing the coefficient of restitution under both resonance and off-resonance conditions.
5. This is a rare (if not first) investigation to consider the influence of most uncertainties on conventional IDs. Different kinds of uncertainty are numerically investigated, according to the optimal design of ID. This leads to a general observation that is invaluable to designers, helping them to roughly estimate the effect of various uncertainties by

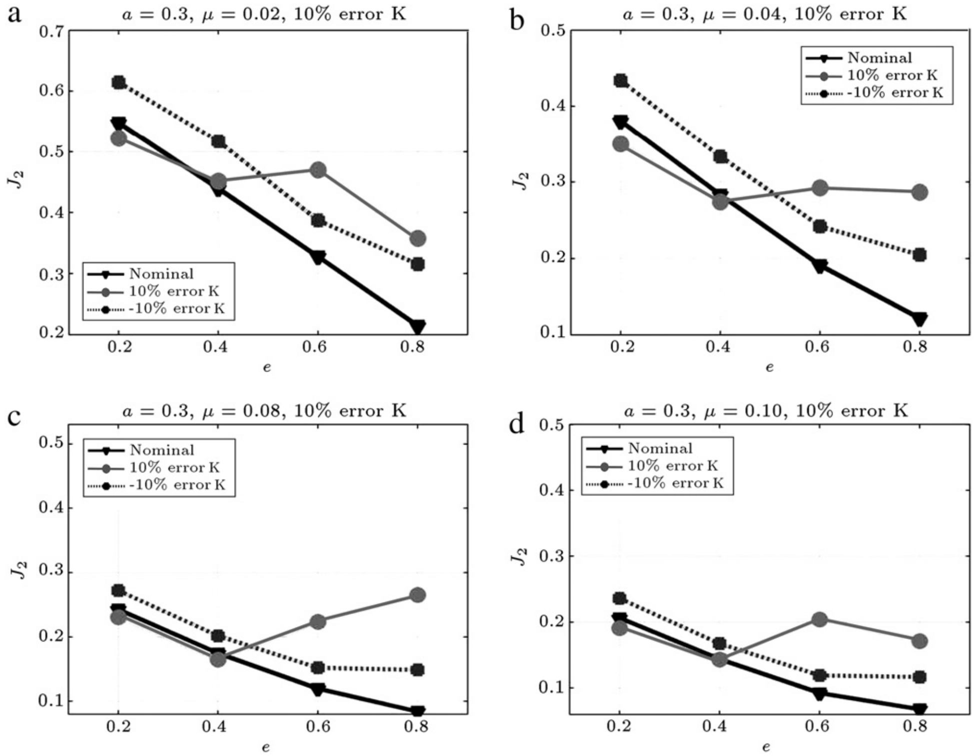

Figure 19: Performance deterioration of optimized ID due to $\pm 10\%$ error in K of structure in resonance condition.

regarding not only the resonance situation, but also a wide range of frequencies.

6. In this paper, two novel criteria are proposed to calculate the vulnerability of optimization to a precise consideration of the initiative parameters of excitation (amplitude and frequency). Accordingly, under both resonance and off-resonance conditions, the optimization of ID with lower values of e and large amounts of μ is insensitive to accurate assumption for excitation parameters (amplitude and frequency). On the contrary, high values of mass ratio may make the ID performance more sensitive to a degradation in optimal clearance (increasing the clearance).
7. In terms of uncertainties in dynamic parameters of the primary structure, the error in C can worsen the efficiency of the optimized ID, but not as much as the error in K. Besides, surprisingly, it is observed that by positive error in C and K, the response of the controlled system in some cases changes for the worse. The physical justification of this behavior is that when displacement reduces, the number of collisions with inappropriate directions may increase, so that the reduction in response is dominated by an accumulation of these undesirable collisions. Accordingly, it is advisable to check the reliability of conventional ID in all cases, which may cause any variation in the maximum uncontrolled frequency response function of the primary system, even if they lead to a reduction in response.

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