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Fractal Dimension Based Texture Analysis of Digital Images

P. Shanmugavadivu[a], V. Sivakumar[b, a*]

[a,b]Department of Computer Science and Applications
Gandhigram Rural Institute – Deemed University
Gandhigram-624 302, Dindigul, Tamil Nadu, India

Abstract

Fractal dimension is an important parameter of Fractal geometry that finds significant applications in various fields including image processing. Image analysis is a high-level image processing technique to identify the image features such as texture, roughness, smoothness, area and solidity. This paper proposes an algorithm to calculate the fractal dimension of digital images, does compare the fractal dimension of such images and proves that fractal dimension is an ideal tool for measuring the roughness/texture of an image.

Keywords: Fractal dimension, Fractal geometry, Box counting, Texture, Roughness.

1. Introduction

An *image* may be defined as a two-dimensional function, *f(x, y)*, where *x* and *y* are *spatial coordinates*, and the amplitude of *f* at any pair of coordinates *(x, y)* is called the *intensity* or *gray level* of the image at that point. When *x*, *y* and the amplitude values of *f* are all finite, discrete quantities, such an image is referred to as a *digital image*. *Texture* is a main characteristic of any image, which defines the special relationship between the gray-scale values of the pixels in a region of the image [1].

There are many techniques to measure the texture of an image that includes *Mean* to measure the average intensity of an image, *Standard Deviation* to measure the average contrast, *Smoothness* to measure the relative smoothness of intensities in a region, *Third Moment* to the measure the skewness of a histogram, *Uniformity* to measure the consistency of intensity values and *Entropy* to measure the randomness [1], [2]. The proposed method quantitatively measures the roughness of a given image, using the principle of fractal dimension.

* Corresponding author. Tel.: +91-9443736780

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E-mail address:psvadivu67@gmail.com.

In this paper, section 2 describes the basics of fractal dimension and its computational methodology, section 3 describes the proposed methodology. Results and discussions are given in section 4 and the conclusions are drawn in section 5.

2 Fractal Dimension

Geometric primitives that are self-similar and irregular in nature are termed as fractals. *Fractal Geometry* was introduced to the world of research in 1982 by Mandelbrot and has gained momentum over the years due to its broad spectrum of application domains [3]. Perusal of literature evidences the fact that fractal analysis, which is branch of aforementioned mathematical approach, is broadly applied to study the feature of an image/object [4]-[7]. However, in most of these applications, the common interest is to determine the Fractal Dimension of an object under investigation.

Review of literature reveals that there are a number of techniques for the estimation of fractal dimension. Pentland (1984) developed a three-dimensional fractal model for image segmentation and estimated the fractal dimension using the Fourier power spectral density where the surfaces were modeled as fractional Brownian Motion (fBM) surfaces. Hartley *et al*., (1984) used the ε-blanket method suggested by Mandelbrot (1982) to estimate the fractal dimension and used it for texture analysis. Dubuc *et al.,* (1989) used the variation method for fractal dimension estimation [8].

2.1 Calculation of Fractal Dimension

The fractal dimension is an important characteristic of fractals because it has got information about their geometric structure. The topological dimension (defined as *d*) of an object would not change whatever be the transformation an object undergoes. In the fractal world, the fractal dimension need not be an integer number. The Fractal dimension (defined as D) of an object is normally greater than its topological dimension (i.e. $D \geq d$).

In a bounded set X considered in Euclidean n-space, the set X is said to be self-similar when X is the union of N_r distinct non-overlapping copies of itself, each of which is similar to X scaled down by a ratio r. Fractal Dimension D of X can be derived from the relation [3], as

$$D = \frac{\log (N_r)}{\log (\frac{1}{r})} \qquad (1)$$

3 Proposed Work

To ascertain the suitability of the proposed method, a few standard images lena, bird, rice, mandrill, peppers, and saturn were chosen. In addition, to validate the obtained results, a few noise-prone images of these standard images were also taken as input images. Fixed-value impulse noise was used to corrupt the images and its density was varied between 10% and 90%. The algorithmic description of the proposed method to calculate the Fractal dimension for those images using the Box Counting method is explained below.

Algorithm : Computation of Fractal Dimension using Box Counting.

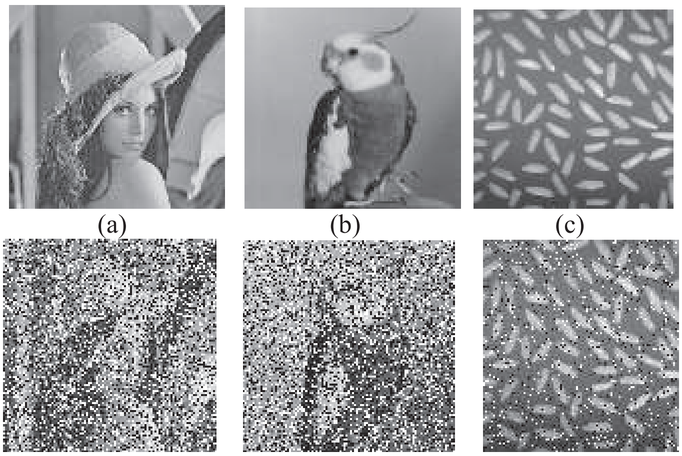
Aim: To Calculate Fractal dimension
Input: A 2-Dimensional image, I
Output: Fractal Dimension, D of I

1. Read a 2-Dimensional input image I
2. [M, N] ← SIZE [I]
3. If M > N then $r \leftarrow$ M
   Else $r \leftarrow$ N
4. Compute fractal dimension using Equation (1).
5. Stop.

In this paper, a comparative study is done on the fractal dimension of those images. Similarly, the fractal dimension is calculated for those images with various levels of salt and pepper noise images using Matlab 7.8.

4 Results and Discussion

Using box counting algorithm, the fractal dimension was calculated for all the input test images among which, lena, bird and rice were chosen for illustrative purpose. The obtained fractal dimensions are enlisted in *Table 1*.



(a) (b) (c) (d) (e) (f)

Figure 1. Standard images: (a) lena (b) bird (c) rice
40% Corrupted images: (d) lena (e) bird (f) rice

Table 1. Fractal dimension obtained by Box Counting Algorithm

Input Image	Fractal Dimension
Lena	2.8687

Bird	2.8713
Rice	2.8455

The fractal dimensions obtained for the corrupted images of lena, bird and rice are given in *Table 2*.

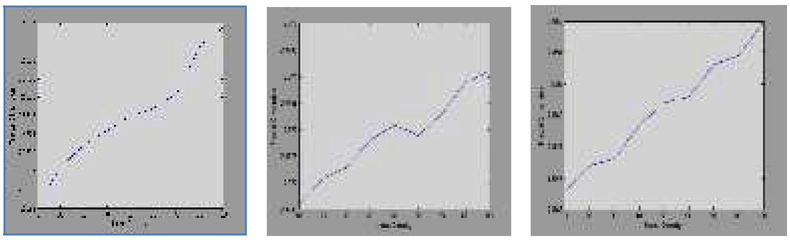
Table 2. Fractal Dimension of corrupted images lena, bird and rice with various noise levels.

Noise Density	Fractal Dimension		
	lena	**bird**	**rice**
10%	2.8691	2.8716	2.8466
20%	2.8701	2.8721	2.8474
30%	2.8707	2.8723	2.8476
40%	2.8711	2.8728	2.8487
50%	2.8715	2.8731	2.8494
60%	2.8717	2.8729	2.8496
70%	2.8721	2.8733	2.8506
80%	2.8734	2.8739	2.8509
90%	2.8739	2.8741	2.8520

The tabulated values of *Table 2* clearly substantiate the fact that, the fractal dimension of any given image will progressively increase with its roughness. The salt and pepper noise especially, fixed-value impulse noise which is characterized by the minimum and the maximum intensity values of the dynamic intensity intervals of the input image tends to increase the roughness of a given image proportionate to the noise density.

It is shown from *Table 1* and *2* that for standard lena image, the fractal dimension was 2.8687 and for the noise density between 10% and 90%, it was found to increase from 2.8691 to 2.8739. Similarly, for standard bird image, the fractal dimension was 2.8713 and for the noise density between 10% and 90%, the same increased from 2.8716 to 2.8741; for standard rice image, the fractal dimension was 2.8455 and for the noise density between 10% and 90%, it increased from 2.8466 to 2.8520.

The estimated values of noise density versus fractal dimension for the images lena, bird and rice are exhibited in *figure 2*.



(a) (b) (c)

Figure 2. Noise density versus Fractal dimension (a) for the image lena (b) for the image bird (c) for the image rice

The *figure 2* clearly indicates that, as the density of noise probability increases, the fractal dimension also increased respectively, indicating the increase in the roughness of the image. To ascertain the authenticity of the proposed work, further it was tested on the following medical mammogram images.

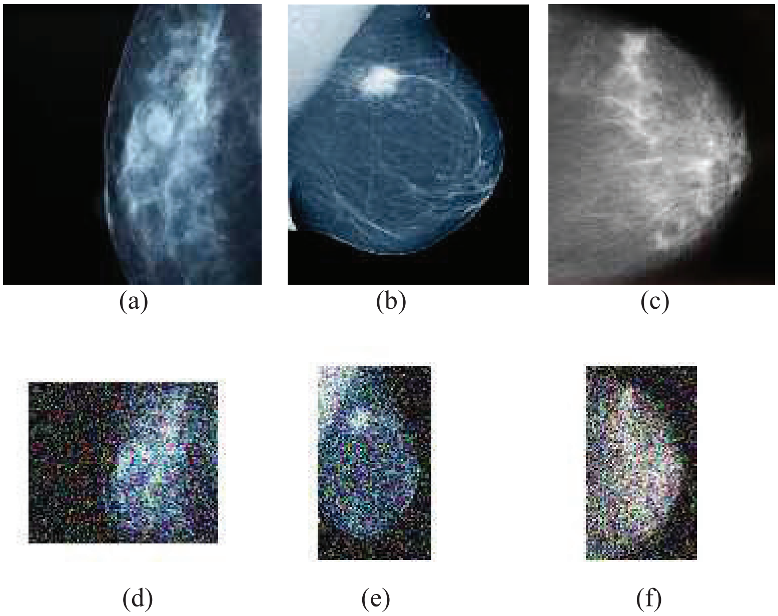


Figure 3.(a), (b), (c) mammogram images ; (d), (e), (f) corrupted mammogram images

The tabulated values of fractal dimension for the original mammogram images and their corrupted images shown in *figure 3* are depicted in the following *table 3*.

Table 3. Fractal Dimension of mammogram images shown in *figure 3*.

Fractal Dimension for Mammogram images	
Uncorrupted	**Corrupted**
(a) - 2.3763	(d) - 2.4129
(b) - 2.5314	(e) - 2.5638
(c) - 2.5961	(f) - 2.6133

From the table 3, it is clearly understood that the results evidenced by the standard images were replicated by those medical images too.

5 Conclusion

From the results and graphs, it is very clear that the fractal dimension of those images increase in line with their noise levels. This shows that the fractal dimension of an image has a correlation with its roughness. Hence, it is concluded that fractal dimension serves as a vital component to measure the roughness of an image. Moreover, this technique can also be used as an alternate approach to noise detection, if the prior knowledge about the uncorrupted image is available.

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