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Solar-neutrino reactions on deuteron in effective field theory

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Abstract

The cross sections for low-energy neutrino–deuteron reactions are calculated within heavy-baryon chiral perturbation theory employing a cut-off regularization scheme. The transition operators are derived up to next-to-next-to-next-to-leading order in the Weinberg counting rules, while the nuclear matrix elements are evaluated using the wave functions generated by a high-quality phenomenological NN potential. With the adoption of the axial-current-four-nucleon coupling constant fixed from the tritium beta decay data, our calculation is free from unknown low-energy constants. Our results exhibit a high degree of stability against different choices of the cutoff parameter, a feature which indicates that, apart from radiative corrections, the uncertainties in the calculated cross sections are less than 1%.

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1. Introduction

This Letter is concerned with a theoretical estimation of the cross sections, $\sigma_{\nu d}$, for the neutrino–deuteron reactions

$$\nu_e + d \to e^- + p + p,$$
$$\bar{\nu}_e + d \to e^+ + n + n \quad \text{(CC)}, \tag{1}$$

$$\nu_l + d \to \nu_l + p + n,$$
$$\bar{\nu}_l + d \to \bar{\nu}_l + p + n \quad \text{(NC)}, \tag{2}$$

where CC and NC stand for the charged-current and neutral-current reaction, respectively, and l denotes the lepton flavor ($l = e, \mu, \tau$). Recent SNO experiments [1,2] have provided strong evidence for ν_e oscillations. In interpreting the existing and future SNO data, accurate estimates of $\sigma_{\nu d}$ in the solar neutrino energy region ($E_\nu \leqslant 20$ MeV) are of great importance.

Recently, two theoretical approaches have been used for evaluating $\sigma_{\nu d}$. One is a traditional method in which nuclear electroweak processes are described in terms of one-body impulse approximation (IA) operators and two-body exchange-current (EXC) operators acting on non-relativistic nuclear wave functions. The

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EXC contributions are derived from one-boson exchange diagrams [3], while the nuclear wave functions are obtained by solving the Schrödinger equation involving high-quality realistic nuclear interactions. For convenience, we refer to this method as the standard nuclear physics approach (SNPA). The successful applications of SNPA are well documented in the literature [4]. A detailed calculation of $\sigma_{\nu d}$ based on SNPA was carried out by Nakamura, Sato, Gudkov and Kubodera (NSGK) [5], and this calculation has recently been updated by Nakamura et al. (NETAL) [6].[2]

The second approach is based on effective field theory (EFT), which has been gaining ground as a new tool for describing low-energy phenomena in few-nucleon systems [9–11]. Butler, Chen, and Kong (BCK) [12] applied EFT to the νd reactions, using the regularization scheme called the power divergence subtraction (PDS) [13]. Their results agree with those of NSGK in the following context. The EFT Lagrangian in PDS involves one unknown low-energy constant (LEC), denoted by L_{1A}, which represents the strength of axial-current-four-nucleon contact coupling. BCK adjusted L_{1A} to optimize fit to the $\sigma_{\nu d}$ of NSGK and found that, after this adjustment, the results of the EFT and SNPA calculations agree with each other within 1% over the entire solar-ν energy region for all of the four reactions in Eqs. (1) and (2). Furthermore, the best-fit value of L_{1A} was found to be of a reasonable magnitude consistent with the "*naturalness*" argument [12].

The fact that the results of an ab initio EFT calculation (with one free parameter fine-tuned) are consistent with those of SNPA is considered to give strong support for the basic soundness of SNPA. At the same time, it highlights the desirability of an EFT calculation of $\sigma_{\nu d}$ free from an adjustable parameter. In this Letter we describe an attempt toward such a goal. We employ here a formalism recently developed in the studies of the solar *hep* process and the solar *pp* fusion reaction [14,15]. In this method, invoking heavy-baryon chiral perturbation theory (HBχPT), we construct transition operators from irreducible diagrams according to Weinberg's counting scheme [9]; the nuclear matrix elements are evaluated by sandwiching the EFT-controlled transition operators between the nuclear wave functions that have been obtained by solving the Schrödinger equation involving high-quality realistic nuclear interactions. For convenience, we refer to this EFT-motivated approach as EFT*. It is known [14] that, for the present purposes, it is sufficient to consider up to next-to-next-to-next-to-leading order (N^{3}LO) in HBχPT, and that to this order there is only one unknown LEC, denoted by $\hat{d}^R$ in [15]. Like L_{1A} in [12], $\hat{d}^R$ controls the strength of the axial-current-four-nucleon contact coupling and subsumes short-distance physics that has been integrated out. An important point noticed in [15] is that, since the tritium β-decay rate Γ_t^β is also sensitive to $\hat{d}^R$, we can determine $\hat{d}^R$ from the well-known experimental value of Γ_t^β. Once $\hat{d}^R$ is determined, we can make a parameter-free calculation of $\sigma_{\nu d}$, and the purpose of this communication is to describe such a calculation.[3] We shall show that, apart from radiative corrections for which we refer to the literature [17–19], $\sigma_{\nu d}$ given here is reliable with $\sim 1\%$ precision.

2. Calculational method

For low-energy processes, we can work with the current–current interaction:

$$H = \frac{G'_F}{\sqrt{2}} \int d^3\vec{x} \left[V_{ud} J_\mu^{(\mathrm{CC})}(\vec{x}) l^{(\mathrm{CC})\mu}(\vec{x}) + J_\mu^{(\mathrm{NC})}(\vec{x}) l^{(\mathrm{NC})\mu}(\vec{x}) \right], \tag{3}$$

where $G'_F = 1.1803 \times 10^{-5}$ (GeV^{-2}) [20] is the weak coupling constant, and $V_{ud} = 0.9746$ is the K–M matrix element. G'_F includes the inner radiative correction: $G'^2_F = G^2_F(1 + \Delta^V_R)$, where $G_F = 1.1166 \times 10^{-5}$ (GeV^{-2}) is the Fermi constant and Δ^V_R is the inner radiative correction [20].[4] The CC- and NC-lepton currents, $l^{(\mathrm{CC})\mu}$ and $l^{(\mathrm{NC})\mu}$, are well known; the CC- and NC-hadronic currents, $J_\mu^{(\mathrm{CC})}$ and $J_\mu^{(\mathrm{NC})}$ are written as

$$J_\mu^{(\mathrm{CC})}(\vec{x}) = V_\mu^{\pm}(\vec{x}) - A_\mu^{\pm}(\vec{x}), \tag{4}$$

[2] For earlier calculations, see, e.g., [7,8].

[3] Similar parameter-free calculations have been carried out for the solar *pp*-fusion reaction and the solar *hep* process [15], and for μ–*d* capture [16].

[4] For more detailed discussion of the radiative correction, see [6].

$$J_\mu^{(\mathrm{NC})}(\vec{x}) = \left(1 - 2\sin^2\theta_W\right) V_\mu^0(\vec{x}) - A_\mu^0(\vec{x}) - 2\sin^2\theta_W V_\mu^S(\vec{x}), \tag{5}$$

where V_μ and A_μ represent the vector and axial current, respectively. The superscripts, $\pm$ and 0, are the isospin indices of the isovector current and S denotes the isoscalar current; θ_W is the Weinberg angle, $\sin^2\theta_W = 0.2312$.

The νd reactions can lead to various values of the relative orbital angular momentum, L, of the final two nucleons. We concentrate here, however, on the $L = 0$ state ($^1S_0$), since it is this partial wave that involves the $\hat{d}^R$ term and since the contributions of higher partial waves are well understood in terms of the one-body operators. The contributions from $L \geqslant 1$ are significant in the upper part of the solar neutrino energy region,[5] but their uncertainty is small enough to be ignored in the present context.

The one-body (1B) currents can be obtained from the phenomenological form factors of the weak-nucleon current.[6] The isovector vector and axial-vector currents are given in momentum space as[7]

$$J_V^{a\mu}(q) = \bar{u}(p')\frac{\tau^a}{2}\left[g_V(q)\gamma^\mu + g_M(q)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]u(p), \tag{6}$$

$$J_A^{a\mu}(q) = \bar{u}(p')\frac{\tau^a}{2}\left[g_A(q)\gamma^\mu\gamma_5 + g_P(q)\frac{q^\mu}{m_\mu}\gamma_5\right]u(p), \tag{7}$$

where 'a' is the isospin index, $u(p)$ is the Dirac spinor for the nucleon, and m_μ (m_N) is the muon (nucleon) mass; $g_V(q)$, $g_M(q)$, $g_A(q)$, and $g_P(q)$ are the vector, magnetic, axial-vector, pseudoscalar form factors, respectively. It is known empirically that the first three form factors can be parametrized very well in the dipole form with the use of effective radii, $r_V^2 = 0.59$, $r_M^2 = 0.80$ and $r_A^2 = 0.42$ fm^2, for $g_V(q)$, $g_M(q)$ and $g_A(q)$, respectively [27]. We are adopting here the usual normalization: $g_V(0) = 1$, $g_A(0) = g_A = 1.267$, and $g_M(0) = \kappa_V = 3.706$. Although $g_P(q)$ is not well known empirically, it is strongly constrained by chiral symmetry; an HBχPT calculation up to NNLO [22] leads to

$$g_P(q) = -\frac{2m_\mu f_\pi g_{\pi N}}{q^2 - m_\pi^2} - \frac{1}{3}g_A m_\mu m_N r_A^2, \tag{8}$$

where $g_{\pi N} = 13.5$. In fact, the contribution of the g_P term is tiny in our case. We apply a non-relativistic expansion of the above expressions and retain terms up to $\mathcal{O}(1/m_N^3)$ (corresponding to N^{3}LO of the chiral order); the details will be described elsewhere [28].

The two-body (2B) current operators are derived from the chiral Lagrangian $\mathcal{L}$, which is expanded as $\mathcal{L} = \sum_{\bar{\nu}} \mathcal{L}_{\bar{\nu}} = \mathcal{L}_0 + \mathcal{L}_1 + \cdots$, where $\mathcal{L}_0$ and $\mathcal{L}_1$ are LO and NLO Lagrangians, respectively. Their explicit expressions are:

$$\mathcal{L}_0 = \overline{N}[iv\cdot D + 2ig_A S\cdot\Delta]N + f_\pi^2\,\mathrm{Tr}\left(-\Delta\cdot\Delta + \frac{\chi_+}{4}\right), \tag{9}$$

$$\begin{aligned}\mathcal{L}_1 = {}& \frac{1}{2m_N}\overline{N}\Big[(v\cdot D)^2 - D^2 + 2g_A\{v\cdot\Delta, S\cdot D\} \\ & - \left(8\hat{c}_2 - g_A^2\right)(v\cdot\Delta)^2 - 8\hat{c}_3\Delta\cdot\Delta \\ & - (4\hat{c}_4 + 1)\left[S^\mu, S^\nu\right][\Delta_\mu, \Delta_\nu] \\ & - 2i(1+\kappa_V)\left[S^\mu, S^\nu\right]f_{\mu\nu}^+\Big]N \\ & + \frac{g_A}{m_N f_\pi^2}\Big[-4i\hat{d}_1\overline{N}S\cdot\Delta N\overline{N}N \\ & + 2i\hat{d}_2\epsilon^{abc}\epsilon_{\mu\nu\alpha\beta}v^\mu \\ & \times \Delta^{a,\nu}\overline{N}S^\alpha\tau^b N\overline{N}S^\beta\tau^c N\Big],\end{aligned} \tag{10}$$

where v^μ is the velocity vector $v^\mu = (1, \vec{0})$ and S^μ is the spin operator $2S^\mu = (0, \vec{\sigma})$. The explicit expressions of the fields, D_μ, Δ_μ, $f_{\mu\nu}^+$, and χ_+, are given in [16], and f_π is the pion decay constant. The LEC's, $\hat{c}_i$, have been determined by Bernard et al. at the tree-level [29];[8]

$$\hat{c}_2 = 1.67 \pm 0.09, \qquad \hat{c}_3 = -3.66 \pm 0.08, \qquad \hat{c}_4 = 2.11 \pm 0.08. \tag{11}$$

[5] The $L \geqslant 1$ contributions increase $\sigma_{\nu d}$ by $\sim 3.8\%$ at $E_\nu = 20$ MeV [6,21].

[6] The low-energy structure of the form factors has been studied in detail within HBχPT [22]. At N^{2}LO, however, we in principle need to consider off-shell form factors [23], a feature that reflects arbitrariness in choosing fields [24]. The influence of off-shell terms, however, should be small at low energies; see, e.g., Ref. [25]. An EFT study of the one-body Gamow–Teller matrix element in the two-nucleon system [26] explicitly shows that the off-shell effects are sufficiently small for our present purposes.

[7] Since we consider the final $^1S_0$ state only, there is no contribution from the isoscalar current.

[8] The relation between our dimensionless LEC's, $\hat{c}_i$'s, and c's used in the literature is $\hat{c}_i = m_N c_i$.

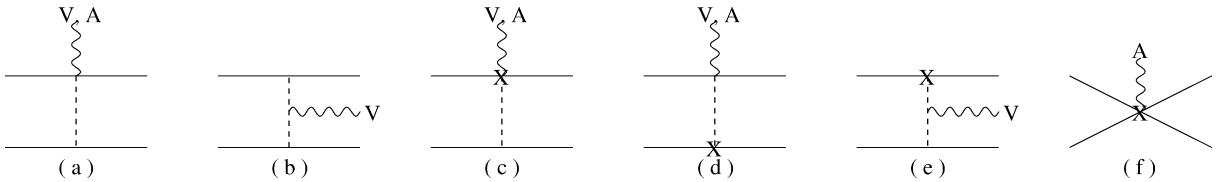

Fig. 1. Diagrams for two-body current operators of order $\nu = 1$ (a), (b) and $\nu = 2$ (c)–(f). The wavy lines with V and A attached denote the vector and axial-vector current, respectively, the dashed line denotes the pion, and vertices without (with) "X" arise from the LO (NLO) Lagrangian.

The LEC's of the contact terms $\hat{d}_{1,2}$ will be discussed later in the text.

We construct 2B transition operators from 2B irreducible Feynman diagrams up to N^{3}LO in Weinberg's counting rule [9]. Since the tree-level 2B operators are higher in chiral counting than the tree-level 1B operators by two orders, we can limit ourselves to tree diagrams for the 2B operators. In addition, since the g_P term is highly suppressed, we do not consider it in the 2B operators.

The diagrams for the 2B operators are given in Fig. 1. Since we only have nucleons and pions in $\mathcal{L}$, the effects involving exchange of heavier mesons such as the σ and ρ mesons are embedded in the contact term, diagram (f) in Fig. 1. We denote by Λ a momentum scale below which our nucleon–pion-only description is expected to be valid. To prevent the exchanged momentum from surpassing Λ, we introduce the cutoff function $S_\Lambda(\vec{k}) = e^{-\vec{k}^2/(2\Lambda^2)}$ in calculating the Fourier transforms of the 2B transition operators [15]. As noted in [15], the short-range part of the 2B contributions can be lumped together into an axial-current-four-nucleon contact coupling term with the strength $\hat{d}^R$, where $\hat{d}^R = \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}$. Then, for a given value of Λ, we can determine $\hat{d}^R$ from the empirical value of Γ_t^β. The results are [15]:

$$\hat{d}^R = 1.00 \pm 0.07,\ 1.78 \pm 0.08,\ 3.90 \pm 0.10, \qquad (12)$$

for $\Lambda = 500, 600, 800$ MeV, respectively. The explicit expressions of the current operators for the CC reaction have been given in [16].[9]

[9] Insofar as the final two-nucleon partial wave is limited to s-wave, one can use the same expression for the NC reaction (with an appropriate change in the coefficient of the vector current).

3. The total cross section

The total cross sections are calculated using the non-relativistic formula

$$\begin{aligned}\sigma_{\nu d}(E_\nu) &= \int dp \int dy\, \frac{1}{(2\pi)^3} \\ &\quad \times \frac{2p^2 k'^2}{k'/E' + (k' - E_\nu y)/(2m_N)} \\ &\quad \times F(Z, E') \frac{1}{3} \sum_{\text{spin}} |T|^2, \end{aligned} \qquad (13)$$

with the energy conservation relation valid up to $1/m_N$,

$$\begin{aligned} &m_d + E_\nu - E' - 2m_N \\ &\quad - \frac{1}{m_N}\left[p^2 + \frac{1}{4}\left(E_\nu^2 + k'^2 - 2E_\nu k' y\right)\right] = 0, \end{aligned} \qquad (14)$$

where E_ν (E') is the energy of the initial neutrino (final lepton), p is the magnitude of the relative three-momentum between the final two nucleons, k' is that of the outgoing lepton ($k' = |\vec{k}'|$), and y is the cosine of the angle between the incoming and outgoing leptons ($y = \hat{k}_\nu \cdot \hat{k}'$). $F(Z, E')$ is the Fermi function and m_d is the deuteron mass. The transition matrix T is decomposed as $T = T_{1\text{B}} + T_{2\text{B}}$, where $T_{1\text{B}}$ and $T_{2\text{B}}$ are the contribution of the 1B and 2B operators, respectively. These will be evaluated with the use of the Argonne V18 potential [30].

Since the calculation of $T_{1\text{B}}$ is standard [28], we give here only the explicit expression for $T_{2\text{B}}$:

$$
\begin{aligned}
&\frac{1}{\sqrt{4\pi}}T_{\rm 2B}\\
&\quad=\beta\chi_{00}^{\dagger}\vec{\Sigma}\chi_{1m_d}\int dr\\
&\qquad\times\Bigg\{\vec{F}_6u_0(r)j_1(qr/2)\frac{y_{1\Lambda}(r)}{r}u_d(r)\\
&\qquad\quad+\vec{F}_7\big[u_0'(r)\big(u_d(r)-\sqrt{2}\,w_d(r)\big)\\
&\qquad\qquad-u_0(r)\big(u_d'(r)-\sqrt{2}\,w_d'(r)\big)\big]\\
&\qquad\quad\times j_0(qr/2)\frac{y_{1\Lambda}(r)}{r}\\
&\qquad\quad+\vec{F}_8u_0(r)j_0(qr/2)\frac{y_{1\Lambda}(r)}{r^2}w_d(r)\\
&\qquad\quad+\vec{F}_9u_0(r)j_0(qr/2)y_{0\Lambda}(r)u_d(r)\\
&\qquad\quad+\vec{F}_{10}u_0(r)j_0(qr/2)y_{2\Lambda}(r)w_d(r)\\
&\qquad\quad+\vec{F}_{11}u_0(r)\delta_{\Lambda}(r)u_d(r)\\
&\qquad\quad+\vec{F}_{12}\int\limits_{-1/2}^{1/2}dy\,u_0(r)j_0(yqr)\\
&\qquad\quad\times\Bigg[y_{0\Lambda}^{L}(r)u_d(r)\\
&\qquad\qquad-\frac{2}{3}y_{1\Lambda}^{L}(r)\bigg(u_d(r)+\frac{w_d(r)}{\sqrt{2}}\bigg)\Bigg]\Bigg\},
\end{aligned}
\tag{15}
$$

where $\beta=(G_F'V_{ud})^2/2$ for CC and $\beta=G_F'^2/4$ for NC. $\vec{\Sigma}=\vec{\sigma}_1-\vec{\sigma}_2$, with $\vec{\sigma}_i$ being the ith nucleon spin operator; χ_{1m_d} and $\chi_{0,0}$ are the spin wave functions for the deuteron and the final two nucleons, respectively. The radial function u_0 corresponds to the final two-nucleon s-wave, while u_d and w_d are the s-wave and d-wave radial functions of the deuteron; $j_L(qr/2)$ is the spherical Bessel function; q^{μ} is a momentum transfer between the currents, $q^{\mu}=k'^{\mu}-k^{\mu}$ and $q=|\vec{q}|$. Furthermore,

$$
\begin{aligned}
\vec{F}_6&=-\frac{g_A}{f_\pi^2}v\cdot\mathcal{J}\hat{q}-\frac{g_A(1+\kappa_V)}{2m_Nf_\pi^2}i\vec{q}\times(\hat{q}\times\vec{\mathcal{J}})\\
&\quad-\frac{g_A}{4m_Nf_\pi^2}\vec{q}\cdot\vec{\mathcal{J}}\hat{q},\\
\vec{F}_7&=\frac{-g_A}{6m_Nf_\pi^2}\vec{\mathcal{J}},\qquad \vec{F}_8=\frac{-g_A}{\sqrt{2}m_Nf_\pi^2}\vec{\mathcal{J}},\\
\vec{F}_9&=-\frac{2g_Am_\pi^2\big(\hat{c}_3+2\hat{c}_4+\frac{1}{2}\big)}{3m_Nf_\pi^2}\vec{\mathcal{J}},\\
\vec{F}_{10}&=\frac{2\sqrt{2}\,g_Am_\pi^2\big(\hat{c}_3-\hat{c}_4-\frac{1}{4}\big)}{3m_Nf_\pi^2}\vec{\mathcal{J}},\\
\vec{F}_{11}&=\frac{2g_A\hat{d}^R}{m_Nf_\pi^2}\vec{\mathcal{J}},\\
\vec{F}_{12}&=\frac{-1}{2}\bigg(\frac{g_A}{f_\pi}\bigg)^2i(\vec{q}\times\vec{\mathcal{J}}),
\end{aligned}
\tag{16}
$$

where $\mathcal{J}^{\mu}$ is the lepton current in momentum space, and

$$
\begin{aligned}
\delta_{\Lambda}(r)&=\int\frac{d^3\vec{k}}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}S_{\Lambda}^2(\vec{k}),\\
y_{0\Lambda}(r)&=\int\frac{d^3\vec{k}}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}\frac{S_{\Lambda}^2(\vec{k})}{\vec{k}^2+m_\pi^2},
\end{aligned}
\tag{17}
$$

and $y_{1\Lambda}=-r\frac{d}{dr}y_{0\Lambda}(r)$, $y_{2\Lambda}=\frac{r}{m_\pi^2}\frac{d}{dr}\big[\frac{1}{r}\frac{d}{dr}y_{0\Lambda}(r)\big]$; $y_{0,1\Lambda}^{L}(r)$ is obtained by exchanging the pion mass m_π to $L=\sqrt{m_\pi^2+(1/4-y^2)\vec{q}^2}$ in Eq. (17). In the above expression we have neglected the small terms proportional to q^2.

4. Numerical results and discussion

As mentioned, we consider in this Letter the contribution from the final two-nucleon s-wave only. The corresponding total cross section is denoted by $\sigma_{\nu d}^{L=0}$. Table 1 gives $\sigma_{\nu d}^{L=0}$ calculated in EFT* for the four reactions in Eqs. (1) and (2).

The results in Table 1 correspond to the case with $\Lambda=600$ MeV, and we now discuss the cutoff dependence. In Figs. 2 and 3 we plot the ratio, $\xi\equiv\sigma_{\rm 1B+2B}/\sigma_{\rm 1B}$, where $\sigma_{\rm 1B+2B}$ represents $\sigma_{\nu d}^{L=0}$ obtained with both the 1B and 2B currents included while $\sigma_{\rm 1B}$ represents $\sigma_{\nu d}^{L=0}$ obtained with the 1B current alone. Fig. 2 gives ξ for CC ($\nu d\to epp$), while Fig. 3 shows ξ for NC ($\nu d\to\nu np$). The three lines in each figure correspond to different choices of Λ. As can be seen from the figures, $\sigma_{\nu d}^{L=0}$ exhibits extremely small Λ dependence, with only 0.02% changes over a wide range of physically reasonable values of Λ ($\Lambda=500$–800 MeV).

We now briefly discuss estimation of higher chiral-order effects. The expansion parameter here is Q/Λ, where Q is the pion mass m_π or the typical external momentum scale $Q_{\rm ext}$, and Λ is the chiral scale cutoff, $\Lambda\approx 600$ MeV. It is common to assume $Q_{\rm ext}\sim m_\pi$,

Table 1
The total cross section $\sigma_{\nu d}^{L=0}$ (in units of 10^{-42} cm^2) for the νd reaction leading to the final two-nucleon s-state. For each of the four reactions in Eqs. (1) and (2), $\sigma_{\nu d}^{L=0}$ calculated in EFT* is shown as a function of the incident neutrino energy E_ν (MeV). For the cutoff parameter, $\Lambda = 600$ MeV has been used

E_ν	$\nu d \to e^- pp$	$\bar{\nu} d \to e^+ nn$	$\nu d \to \nu np$	$\bar{\nu} d \to \bar{\nu} np$
2	0.004	0	0	0
3	0.047	0	0.003	0.003
4	0.158	0	0.031	0.031
5	0.348	0.029	0.096	0.094
6	0.625	0.120	0.204	0.198
7	0.996	0.284	0.357	0.346
8	1.463	0.525	0.558	0.538
9	2.030	0.846	0.808	0.774
10	2.697	1.247	1.106	1.054
11	3.468	1.727	1.455	1.378
12	4.342	2.286	1.853	1.746
13	5.321	2.922	2.302	2.157
14	6.405	3.633	2.800	2.610
15	7.596	4.418	3.349	3.104
16	8.892	5.274	3.947	3.638
17	10.29	6.200	4.594	4.212
18	11.80	7.194	5.291	4.824
19	13.41	8.252	6.036	5.474
20	15.13	9.374	6.830	6.161

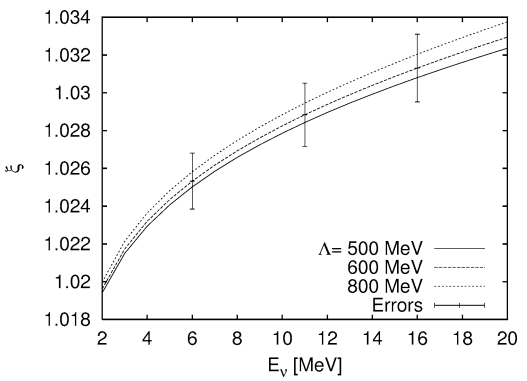

Fig. 2. The ratio ξ for CC defined in the text. The results for three different choices of Λ are plotted. The vertical bars represent changes in ξ as $\hat{d}^R$ is varied within a range allowed by the existing experimental errors in Γ_t^β; the representative results obtained for $\Lambda = 600$ MeV are shown for three values of E_ν.

but $Q_{\rm ext}$ in our case is the incident neutrino energy E_ν, whose maximum value is $E_\nu^{\max} \sim 20$ MeV; thus $E^{\max}/\Lambda \sim 0.03 \ll 0.23 \simeq m_\pi/\Lambda$. The actual numerical behavior of the chiral expansion in the present case may be typified by the results for the CC reaction ($\nu d \to e^- pp$) at $E_\nu = 20$ MeV. As far as the

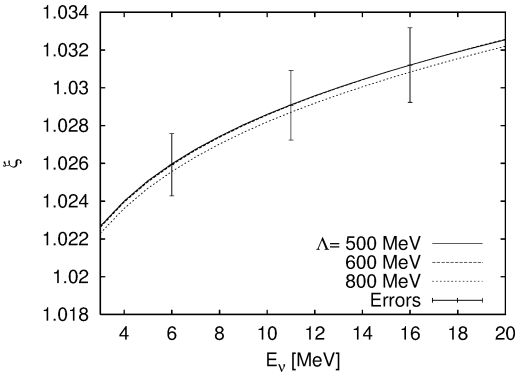

Fig. 3. The ratio ξ for NC defined in the text. See also the caption for Fig. 2.

1B operators are concerned, the contribution to $\sigma^{L=0}$ of the LO terms amounts to 88.5%, while the corrections due to the NLO, N^{2}LO and N^{3}LO terms are 8.8%, -0.5% and $\sim 0.001\%$, respectively. As for the 2B operators, the N^{2}LO terms give a $\sim 0.3\%$ correction, whereas the N^{3}LO terms give a $\sim 2.9\%$ correction. Thus, the overall behavior is consistent with convergence with respect to the expansion parameter, m_π/Λ; the rather conspicuous 2.9% correction of the N^{3}LO 2B terms is comparable to $(m_\pi/\Lambda)^3 \simeq 1.2\%$, while the other terms are decreasing faster (almost in powers of $E_\nu^{\max}/\Lambda$.). Therefore a possible measure of corrections due to N^{4}LO or higher-order terms is $2.9\% \times (m_\pi/\Lambda) \sim 0.6\%$.

The convergence property, however, can in fact be better than this. Since in our approach the overall strength, $\hat{d}^R$, of the 2B operator is adjusted to reproduce Γ_t^β, the bulk of higher order corrections have already been effectively taken into account. In particular, the chiral-symmetry breaking terms (proportional to m_π) give energy-independent contributions, which are essentially incorporated into the effective $\hat{d}^R$. The derivative terms acting on the wavefunctions or the two-body operators may pick up the pion mass scale, but their effects at the tritium β-decay energy are again essentially subsumed in $\hat{d}^R$. The remaining pieces of higher-order contributions are E_ν-dependent effects, and hence they are likely to be controlled by the parameter E_ν/Λ rather than m_π/Λ. From this viewpoint it seems reasonable to adopt $2.9\% \times (E^{\max}/\Lambda) \sim 0.1\%$ as a measure of the higher-order corrections. Another measure of convergence is obtained as follows. A tenet of a cutoff EFT (such as used here) demands that, pro-

vided a large enough number of terms are included in chiral expansion, the calculational results should be independent of choices of the cutoff parameter Λ (within a reasonable range). Thus, the sensitivity of the calculated $\sigma_{vd}^{L=0}$ to Λ serves as an indicator of the importance of the contributions of the neglected higher order terms. This sensitivity, however, has been found to be extremely small (0.02% variation) in our case.

Although the above discussion suggests that higher-order effects (N^4LO or higher) are reassuringly small, we make a brief comment on three-body (3B) operators, which represent a particular class of higher-order contributions. It is known (see Table 1 of the last article in Ref. [15]) that, at N^4LO, there is a contribution to the GT transition from the 3B-operator, which we denote here by $\mathcal{O}_{\rm GT}(3B)$. Obviously, although $\mathcal{O}_{\rm GT}(3B)$ contributes to Γ_t^β, it plays no role in the two-nucleon systems. At N^4LO, therefore, in renormalizing $\hat{d}^R$ with the use of Γ_t^β, one would need to subtract the contribution of $\mathcal{O}_{\rm GT}(3B)$. Formally speaking, our present treatment is free from this complication, since both the determination of $\hat{d}^R$ and the calculation of σ_{vd} are carried out within N^3LO. However, to the extent that $\hat{d}^R$adjusted to reproduce Γ_t^β effectively includes higher order contributions, the above-mentioned subtraction is still needed. Although a full solution of this problem would require a systematic N^4LO calculation, it is reasonable to expect that the contributions of the 3B operators, and hence the uncertainties due to them also, lie within the above-discussed overall range of higher-order effects.

These considerations lead to the estimation that the corrections due to the N^4LO or higher-order terms should be of the order of $\sim 0.1\%$. We also note that, within SNPA, the 3B contribution to Γ_t^β was calculated explicitly and found to be negligibly small compared with the leading 2B terms [31].

Figs. 2 and 3 also show the uncertainty in ξ due to the finite precision with which $\hat{d}^R$ can be fixed from Γ_t^β. In fact, the largest uncertainty in our present calculation comes from this origin, and yet it only amounts to $\sim 0.5\%$ ambiguity in ξ. Based on these observations, we consider it safe to conclude that $\sigma_{vd}^{L=0}$'s calculated here are reliable at the $\sim 1\%$ level.

Comparison of our EFT* results with those of the latest SNPA calculation by NETAL [6] has already been described in [6]. We therefore only mention here that $\sigma_{vd}^{L=0}$ in Table 1 agrees with $\sigma_{vd}^{L=0}$ of NETAL within 1% accuracy (see Table 4 in [6]). As discussed, to the chiral order we are concerned with, $\sigma_{vd}^{L\geqslant 1}$ calculated in EFT* should agree with that obtained in SNPA. Therefore σ_{vd} (including all final partial waves) in EFT* can be identified, within 1% accuracy, with σ_{vd} given in NETAL [6].

There have been attempts to directly apply EFT to nuclear systems with mass number $A \geqslant 3$ [10,32]. Here, "directly" means that the nuclear wave functions are obtained in the framework of EFT instead of using phenomenological potentials. It will be interesting to employ this "direct" EFT approach for determining $\hat{d}^R$ (or L_{1A}) from Γ_t^β and use the resulting value of $\hat{d}^R$ for recalculating σ_{vd}.

To summarize, we have carried out an EFT* calculation (up to N^3LO) to estimate $\sigma_{vd}^{L=0}$, the cross sections of the vd reactions leading to the final two-nucleon s-wave state. Our results agree, within 1% accuracy, with those of the most recent SNPA calculation reported in [6]. In addition, we have found that the calculated $\sigma_{vd}^{L=0}$ exhibits very small cut-off dependence (only $\sim 0.02\%$ variation). The corrections due to higher chiral order terms are estimated to be of the order of $\sim 0.1\%$. The prime uncertainties in the calculated $\sigma_{vd}^{L=0}$ stem from the experimental errors in Γ_t^β; this uncertainty, however, is less than $\sim 0.5\%$. We therefore conclude that, apart from the radiative corrections for which we refer to the literature, the uncertainties in the calculated $\sigma_{vd}^{L=0}$ are less than 1%.

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